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A STUDY ON INTUITIONISTIC FUZZY MODEL FOR CLASSICAL QUEUING SYSTEM

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Abstract

The performance measures of classic queueing models with intuitionistic fuzzy data are studied in this paper. This paper combines the Intuitionistic Fuzzy Sets and Queueing models to represent practical situations which are clear but the assumptions are frequently too far from reality. It is shown that this approach can use the advantages of both the intuitionistic fuzzy and queueing model approaches to make the model more realistic. Trapezoidal Intuitionistic Fuzzy Number are used to construct the membership functions and non-membership functions of the performance measures of queueing models *M/IF/1*, *IF/M/1* and *IF/IF/1* and the analytical and simulation results are discussed based on actual examples.

Keywords: (α,β)-cut, Intuitionistic Fuzzy Queuing, Zadeh's extension principle, Trapezoidal intuitionistic Fuzzy Number.

Mathematics Subject Classification: 03E72, 03F55, 03B20, 60K20.

1. Introduction

A queue is the customers waiting for service from a server and also called the waiting line. The queue is formed in several situations, for example, when demand for service exceeds the number of servers. Queueing models can be divided into two broad groups. One is descriptive model which provides the probabilities of performance measures that describe the system when patterns of arrivals and services, the number of servers, system capacity and queue discipline so on. The second group called queuing decision models (design and control models), attempts to find the parameters which optimize the models. Models should be optimized when there is uncertainty about input data. In this paper we try to find the parameters, which are uncertain and provide optimizing intuitionistic fuzzy queueing systems. Classic queuing model assumes that arrivals follow a Poisson process and service times are exponentially distributed. But the arrival rate λ in many real life situations are more possibilitic than probabilistic, and the parameters like λ and μ in M/M/1queueing model are often fuzzy and cannot be expressed in exact terms. Pardo and David de la Fuente [28] developed two fuzzy queueing models (M/M/1 and M/F/1) with priority-discipline. Chen [29] proposes parametric programming approach to construct the membership functions of the fuzzy objective function in which the cost coefficients and arrival rates are fuzzy numbers for modeling cost based queuing decision problem. Ritha and Lilly Robert [30] described a fuzzy priority queueing model and obtained the total cost of the given problem. Many methods were designed to solve problems when cost coefficients and arrival or service patterns are known and precise.

The arrival of customers does not know in advance; we can't predict at a particular point of time how many customers will arrive. Similarly, we are uncertain about the service rate. We can resolve the uncertainty by using a fuzzy set. Fuzzy queues are more realistic than the crisp lines in many practical situations. Here the arrival rate and service rate is assumed to be a fuzzy number. For example, denote the arrival as very slow, slow, medium, fast, very fast. As in crisp queueing system, the main aim of the fuzzy queue is to find fuzzy performance measures with fuzzified exponential arrival rate and service rate. However, in real-life systems, the server may experience unpredictable breakdowns. The objective is to determine the optimal number of servers, which can reduce their operational cost and reduce the waiting time of customers. In fuzzy queueing system literature, most of the researchers analyzed fuzzy queues characteristics, with the help of Zadeh’s extension principle and α-cuts approach to construct the membership functions of fuzzy queues characteristics.

Intuitionistic fuzzy set (IFS), developed by Krassimiri T. Atanassov [1] is a powerful tool to deal with vagueness compared to fuzzy set as it considers membership, non-membership and hesitancy degree of an object simultaneously. IFS constitutes an extension of Zadeh's fuzzy set, which only assigns to each item a membership degree. Kao et al. [4] constructed the membership functions of the system characteristic for fuzzy queues using parametric linear programming. Li and Lee [12] have derived analytical results for two fuzzy queueing systems based on Zadeh's extension principle [25, 26]. Negi and Lee [17] proposed the α-cut and the variable approaches to analyse fuzzy queues, which only provides crisp solutions. The membership of the performance measure of the input information can be used to represent the fuzzy system more accurately. They applied α-cuts and transformed the fuzzy bulk arrival queues to a family of crisp bulk arrival queues. Atanassov [1] discussed open problems in IFS theory. Atanassov and Kreinovich [2] implemented IF interpretation of interval data. An alternate solution for fuzzy queuing problems was given by Prade [18]. Mukeba [15] analyzed fuzzy queue characteristics using fuzzy arithmetic based on extension principle and α-cuts and intervals arithmetic. He also proposes the process of the flexible a-cuts method.

Claudilene G. da Costa [6] introduced the notion of fuzzy probabilities by representing probabilities through the Atanassov intuitionistic fuzzy numbers. Intuitionistic fuzzy number plays an important role in real life problems involving uncertainty. G.S. Mahapatra, T.K. Roy [14] analyzed the fuzzy system reliability, the reliability of each component of the system as a triangular intuitionistic fuzzy number. Sujatha [23] introduced the concept of intuitionistic Markov chain on intuitionistic possibility space. Nagoor Gani and Ashok Kumar [16] constructed the membership functions for performance measures in bulk arrival queuing systems with varying fuzzy batch sizes using fuzzy numbers. The basic idea here is to transform fuzzy queue with bulk arrival to a family of conventional crisp queue with bulk arrival using α-cut approach.

The research in queueing theory mainly focuses on any one of the following category:

1. Evaluation of various performance measures.
2. Finding solution like exact, transform, algorithmic, asymptotic, numerical, approximations
3. Nature of solution – stationery, transient etc.
4. Control and design of queues - comparison of behavior and performances under various situations, as well as queue discipline, service rules etc
5. Optimization of the objective functions in terms of performance measures, associated cost functions.

In this paper we are going to model queueing systems using intuitionistic fuzzy sets and find the various performance measures. We describe M/IF/1, *IF/M/1* and *IF/IF/1* in terms of Intuitionistic fuzzy set.

2. Preliminaries:

Basic concepts and definitions are taken from standard references papers.

Definition 2.1: Let X be a universal set. Then the fuzzy subset A of X is defined by its membership function $\mu_A: X \rightarrow [0,1]$ which assign a real number $\mu_A(x)$ in the interval [0, 1], to each element $x \in X$, where the value of $\mu_A(x)$ at x shows the grade of membership of x in A. The membership function of a fuzzy set is known as a possibility distribution.

Definition 2.2: Given a fuzzy set A in X and any real number $\alpha \in [0, 1]$, then the α-cut or α-level or cut worthy set of A, denoted by α_A is the crisp set $\alpha_A = \{x \in X: \mu_A(x) \geq \alpha\}$. The strong α-cut, denoted by α^+_A is the crisp set $\alpha^+_A = \{x \in X: \mu_A(x) > \alpha\}$. For example, let A be a fuzzy set whose membership function is given as ,

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \dfrac{c-x}{c-b} & \text{if } b \leq x \leq c \end{cases} \qquad (2.1)$$

To find the α-cut of A, we first set $\alpha \in [0,1]$ to both left and right reference functions of A. That is, $\alpha = (x - a) / (b - a)$ and $\alpha = (c - x) / (c - b)$. That is, x can be expressed in terms of α, as

$x = (b - a)\alpha + a$ and $x = c - (c-b)\alpha$ which gives α-cut of A as $\alpha A = [(b - a)\alpha + a, c - (c-b)\alpha]$. $x \in [a,b]$ is called the left reference function, which is right continuous, monotone and non-decreasing, while the $x \in [b, c]$ is called right reference function, which is left continuous, monotone and non- increasing. The above definition of a fuzzy number is known as an L-R fuzzy number.

Definition 2.3: A triangular fuzzy number TFN for a fuzzy set A can be defined as a triplet [a, b, c]. Its membership function is defined as in equation (2.1).

Definition 2.4: A trapezoidal fuzzy number TRFN for a fuzzy set A can be expressed as [a, b, c, d] and its membership function is defined as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } b \leq x \leq c \\ \dfrac{d-x}{d-c} & \text{if } c \leq x \leq d \end{cases}$$

Definition 2.5: Let X be a universal set. An Intuitionistic fuzzy set (IFS) Ã in X is an object of the following form:

$$\tilde{A} = \{(x, \mu_A(x), \nu_A(x)): x \in X\},$$

where the function $\mu_A: X \rightarrow [0.1]$ and $\nu_A: X \rightarrow [0,1]$ are the membership and non membership functions. For simplicity we denote Ã as A itself. It should satisfy the condition that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$, When $\nu_A(x) = 1 - \mu_A(x)$, for all $x \in X$, is an ordinary fuzzy set. In addition, for each IFS A in X, if

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

Then $\pi_A(x)$ is called the intuitionistic fuzzy index (hesitancy margin) of x to A. The number $\pi_A(x)$ may be treated as the indeterminacy (indefiniteness) degree of the membership of the element x to the intuitionistic fuzzy set A. In particular if $\pi_A(x) = 0$, for all $x \in X$, then the IFS, A is reduced to a fuzzy set.

Definition 2.6: (α, β)-cut of IFS A is the crisp subset

$$(\alpha,\beta)_A = \{ x \in X : \mu_A(x) \geq \alpha , \nu_A(x) \leq \beta \}$$

of X, where $\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$. For an IFS A, the α-cut of A is defined by

$$\alpha_A = \{x \in X / \mu_A(x) \geq \alpha , \nu_A(x) \geq 0 \text{ such that } \mu_A(x) + \nu_A(x) \leq 1\}.$$

For an IFS A, the strong α-cut of A is defined by

$$\alpha_A = \{x \in X / \mu_A(x) > \alpha , \nu_A(x) > 0 \text{ such that } \mu_A(x) + \nu_A(x) \leq 1\}.$$

Note that $(\alpha,\beta)_A$ defined as the crisp set of elements x which belong to A at least to the degree α and which does belong to A at most to the degree β

Definition 2.7: Intuitionistic Fuzzy Number IFN Ã is defined as follows:

i) intuitionistic fuzzy subset of the real line

ii) normal, i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$, $\nu_{\tilde{A}}(x) = 0$

iii) a convex set for the membership function $\mu_{\tilde{A}}(x)$. That is

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \text{ for all } x_1, x_2 \in R \text{ and } \lambda \in [0,1]$$

iv) a concave set for the non-membership function $\nu_{\tilde{A}}(x)$. That is

$\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq \max(\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2))$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$

Definition 2.8: **Triangular Intuitionistic Fuzzy Number** TIFN is defined as follows:

Let A be a Intuitionistic fuzzy set defined by $A = \{(x, \mu_A(x), \nu_A(x)): x \in X\}$. Let the membership function

μ_A is defined as

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{b_1 - a_1} & \text{if } a_1 \leq x \leq b_1 \\ \dfrac{c_1 - x}{c_1 - b_1} & \text{if } b_1 \leq x \leq c_1 \\ 0 & \text{otherwise} \end{cases}$$

and non membership function ν_A is given by

$$\nu_A(x) = \begin{cases} \dfrac{b_1 - x}{b_1 - a_1'} & \text{if } a_1' \leq x \leq b_1 \\ \dfrac{x - b_1}{c_1' - b_1} & \text{if } b_1 \leq x \leq c_1' \\ 1 & \text{otherwise} \end{cases}$$

where a’ < a < b < c < c’ and $\mu_A(x), \nu_A(x) \leq 0.5$

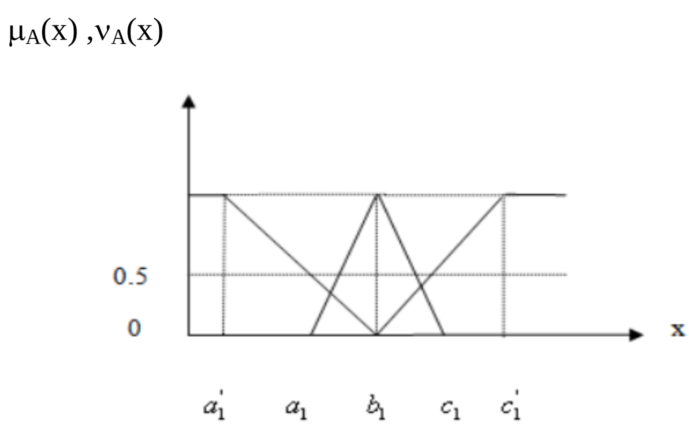

Figure 2.1 Membership and Non membership values of Triangular Intuitionistic Fuzzy Number.

If we put a’ =a, and c’ = c and $\nu_A(x) = 1 - \mu_A(x)$ for all $x \in X$, then TIFN becomes Triangular Fuzzy Number TFN.

Definition-2.9. **Trapezoidal Intuitionistic Fuzzy Number (TrIFN)**: $\tilde{A}$ is a subset of IFS in R with membership function and non-membership function as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ 1 & \text{for } a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{for } a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1'} & \text{for } a_1' \le x \le a_2 \\ 0 & \text{for } a_2 \le x \le a_3 \\ \dfrac{x - a_3}{a_4' - a_3} & \text{for } a_3 \le x \le a_4' \\ 1 & \text{otherwise} \end{cases}$$

where $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$ and is denoted by $\tilde{A}_{TrIFN} = (a_1, a_2, a_3, a_4;\ a_1', a_2, a_3, a_4')$

$\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)$

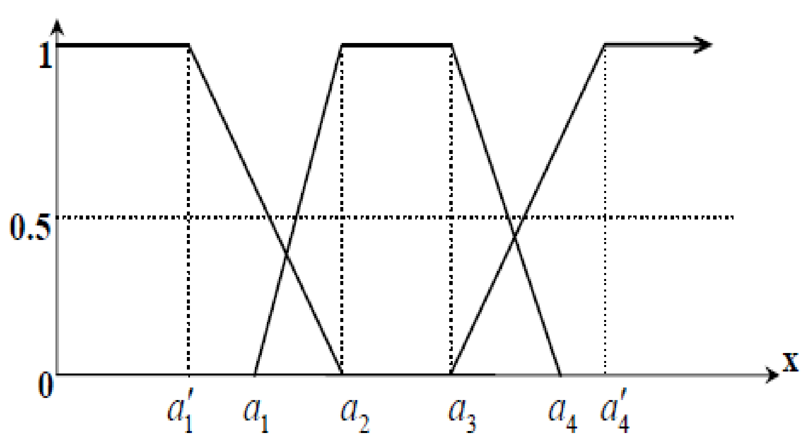

Figure 2.2 Membership and Non membership values of Trapezoidal Intuitionistic Fuzzy Number.

Definition2.10. Let Γ be the universe of discourse and $\wp$ be the power set of Γ. An intuitionistic possibility measure is the pair σ = (μ, ν) where $\mu: \wp \rightarrow [0,1]$ and $\nu : \wp \rightarrow [0,1]$ such that $0 \leq \mu(x) + \nu(x) \leq 1$ with the following properties:

1. $\mu(\phi) = 0; \nu(\phi) = 1$.
2. $\mu(\Gamma) = 1$; $\nu(\Gamma) = 0$.
3. $\mu(\cup A_\alpha) = \max_\alpha\{\mu(A_\alpha)\}$; $\nu(\cup A_\alpha) = \min_\alpha\{\nu(A_\alpha)\}$.Then(Γ, σ) is called intuitionistic possibility space.

Definition 2.11. A intuitionistic random process{X(n): n ∈ N}is said to an intuitionistic Markov chain if it satisfies the Markov property

$$\sigma(X_{n+1} = j \mid Xn = i, X_{n-1} = k, \ldots, X_0 = m) = \sigma(Xn+1 = j \mid Xn = i)$$

where i, j, k, ...constitute the state space S of the process.

Here $\tilde{P}ij = \sigma(X_{n+1} = j \mid X_n = i)$ are called intuitionistic transition possibilities of moving from state i to state j in one step. So $\tilde{P}ij = (\tilde{\mu}_{Pij}, \tilde{\nu}_{Pij})$,where $\tilde{\mu}_{Pij}$ denotes the membership of transition from state i to state j and $\tilde{\nu}_{Pij}$ denotes the non-membership of transition from state i to state j. The matrix $\tilde{P} = (\tilde{P}ij)$ is called the intuitionistic transition possibility matrix.

3. Problem Formulation: Consider a general queueing system with one server. Let X and Y denote crisp universal sets of interarrival time and service time respectively. The interarrival time Ã and Service time Š are IFS where it is represented as

$$\tilde{A} = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}, \check{S} = \{(x, \mu_S(y), \nu_S(y)) : y \in Y\}, \quad (3.1)$$

where $\mu_A(x)$ and $\mu_S(y)$ are corresponding membership functions and $\nu_A(x)$ and $\nu_S(y)$ are non-membership functions. The (α, β)-cut of Ã and Š are defined as

$$(\alpha,\beta)_{\tilde{A}} = \{ x \in X : \mu_A(x) \geq \alpha , \nu_A(x) \leq \beta \} \quad (3.2)$$

$$(\alpha,\beta)_{\check{S}} = \{ y \in Y : \mu_S(y) \geq \alpha , \nu_S(y) \leq \beta \} \quad (3.3)$$

where the $(\alpha,\beta)_{\tilde{A}}$ and $(\alpha,\beta)_{\check{S}}$ are the crisp sub sets of X and Y respectively. α-cut of Ã and Š are defined as

$$\alpha_{\tilde{A}} = \{ x \in X : \mu_A(x) \geq \alpha , \nu_A(x) \geq 0 \text{ such that } \mu_A(x) + \nu_A(x) \leq 1 \} \quad (3.4)$$

$$\alpha_{\check{S}} = \{ y \in Y : \mu_{\check{S}}(y) \geq \alpha , \nu_{\check{S}}(y) \geq 0 \text{ such that } \mu_{\check{S}}(y) + \nu_{\check{S}}(y) \leq 1 \} \quad (3.5)$$

Using (α, β) -cuts, the interarrival time and service time can be represented by different levels of confidence intervals. Consequently, a intuitionistic fuzzy queue can be reduced to a family of crisp queues with different (α,β)-cuts$\{(\alpha,\beta)_{\tilde{A}} : 0 < \alpha \le 1, 0 \le \beta < 1\}$ and $\{(\alpha,\beta)_{\check{S}} : 0 < \alpha \le 1, 0 \le \beta < 1\}$.

Suppose Ã and Š follows TIFN. Denote confidence intervals by $[l_{(\alpha,\beta)\tilde{A}}\ , u_{(\alpha,\beta)\tilde{A}}]$ of Ã and $[u_{(\alpha,\beta)\check{S}}, u_{(\alpha,\beta)\check{S}}]$ of Š, We can use the idea of Chiang Kao et.al [1] for deriving different performance measures analytically for different possibility level (α,β). In this paper we discuss models such as: i) interarrival time is Intuitionistic fuzzy and the service time follows a crisp probability distribution (or) service time is Intuionistic fuzzy and interarrival time follows poisson distribution ii) both interarrival time and service time is intuitionistic fuzzy.

3.1 Interarrival Time is Intuitionistic Fuzzy and The Service Time Follows a Crisp Probability Distribution.

We assume that the interarrival time is a IFN and the service time follows a crisp probability distribution. The case when service time is IFN and interarrival time is crisp is similar.

The (α,β)-cut of Ã defines an interval, which indicates the constant interarrival time lies at possibility (α,β) and can be expressed as

$$[l_{(\alpha,\beta)\tilde{A}}\ , u_{(\alpha,\beta)\tilde{A}}] = [\min_{a \in X}\{ a:\ \mu\tilde{A}(a) \ge \alpha, \nu_{\tilde{A}}(a) \le \beta\}, \max_{a \in X}\{ a:\ \mu\tilde{A}(a) \ge \alpha, \nu_{\tilde{A}}(a) \le \beta\}.$$

By the convex and concave nature of membership and non-membership function of IFN, the bounds of the interval can be obtained as

$$l_{(\alpha,\beta)\tilde{A}} = \min [\mu^{-1}_{\tilde{A}}(\alpha) \wedge \nu^{-1}_{\tilde{A}}(\beta)] \text{ and } u_{(\alpha,\beta)\tilde{A}} = \max[\mu^{-1}_{\tilde{A}}(\alpha) \wedge \nu^{-1}_{\tilde{A}}(\beta)]$$

Let p(a,b) denote the system performance measure of interest. Let us assume that the performance measure of interest is the expected number of customers in the queue Lq. By Little's formula we can find other performance measures. When Ã is intuitionstic fuzzy, p(Ã, b) is a IFN. According to extension principle, the membership function of p(Ã, b) is:

$$(\mu_{p(\tilde{A}, b)})(z) = \{\sup_{a\in X} [\mu_{\tilde{A}}(a)]: z = p(a, b) \} \qquad (3.6)$$

The non-membership function of p(Ã, b) is:

$$(\nu_{p(\tilde{A}, b)})(z) = \{\inf_{a\in X} [\nu_{\tilde{A}}(a)] : z = p(a, b)\} \qquad (3.7)$$

which is parameterixed by (α,β). Lets find the bounds of the intervals of p(Ã, b) at (α,β) as

$$l_{p(\alpha,\beta)} = \min_{a\in X}\{p(a, b)\} \text{ such that } l_{A(\alpha,\beta)} \leq a \leq u_{A(\alpha,\beta)} \qquad (3.8)$$

$$u_{p(\alpha,\beta)} = \max_{a\in X}\{p(a, b)\} \text{ such that } l_{A(\alpha,\beta)} \leq a \leq u_{A(\alpha,\beta)} \qquad (3.9)$$

This pair gives the idea of how the optimal solutions can be found when the bounds $l_{(\alpha,\beta)\tilde{A}}$ and $u_{(\alpha,\beta)\tilde{A}}$ varies over $\alpha\in (0, 1]$ and $\beta\in[0,1)$ such that $\alpha + \beta \leq 1$. If both $l_{p(\alpha,\beta)}$ and $u_{p(\alpha,\beta)}$ are invertible with respect to (α,β), then a left shape function $L(z) = l^{-1}{}_{p(\alpha,\beta)}$ and a right shape function $R(z)= u^{-1}{}_{p(\alpha,\beta)}$ can be obtained, from which the membership function $\mu_{p(\tilde{A}, b)}$ is constructed as follows:

$$(\mu_{p(\tilde{A}, b)})(z) = \begin{cases} L(z) & z_1 \leq z \leq z_2 \\ 1 & z_2 \leq z \leq z_3 \\ R(z) & z_3 \leq z \leq z_4 \end{cases}$$

where $z_1 \leq z_2 \leq z_3 \leq z_4$ and $L(z_1) =0$, $R(z_4) = 0$. Similarly the non-membership function $\nu_{p(\tilde{A}, b)}$ is constructed as follows:

$$(\nu_{p(\tilde{A}, b)})(z) = \begin{cases} L(z)^c & z_1' \leq z \leq z_2 \\ 0 & z_2 \leq z \leq z_3 \\ R(z)^c & z_3 \leq z \leq z_4' \end{cases}$$

Pictorial Representation is given by Figure 3.1

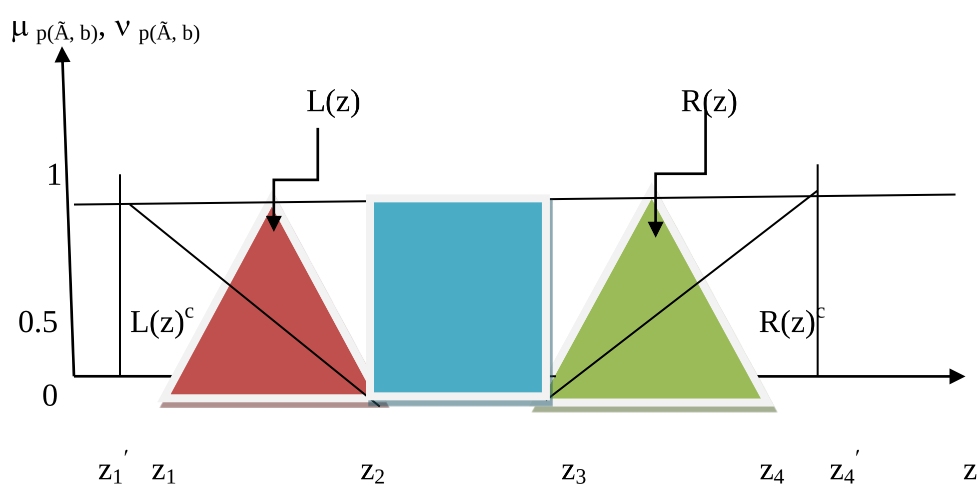

Figure 3.1 Membership and Non-membership values of performance measure of L_q

When arrival time follows a crisp probability distribution and service time is intuitionistic fuzzy, then p(a, Š) is a IFN. According to extension principle, the membership function of p(a, Š) is:

$$(\mu_{p(a,\,Š)})(z) = \{\sup_{b\in Y} [\mu_{Š}(b)] : z = p(a, b)\} \qquad (3.6)'$$

The non-membership function of p(a, Š) is:

$$(\nu_{p(a,}\, p(a,_{Š)})(z) = \{\inf_{b\in Y} [\nu_{\tilde{A}}(b)] : z = p(a, b)\} \qquad (3.7)'$$

which is parameterixed by (α,β). The bounds of the intervals of p(a, Š) at (α,β) as

$$l_{p(\alpha,\beta)} = \min_{b\in Y}\{p(a, b)\} \text{ such that } l_{Š(\alpha,\beta)} \le b \le u_{Š(\alpha,\beta)} \qquad (3.8)'$$

$$u_{p(\alpha,\beta)} = \max_{b\in Y}\{p(a, b)\} \text{ such that } l_{Š(\alpha,\beta)} \le b \le u_{Š(\alpha,\beta)} \qquad (3.9)'$$

and explanation about p(a, Š) is similar to p(Ã, b).

3.2. Both Interarrival Time and Service Time is Intuitionistic Fuzzy

Let us denote the performance measure by p(Ã,Š) and the membership function and non-membership function p(Ã,Š) can be defined as:

$$(\mu_{p(\tilde{A},Š)})(z) = Sup\{\min_{a\in X, a'\in Y}(\mu\tilde{A}(a), \muŠ(a')) : z = p(a, a')\}$$

and

$$(\nu_{p(\tilde{A},Š)})(z) = inf\{\max_{a\in X, a'\in Y}(\mu\tilde{A}(a), \muŠ(a')) : z = p(a, a')\}$$

We can find the lower and upper bounds of the (α,β) cuts of $\mu_{p(\tilde{A},Š)}$ as follows:

$$l_{p(\alpha,\beta)} = \min p(a, a') \text{ such that } l_{(\alpha,\beta)\tilde{A}} \le a \le u_{(\alpha,\beta)\tilde{A}},\ l_{(\alpha,\beta)\tilde{A}} \le a \le u_{(\alpha,\beta)\tilde{A}},$$

$$u_{p(\alpha,\beta)} = \max p(a, a') \text{ such that } l_{(\alpha,\beta)\tilde{A}} \le a \le u_{(\alpha,\beta)\tilde{A}},\ l_{(\alpha,\beta)Š} \le a' \le u_{(\alpha,\beta)Š}$$

provided a∈Ã(α,β) and a′ ∈ Š(α,β). From $[l_{p(\alpha,\beta)}, u_{p(\alpha,\beta)}]$ the membership function $\mu_{p(\tilde{A},Š)}$ and non-membership can be constructed as in Section 3.1

4. Intuitionistic Fuzzy Queues (IFQ)

In this paper we investigate four types of IFQ. That is *M/IF/1,IF/M/1* and *IF/IF/1*. All three queues adopt a first-come-first-served discipline and consider an infinite source population. The first system has an exponentially distributed interarrival time with mean π and a intuitionistic fuzzy service time Š. The second system, has a intuitionistic fuzzy interarrival time Ã and an exponentially distributed service time with mean σ. These two systems have only one intuitionistic fuzzy variable. For the third queue, both the interarrival time Ã and the service time Š are intuitionistic fuzzy. In the fourth queue, both the interarrival time and the service time follow exponential distributions with rates λ and μ respectively, which are fuzzy variables rather than crisp values. These four queues are representative in IFQ. Therefore, to gain some insight into IFQ, it suffices to study these three systems.

Let us take a queuing system with one server and with Poisson arrival. The Poisson case is a convenient mathematical model of many real life queuing systems and is described by a single parameter - the average arrival rate. The mean of the arrival rate is π and the service discipline is first come-first served. The service times are fixed (deterministic). Service times are deterministic time D (serving at rate $\mu = 1/D$).That is, consider (M/D/1): (∞/FCFS). Suppose that Y_1, Y_2, . . . are independent random variables, that are all exponentially distributed with mean $1/\pi$, i.e., $P(Y_i > t) = e^{-\pi t}$, $t \geq 0$. Then by D.Gross and C.M.Harris [8], we get

$$P(Y_1 + \cdots + Y_{k+1} \leq t) = 1 - \sum_{j=0}^{k} \frac{(\pi t)^j}{j!} (e^{-\pi t}) \quad t \geq 0,\ k = 0,1,\ldots,$$

$$P(Y_1 + \cdots + Y_{k+1} > t) = \sum_{j=0}^{k} \frac{(\pi t)^j}{j!} (e^{-\pi t}) \quad t \geq 0,\ k = 0,1,\ldots,$$

That is, Y_1, Y_2, . . . are the intervals between successive occurrences of events, like arrivals of customers at some service facility. Let N(t) be the number of arrivals in [0, t]. Then

$$P(N(t) > t) = \sum_{j=0}^{k} \frac{(\pi t)^j}{j!} (e^{-\pi t}) \quad t \geq 0,\ k = 0,1,\ldots,$$

The process $\{N(t), t \geq 0\}$ is called Poisson Process with rate λ. Let $p = (p_0, p_1, \ldots)$ denote the stationary distribution of P, and define the probability generating function p(z) of p by

$$p(z) = \sum_{n=0}^{\infty} p_n z^n$$

The explicit form of p_n is given by the Taylor expansion of p(z),

$$p_0 = 1 - \pi t \text{ where } t < 1/\pi,\ t, \lambda \in R^+ \qquad (4.1)$$

$$p_1 = (1 - \pi t)(e^{\pi t} - 1) \qquad (4.2)$$

$$p_n = (1 - \pi t) \sum_{k=1}^{n} (-1)^{n-k} e^{k\pi t} \left\{ \frac{(k\pi t)^{n-k}}{(n-k)!} + \frac{(k\pi t)^{n-k-1}}{(n-k-1)!} \right\} \qquad (4.3)$$

for $n \geq 2$

where p_n denotes the steady-state probability of n customers in the system at a departure point.

Then we get

$$L_q = \frac{\pi t(2 - \pi t)}{2(1 - \pi t)}, \quad W_q = \frac{L}{\pi} = \frac{t(2 - \pi t)}{2(1 - \pi t)} \qquad (4.4)$$

The Pollaczek-Khinchin formula

$$p(z) = \frac{(1 - \pi t)(z - 1)\exp(\pi t(z - 1))}{z - \exp(\pi t(z - 1))} \qquad (4.5)$$

We can solve the performance measures by two basic approaches: i) analytic methods ii) simulation

4.1. (M/IF/1): (∞/FCFS) Consider the case when arrival follows a Poisson distribution and the service time Š is uncertain and is represented by a possibility distribution $p(t) = \mu_{Š}(t)$ which restricts its more or less possible values. Let $\tilde{P}_i$ be the probability of i arrivals during service time Š. For the Poisson arrival process with parameter π, the intuitionistic fuzzy probability function can be defined by

$$\mu_P(x) = \sup_{t \in R} \left\{ \mu_{Š}(t) : x = \frac{e^{-\pi t}(\pi t)^i}{i!} \right\} \text{ and } \nu_P(x) = \inf_{t \in R} \left\{ \nu_{Š}(t) : x = \frac{e^{-\pi t}(\pi t)^i}{i!} \right\}$$

The one-step transition matrix for a intuitionistic fuzzy Markov chain can be constructed as in Definition 2.12 . Note that a transition from state zero to state j or a transition from state i to state j both require the arrival of j customers during the service interval. Moreover, moving from state i to state j where $j \geq i - 1 > 0$ requires that (j - i + 1) customers arrived during a service interval, the extra arrival being necessary to account for the customer known to be departing at the transition point. Let the intuitionistic transition possibility matrix be $\tilde{P} = (\tilde{P}_{ij})$. We can then write for all $j \geq i - 1, i \geq 1$ as

$$\tilde{P} = (\tilde{P}_{ij}) = \begin{pmatrix} \tilde{P}_0 & \tilde{P}_1 & \tilde{P}_2 & \tilde{P}_3 & \ldots \\ \tilde{P}_0 & \tilde{P}_1 & \tilde{P}_2 & \tilde{P}_3 & \ldots \\ 0 & \tilde{P}_0 & \tilde{P}_1 & \tilde{P}_2 & \ldots \\ 0 & 0 & \tilde{P}_0 & \tilde{P}_1 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \end{pmatrix}$$

where $\tilde{P}_{ij}$ are defined by $\mu_{\tilde{P}ij}(x_{ij}) = \sup_{t\in R, \, t < 1/\pi} \left\{ \mu_{\check{s}}(t) : x = \frac{e^{-\pi t} (\pi t)^{j-i+1}}{(j-i+1)!} \right\}$ and

$$\nu_{\tilde{P}ij}(x_{ij}) = \inf_{t\in R, \, t < 1/\pi} \left\{ \nu_{\check{s}}(t) : x = \frac{e^{-\pi t} (\pi t)^{j-i+1}}{(j-i+1)!} \right\}$$

By means of the extension principle the equations (4.1)-(4.3) for the intuitionistic fuzzy case can be defined by their membership functions and non-membership functions for the steady-state solutions are: For $t, \pi \in R^+$ $t < 1/\pi$,

$$\mu_{\tilde{p}0}(x) = \sup \left\{ \mu_{\check{s}}(t) : x = 1-\pi t \right\} \text{ and } \nu_{\tilde{p}0}(x) = \inf \left\{ \nu_{\check{s}}(t) : x = 1-\pi t \right\}$$

$$\mu_{\tilde{p}1}(x) = \sup \left\{ \mu_{\check{s}}(t) : x = (1-\pi t)(e^{\pi t} - 1) \right\} \text{ and}$$

$$\nu_{\tilde{p}1}(x) = \inf \left\{ \nu_{\check{s}}(t) : x = (1-\pi t)(e^{\pi t} - 1) \right\}$$

$$\mu_{\tilde{p}n}(x) = \sup \left\{ \mu_{\check{s}}(t) : x = (1-\pi t) \sum_{k=1}^{n} (-1)^{n-k} e^{k\lambda t} \left[\frac{(k\pi t)^{n-k}}{(n-k)!} + \frac{(k\pi t)^{n-k-1}}{(n-k-1)!} \right] \right\}$$

$$\nu_{\tilde{p}n}(x) = \inf \left\{ \nu_{\check{s}}(t) : x = (1-\pi t) \sum_{k=1}^{n} (-1)^{n-k} e^{k\lambda t} \left[\frac{(k\pi t)^{n-k}}{(n-k)!} + \frac{(k\pi t)^{n-k-1}}{(n-k-1)!} \right] \right\}$$

The system performance measures are given by:

$$\mu_{\tilde{L}}(x) = \sup \left\{ \mu_{\check{s}}(t) : x = \frac{\pi t(2 - \pi t)}{2(1-\pi t)} \right\} \text{ and } \nu_{\tilde{L}}(x) = \inf \left\{ \nu_{\check{s}}(t) : x = \frac{\pi t(2 - \pi t)}{2(1-\pi t)} \right\} \quad (4.6)$$

$$\mu_{\tilde{\omega}}(x) = \sup\left\{\mu_{\check{s}}(t):\ x = \frac{t(2-\pi t)}{2(1-\pi t)}\right\} \quad \text{and} \quad \nu_{\tilde{\omega}}(x) = \inf\left\{\nu_{\check{s}}(t):\ x = \frac{t(2-\pi t)}{2(1-\pi t)}\right\} \qquad (4.7)$$

for t, $\pi \in R^+$ and $t < 1/\pi$. Let us take p(a, Š) = $\tilde{L}_q$. The notations in equations (3.6)′, (3.7)′ substituted in equations (4.6), (4.7) leads to

$$\mu_{\tilde{L}q}(z) = \sup_{b\in Y,\ b/\pi < 1}\left\{\mu_{\check{S}}(b):\ z = \frac{[b(2-b/\pi)]}{[2\pi(1-b/\pi)]}\right\} \text{ and } \nu_{\tilde{L}q}(z) = \inf_{b\in Y,\ b<\lambda}\left\{\nu_{\check{S}}(b):\ z = \frac{[b(2-b/\pi)]}{[2\pi(1-b/\pi)]}\right\} \quad (4.6)'$$

$$\mu_{\tilde{\omega}}(z) = \sup_{b\in Y,\ b<\pi}\left\{\mu_{\check{S}}(b):\ z = \frac{b(2-b/\pi)}{[2(1-b/\pi)}\right\} \quad \text{and} \quad \nu_{\tilde{\omega}}(x) = \inf_{b\in Y,\ b<\pi}\left\{\nu_{\check{S}}(t):\ z = \frac{b(2-b/\pi)}{2(1-b/\pi)}\right\} \qquad (4.7)'$$

By (4.5) we find the expected number of customers in the system, $\tilde{L}_S$ as:

$$\mu_{\tilde{L}S}(z) = \sup_{b\in Y,\ b<\pi}\left\{\mu_{\check{S}}(b):\ z = b/\pi + b^2/['2\pi^2(1-b/\pi)]\right\} \qquad (4.8)$$

$$\nu_{\tilde{L}S}(z) = \inf_{b\in Y,\ b<\pi}\left\{\nu_{\check{S}}(b):\ z = b/\pi + b^2/['2\pi^2(1-b/\pi)]\right\} \qquad (4.9)$$

Example 4.1: Consider a single server queuing system with Poisson arrival and intuitionistic fuzzy service time. The interarrival time follows an exponential distribution with mean π = 15 minutes and the intuitionistic fuzzy service time represented as trapezoidal intuitionistic fuzzy number Š ={(4,6,8,10),(2,5,9,12)} as in Definition 2.10, where

$$\mu_{\check{S}}(b) = \begin{cases} (b-4)/2 & 4 < b < 6 \\ 1 & 6 < b < 8 \\ (10-b)/2 & 8 < b < 10 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_{\check{S}}(b) = \begin{cases} (b-5)/-3 & 2 < b < 5 \\ 0 & 5 < b < 9 \\ (b-9)/3 & 9 < b < 12 \\ 1 & \text{otherwise} \end{cases}$$

The (α,β) cut of Š is [min $(\mu_{\check{S}}^{-1}(\alpha) \wedge \nu_{\check{S}}^{-1}(\beta))$, max$(\mu_{\check{S}}^{-1}(\alpha) \wedge \nu_{\check{S}}^{-1}(\beta))$] = [$(2\alpha+4) \wedge (3\beta + 2)$, $(10-2\alpha) \wedge (12-3\beta)$]. To find membership and non-membership for p(a, Š) (That is expected system length) = $(\mu_L(b), \nu_L(b))$ based on equations (3.8)′ and (3.9)′ and (4.8) and (4.9) we get

$l_{L(\alpha,\beta)}$ = min [(b/15) + b^2/[450 (1 - b/15)] with $2\alpha + 4 \le b \le 10-2\alpha$, $3\beta + 2 \le b \le 12-3\beta$, $\alpha, \beta \in [0,1]$ (4.10)

$u_{L(\alpha,\beta)}$ = max [(b/15) + b^2/[450 (1 - b/15)] with $2\alpha+4 \le b \le 10-2\alpha$, $3\beta + 2 \le b \le 12-3\beta$, $\alpha, \beta \in [0,1]$ (4.11)

We can find lower limit and upper limit for different α, β values: 0, 0.1, 0. 2, . . . , 1.0. For each (α, β)-cut set we determine the queue characteristics. For example, at α = 0.5, β = 0.5, the membership value of b ranges between 5 and 9, non-membership of b ranges between 3.5 and 10.5.

Table 4.1: (α,β) cuts for membership and non-membership values of L_S.

α	$\mu_{L(S)}$		β	$\nu_{L(S)}$	
	$l\mu_{L(\alpha,\beta)}$	$u\mu_{L(\alpha,\beta)}$		$l\nu_{L(\alpha,\beta)}$	$u\nu_{L(\alpha,\beta)}$
0	0.31515	1	0	0.41667	1
0.1	0.33444	1	0.1	0.38482	1
0.2	0.35421	1	0.2	0.35421	1
0.3	0.37449	1	0.3	0.32474	1
0.4	0.39529	1	0.4	0.29631	1
0.5	0.41667	1	0.5	0.26884	1
0.6	0.43864	1	0.6	0.24226	1
0.7	0.46125	0.95854	0.7	0.21650	1
0.8	0.48454	0.91636	0.8	0.19151	1
0.9	0.50855	0.87627	0.9	0.16722	1

For example, at α=0.5 possibility, the expected number of customers waiting in the systems is 15.75 ≈ 16 and for β = 0.5 non-possibility, the expected number of customers waiting in the systems is approximately 6 according to (4.7)′. Obviously, the values of L_S, W_S, and L_q, Wq as a function of (α,β) depends on the shape of the TrIFN. It should be noted that the results obtained as above corresponds to intuitionistic fuzzy sets, not crisp numbers. For example, if the designer wishes a possibility level of $\alpha = 0.8$, $\beta = 0.1$, in his design than the system characteristic is known.

4.2. (IF/M/1: ∞/FCFS): In (D/M/1: ∞/FCFS) queue the arrivals occur deterministically at fixed times γ apart. Service times are exponentially distributed (with parameter σ). A single server serves customers one at a time from the front of the queue. When the service is complete the customer leaves the queue and the number of customers in the system reduces by one.

When $\sigma\gamma > 1$, the queue has stationary distribution given by

$$p_i = \begin{cases} 0 \text{ when } i = 0 \\ (1-\delta)\delta^{i-1} \text{ when } i > 0 \end{cases}$$

where δ is the root of the equation $\delta = e^{-\sigma\gamma(1-\delta)}$ with smallest absolute value. The performance measure is given by

$$W_q = (1/\sigma)\,(\delta/(1-\delta)),\ L_S = \sigma/[a(1-\delta)],\ W_S = \sigma/(1-\delta),\ \delta \in (0,1) \qquad (4.10)$$

The I*F/M/I* queue to generalization of D/M/1 queue. At each (α,β)-level the intuitionistic fuzzy interarrival time is a crisp value lying in the range of $[\, l_{\tilde{A}(\alpha,\beta)}, u_{\tilde{A}(\alpha,\beta)}]$.

The performance measure for the IF/M/1 can be obtained as in Section 4.1 by substituting (3.6), (3.7) in (4.10).

Example 4.2: Consider an I*F/M/1* queue, where the interarrival time is a intuitionistic trapezoidal fuzzy number $\tilde{A}_{TrIFN}$ = {(4,5,6,7),(3,5,6,8)}and the service time is exponentially distributed with mean $\sigma = 2$. Then $[l_{\tilde{A}(\alpha,\beta)}, u_{\tilde{A}(\alpha,\beta)}] = [\alpha + 4, 7-\alpha] \wedge [2\beta+3, 8-2\beta]$. Then

$$l_{LS(\alpha,\beta)} = \text{min. } [2/[a(1-\delta)]\,] \text{ for } a \in [\alpha + 4, 7-\alpha] \wedge [2\beta+3, 8-2\beta],\ \alpha, \beta \in [0,1]$$

$$u_{LS(\alpha,\beta)} = \text{max } [2/[a(1-\delta)]\,] \text{ for } a \in [\alpha + 4, 7-\alpha] \wedge [2\beta+3, 8-2\beta],\ \alpha, \beta \in [0,1]$$

$$l_{wS(\alpha,\beta)} = \text{min.}[2/[1-\delta]\,] \text{ for } a \in [\alpha + 4, 7-\alpha] \wedge [2\beta+3, 8-2\beta],\ \alpha, \beta \in [0,1]$$

$u_{WS(\alpha,\beta)}$ = max [2/(1 - δ]] for a ∈ [α + 4, 7- α] ∧ [2β+3, 8-2β], α, β ∈[0,1]

for $\delta = e^{-2\gamma(1-\delta)}$ and γ = 1 for membership of Ã.

Let $f(\delta) = e^{-2\gamma(1-\delta)}$ and g(δ) = δ. As *a* increases from its lower bound α+ 4 to its upper bound 7- α, the intercept of $f(\delta) = e^{-2\gamma(1-\delta)}$ drops from $e^{-2(\alpha+4)}$ to $e^{-2(7-\alpha)}$, and the intersection of g(δ) = δ and $f(\delta) = e^{-2\gamma(1-\delta)}$ decreases from lower bound of δ to upper bound of δ. The objective function 2/[a(1 - δ)] attains its minimum when *a* reaches its upper bound and its maximum when *a* reaches its lower bound. That is, when δ∈(0,1), and if α, β ∈ [0, 1] , the membership value and non-membership value of lower bound and upper bound of expected length of the system L_S is given in Table 4.1 & Table 4.2. In particular for α = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8,0.9, 1.0, and δ = 0.1, 0.2 the result is given.

Table 4.2 The lower bound and upper bound of membership values for different values of α, δ.

δ	α	$l\mu_L$	$u\mu_L$	δ	α	$l\mu_L$	$u\mu_L$	δ	α	$l\mu_L$	$u\mu_L$
0.1	0	0.31746	0.555556	0.2	0	0.357143	0.625	0.9	0	2.857143	5
	0.1	0.32206	0.542005		0.1	0.362318	0.609756		0.1	2.898	4.8780
	0.2	0.326797	0.529101		0.2	0.367647	0.595238		0.2	2.941176	4.761905
	0.3	0.331675	0.516796		0.3	0.373134	0.581395		0.3	2.985075	4.651163
	0.4	0.3367	0.505051		0.4	0.378788	0.568182		0.4	3.030303	4.545455
	0.5	0.34188	0.493827		0.5	0.384615	0.555556		0.5	3.076923	4.444444
	0.6	0.347222	0.483092		0.6	0.390625	0.543478		0.6	3.125	4.347826
	0.7	0.352734	0.472813		0.7	0.396825	0.531915		0.7	3.174603	4.255319
	0.8	0.358423	0.462963		0.8	0.403226	0.520833		0.8	3.225806	4.166667
	0.9	0.364299	0.453515		0.9	0.409836	0.510204		0.9	3.278689	4.081633
	1.0	0.37037	0.444444		1.0	0.416667	0.5		1.0	3.333333	4

Table 4.3 The lower bound and upper bound of non-membership values for different values of β, δ.

δ	β	$l\mu_L$	$u\mu_L$	δ	β	$l\mu_L$	$u\mu_L$	δ	β	$l\mu_L$	$u\mu_L$
0.1	0	0.277778	0.740741	0.2	0	0.3125	0.833333	0.9	0	2.5	6.666667
	0.1	0.284900	0.694444		0.1	0.320512	0.78125		0.1	2.564102	6.25
	0.2	0.292398	0.653595		0.2	0.328947	0.735294		0.2	2.631579	5.882353
	0.3	0.3003	0.617284		0.3	0.337838	0.694444		0.3	2.702703	5.555556
	0.4	0.308642	0.584795		0.4	0.347222	0.657895		0.4	2.777778	5.263158
	0.5	0.31746	0.555556		0.5	0.357143	0.625		0.5	2.857143	5
	0.6	0.326797	0.529101		0.6	0.367647	0.595238		0.6	2.941176	4.761905
	0.7	0.3367	0.505051		0.7	0.378788	0.568182		0.7	3.030303	4.545455
	0.8	0.347222	0.483092		0.8	0.390625	0.543478		0.8	3.125	4.347826
	0.9	0.358423	0.462963		0.9	0.403226	0.520833		0.9	3.225806	4.166667
	1.0	0.37037	0.444444		1.0	0.416667	0.5		1.0	3.333333	4

IF /IF/1 queuing system: This queue is extension of D/D/1 queue. Here deterministic arrivals: constant interarrival time (*a*), and deterministic service: each customer has the same service demand (*b*). In stationary distribution, we get the number of customers in the queue

$$L_q = \begin{cases} b/a & \text{if } b \le a, \\ \infty & \text{otherwise.} \end{cases}$$

$$Prob(n=0) = (a-b)/a \; ; \;\; Prob(n=1) = b/a$$

Waiting time in the queue $W_q = 0$; Waiting time in the system $W_S = 0$; $U = b/a$ (utilization).

Based on the (α,β)-cut approach, when the IFN Ã and Š are at possibility levels α and α′, non possibility level β, β′ respectively, they become crisp lying in the range of $[l_{(\alpha,\beta)\tilde{A}}, u_{(\alpha,\beta)\tilde{A}}]$,

$[l_{(\alpha',\beta')\check{S}}, u_{(\alpha',\beta')\check{S}}]$, section 3.2 can be applied to construct the membership function and non membership for L_q.

Example 4.3. Consider a single server queue with intuitionistic fuzzy arrival and intuitionistic fuzzy service represented by the following trapezoidal fuzzy numbers $\tilde{A}_{TrIFN}$ = [(4, 5, 6, 8), (3,5,6,8)] and $\check{S}$ = [(4, 4.5, 5, 5.5), (3, 4.5, 5, 6)]. Then $[l_{\tilde{A}(\alpha,\beta)}, u_{\tilde{A}(\alpha,\beta)}] = [\alpha + 4, 8- 2\alpha] \wedge [2\beta+3, 8-2\beta]$ and $[l_{(\alpha',\beta')\check{S}}, u_{(\alpha',\beta')\check{S}}] = [4 + 0.5\alpha', 5.5 - 0.5\alpha'] \wedge [1.5\beta'+3, 6-\beta']$. Then

$$l_{L(\alpha,\beta)} = \min[b/a] \quad \text{such that} \quad \alpha + 4 \le a \le 8- 2\alpha \text{ and } 2\beta+3 \le a \le 8-2\beta,$$
$$4 + 0.5\alpha' \le b \le 5.5 - 0.5\alpha' \text{ and } 1.5\beta'+3 \le b \le 6- \beta'.$$

$$u_{L(\alpha,\beta)} = \max[b/a] \quad \text{such that} \quad \alpha + 4 \le a \le 8- 2\alpha \text{ and } 2\beta+3 \le a \le 8-2\beta,$$
$$4 + 0.5\alpha' \le b \le 5.5 - 0.5\alpha' \text{ and } 1.5\beta'+3 \le b \le 6- \beta'.$$

To minimize *b/a,* the smallest value of *b* and the largest value of *a*, and to maximize *b/a* the largest value of *b* and the smallest value of *a* are desired; Therefore the membership and non membership for the lower bound and upper bound of L_S is given by:

$$l\mu_{L(\alpha,\beta)} = [4 + 0.5\alpha']/[8- 2\alpha] \text{ and } l\nu_{L(\alpha,\beta)} = [1.5\beta'+3]/[8-2\beta]$$

and

$$u\mu_{L(\alpha,\beta)} = [5.5 - 0.5\alpha']/[\alpha + 4] \text{ and } u\nu_{L(\alpha,\beta)} = [6-\beta']/[2\beta+3]$$

Let $\alpha = \alpha'$, $\beta = \beta'$. From the Figure 4.1, it is observed that $\alpha = 0.54$, then the largest service time is greater than the smallest interarrival time. In this case the average number of customers in the queue is ∞. That is, membership of L_q is

$$= \begin{cases} ([4 + 0.5\alpha']/[8- 2\alpha],\ [5.5 - 0.5\alpha']/[\alpha + 4]) & \text{for } 0.54 \le \alpha \le 1 \\ ([4 + 0.5\alpha']/[8- 2\alpha],\ \infty) & \text{for } 0 \le \alpha \le 0.54 \end{cases}$$

Similarly for $\beta = 0.66$, the non – membership of L_q is

$$= \begin{cases} ([1.5\beta'+3]/[8-2\beta],\ [6-\beta']/[2\beta+3]) & \text{for } 0.66 \le \beta \le 1 \\ ([1.5\beta'+3]/[8-2\beta],\ \infty) & 0 \le \beta \le 0.66 \end{cases}$$

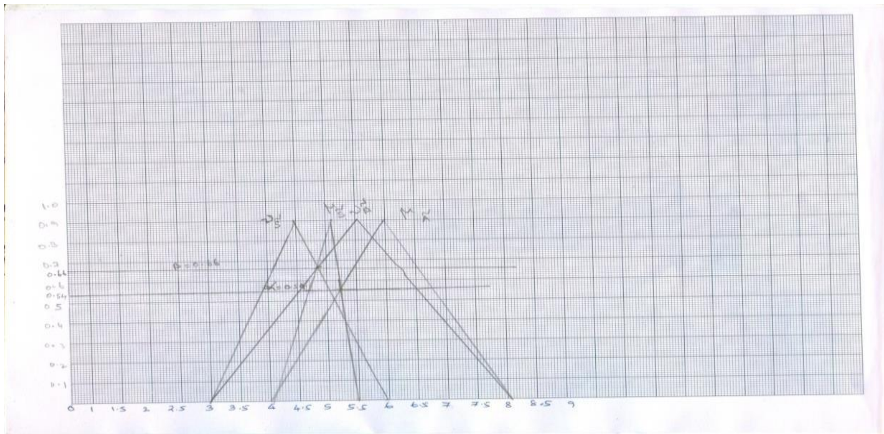

Figure 4.1: The membership and non-membership of L_S.

From (4.6)′, $\tilde{\mu}_{Lq}(z)$ and $\tilde{\nu}_{Lq}(z)$ can be found out easily.

5. Conclusion

The papers such as [4], [7], [12], [13], [15],[16], [17] and [18] have applied fuzzy set concept to classical queueing systems and studied performance measures. When the interarrival time and service time are fuzzy variables, according to Zadeh's extension principle, the performance measures such as the average system length, the average waiting time, etc., will be fuzzy as well. In this paper, we introduce the concept of Intuitionistic fuzzy set concept to classical queue models and analyzed the system performance using (α,β)-cut to reduce an intuitionistic fuzzy queue into a family of crisp queues which can be described by membership functions and non-membership functions of the performance measures. Since the performance measures are expressed by membership functions and non-membership functions rather than possible values, they provide more information. Also in this paper, the three intuitionistic fuzzy queues namely M/IF/1, IF/M/1, IF/IF/1 with infinite capacity are investigated. Other intuitionistic fuzzy queues such as M/IF/c, IF/M/c, where c denotes finite capacity, or bulk service are also possible to analyze. In future, this work may be extended to systems with more than two intuitionistic fuzzy variables, for instance, k servers each has an intuitionistic fuzzy service time, the approach proposed in this paper can be extended using (α,β) approach.

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