ISSN: 0975-766X
CODEN: IJPTFI
Available Online through
www.ijptonline.com
Research Article

NUMERICAL STUDY OF MHD FLOW AND HEAT TRANSFER OVER AN EXPONENTIALLY PERMEABLE STRETCHING SURFACE OF A CASSON FLUID

T.Hymavathi[1*], Pavana Kumari[2]
[1,2]Department of Mathematics, Adikavi Nannaya University, Rajamundry, A.P., India.
Email: talla.hymavathianur@gmail.com

Received on: 20-09-2018 *Accepted on: 10-11-2018*

Abstract: The present paper contains the study of boundary layer flow and heat transfer of a non-Newtonian MHD Casson fluid over an exponentially stretching permeable surface. Using similarity transformations the governing partial differential equations corresponding to the equation of continuity, momentum, and energy equations are converted into nonlinear ordinary differential equations and numerical solutions to these equations are obtained using an implicit finite difference scheme known as Keller Box method. It is observed that the increase in Casson parameter results in decrease of velocity whereas temperature increases. Also suction/blowing shows opposite effect on velocity and temperature. Increase in Prandtl number decreases temperature. Skin friction is examined and compared with previous results. The results are found to be good in agreement.

Keywords: Casson fluid, MHD, heat transfer, Keller Box method, exponentially stretching sheet, suction / blowing.

1. Introduction

In many engineering problems, boundary layer flow over a stretching surface is encountered. Sakiadis[1] examined the boundary layer flow over a continuous solid surface. L.J.Crane[2] studied flow past a stretching surface. Flow and Heat transfer over an exponentially stretching sheet has very important applications in industry. For example, In case of annealing and thinning of copper wire, the final product depends upon the rate of heat transfer at the stretching continuous surface with variations in stretching velocity, temperature distribution exponentially. The quality of the final product depends on these parameters. Magyari et. al.[3] studied the heat and mass transfer on boundary layer flow due to an exponentially stretching sheet.

Elbashbeshy[4] investigated the flow of an exponentially stretching sheet. Sajid et al[5] examined the influence of thermal radiation on boundary layer flow due to exponentially stretching sheet using HAM. Bidin et al[6] analyzed the effect of thermal radiation on the study of laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet. Hymavathi et al[7] studied numerically the flow and heat transfer of a Casson fluid over an exponentially permeable stretching surface using Keller Box method. Swathy Mukhopadhyay[8] using shooting method discussed the Casson fluid flow and heat transfer at an exponentially stretching permeable surface.

Swathi Mukhopadhyay[9] studied MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium. Swathi Mukhopadhyay[10] also examined the slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation. The technique for keller box scheme was discussed in-detailed by Cebeci et al[11] in physical and computational aspects of convective Heat Transfer. The best examples of Casson fluid are soup, tomato sauce, jelly,honey, human blood etc. It can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear; a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear [12]. Eldabe and Salwa [13] have studied the Casson fluid for the flow between two rotating cylinders and Boyd et al. [14] investigated the Casson fluid flow for the steady and oscillatory blood flow. Hymavathi et al.[15] discussed the numerical solution to mass transfer on MHD flow of Casson fluid with suction and chemical reaction. Mustafa et al[16] discussed the Stagnation-point flow and heat transfer of a Casson fluid towards a stretching sheet. Bachok et al[17] examined the flow and heat transfer over an unsteady stretching sheet in a micro polar fluid with prescribed heat flux. In this present study of work, numerical solution to MHD Casson fluid flow and Heat transfer at an exponentially stretching surface is examined. The Keller Box method is discussed for numerical computation. The velocity and Temperature profiles were examined for various parameters like casson fluid parameter, magnetic parameter, and suction/blowing parameter.

2. Equations of Motion

Consider the boundary layer flow of a steady incompressible viscous fluid past a flat sheet coinciding with the plane $y = 0$. The fluid flow is confined to $y > 0$. Two equal and opposite forces are applied along the x axis so that the wall is stretched keeping the origin fixed (shown in Fig.1). The rheological equation of state for an isotropic and incompressible Casson fluid is

$$\tau_j = \begin{cases} 2(\mu_B + P_y/\sqrt{2\pi})e_{ij}, & \pi > \pi_c \\ 2(\mu_B + P_y/\sqrt{2\pi_c})e_{ij}, & \pi < \pi_c \end{cases} \quad (1)$$

Here $\pi = e_{ij}e_{ij}$ and e_{ij}, is the (i,j)th component of the deformation rate, π is the product of the component of the deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, and P_y is the yield stress of the fluid. The continuity, momentum [24], and energy [7,10] equations governing such a type of flow problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B_0^2}{\rho}u \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} \quad (4)$$

Where, u and v are the components of velocity respectively in the x and y directions, v is the kinematic viscosity, ρ is the fluid density (assumed constant), $\beta = \mu_B\sqrt{2\pi_C}/P_y$ is the non-Newtonian parameter of the Casson fluid, k is the thermal diffusion coefficient of the fluid.

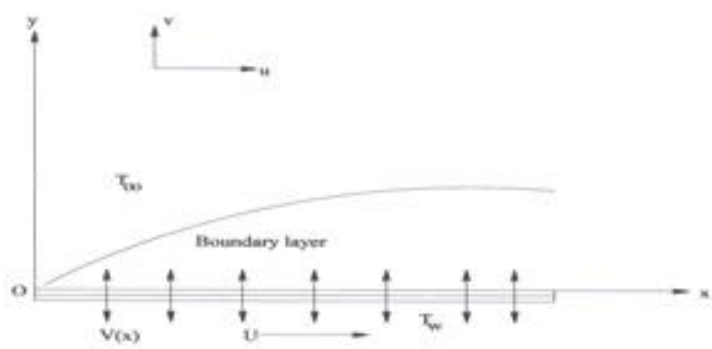

Fig. 1 Sketch of the physical problem

2.1 Boundary Conditions

The appropriate boundary conditions for the problem are given by

$$u = U, v = -v_0(x), T = T_w \ at \ y = 0 \quad (5)$$

$$as \ y \to \infty, u \to 0, T \to T_\infty \quad (6)$$

Here $U = U_0 e^{\frac{x}{L}}$ is the stretching velocity, $T_w = T_\infty + T_0 e^{\frac{x}{2L}}$ is the temperature at the sheet, U_0 and T_0 are the reference velocity and temperature respectively. $V(x) = V_0 e^{\frac{x}{2L}}$ is a special type of velocity at the wall with V_0 as constant. The meaning of V_0 :V(x)>0 is the velocity suction and V(x)<0 is the velocity blowing.

Equations (2)-(6) can be made dimensionless by introducing the following change of variables

$$\eta = \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}} y \qquad (7a)$$

$$U = U_0 e^{\frac{x}{L}} f'(\eta) \qquad (7b)$$

$$\upsilon = -\sqrt{\frac{\upsilon U_0}{2L}} e^{\frac{x}{2L}} \left[f(\eta) + \eta f'(\eta)\right] \qquad (7c)$$

$$T = T_\infty + T_0 e^{\frac{x}{2L}} \theta(\eta) \qquad (7d)$$

The dimensionless problem satisfies

$$\left(1 + \frac{1}{\beta}\right) f''' + f f'' - 2f'^2 - M f' = 0 \qquad (8)$$

$$\theta'' + \Pr(f\theta' - f'\theta) = 0 \qquad (9)$$

and the boundary conditions take the following form

$$at\ \eta = 0, f = S, \theta = 1, f' = 1 \qquad (10)$$

$$as\ \eta \to \infty, f' \to 0, \theta \to 0 \qquad (11)$$

Where $S = v_0 / \sqrt{\frac{U_0 \upsilon}{2L}}$ >0 or (<0) is the suction or (blowing) parameter and $Pr = \frac{\upsilon}{k}$ is the Prandtl number.

2.2 Numerical Procedure

Equation subject to boundary conditions is solved numerically using an implicit-finite difference scheme known as Keller Box method, as described by Cebeci and Bradshaw[11]. The steps followed are

1. Reduce (8)-(9) to a first order equation
2. Write the difference equations using central differences
3. Linearize the resulting algebraic equation by Newton’s method and write in matrix vector form
4. Use the block tridiagonal elimination technique to solve the linear system.

Consider the flow equation and concentration equations (8) and (9) and the boundary conditions (10) and (11)

Introduce $f'=p$, (12)

$p'=q$, (13)

$g'=t$ $(g=\phi)$ (14)

eqn(8) and (9) reduces to

$$\left(1+\frac{1}{\beta}\right)q' + fq - 2p^2 - Mp = 0 \quad (15)$$

$$t' + Pr(ft - pg) = 0 \quad (16)$$

consider the segment η_{j-1},η_j with $\eta_{j-1/2}$ as the mid point $\eta_0=0$, $\eta_j = \eta_{j-1}+h_j$, $\eta_j=\eta_\infty$ (17)

where h_j is the $\Delta\eta$ spaces and j=1,2,……..J is a sequence number that indicates the coordinate locations.

Introducing finite differences

$$\frac{f_j - f_{j-1}}{h_j} = \frac{p_j + p_{j-1}}{2} = p_{j-1/2} \quad (18)$$

$$\frac{p_j - p_{j-1}}{h_j} = \frac{q_j + q_{j-1}}{2} = q_{j-1/2} \quad (19)$$

$$\frac{g_j - g_{j-1}}{h_j} = \frac{t_j + t_{j-1}}{2} = n_{j-1/2} \quad (20)$$

$$\left(1+\frac{1}{\beta}\right)\frac{q_j - q_{j-1}}{h_j} + \left(\frac{f_j + f_{j-1}}{2}\right)\left(\frac{q_j + q_{j-1}}{2}\right) - 2\left(\frac{p_j + p_{j-1}}{2}\right)^2 - M\left(\left(\frac{p_j + p_{j-1}}{2}\right)\right) = 0 \quad (21)$$

$$\frac{t_j - t_{j-1}}{h_j} + \Pr\left(\frac{f_j + f_{j-1}}{2}\right)\left(\frac{t_j + t_{j-1}}{2}\right) - .\Pr\left(\frac{p_j + p_{j-1}}{2}\right)\left(\frac{g_j + g_{j-1}}{2}\right) = 0 \quad (22)$$

2.3 Newton's method

Linearizing the nonlinear system of equations (18) to (22)

Introduce

$$f_j^{(k+1)} = f_j^{(k)} + \delta f_j^{(k)}$$

$$p_j^{(k+1)} = p_j^{(k)} + \delta p_j^{(k)}$$

$$q_j^{(k+1)} = q_j^{(k)} + \delta q_j^{(k)}$$

$$g_j^{(k+1)} = g_j^{(k)} + \delta g_j^{(k)}$$

$$t_j^{(k+1)} = t_j^{(k)} + \delta t_j^{(k)} \qquad (23)$$

Substitute in equations (12) to (16)

Write

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}(\delta p_j + \delta p_{j-1}) = (r_1)_{j-\frac{1}{2}} \qquad (24)$$

$$\delta p_j - \delta p_{j-1} - \frac{h_j}{2}(\delta q_j + \delta q_{j-1}) = (r_2)_{j-\frac{1}{2}} \qquad (25)$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2}(\delta n_j + \delta n_{j-1}) = (r_3)_{j-\frac{1}{2}} \qquad (26)$$

$$(a_1)_j \delta q_j + (a_2)_j \delta q_{j-1} + (a_3)_j \delta f_j + (a_4)_j \delta f_{j-1} + (a_5)_j \delta p_j + (a_6)_j \delta p_{j-1} = (r_4)_{j-\frac{1}{2}} \qquad (27)$$

$$(b_1)_j \delta t_j + (b_2)_j \delta t_{j-1} + (b_3)_j \delta f_j + (b_4)_j \delta f_{j-1} + (b_5)_j \delta p_j + (b_6)_j \delta p_{j-1} + (b_7)_j \delta g_j + (b_8)_j \delta g_{j-1} = (r_5)_{j-\frac{1}{2}} \qquad (28)$$

Where,

$$\left.\begin{aligned} (a_1)_j &= 1 + \frac{\beta h_j}{4(\beta+1)}(f_j + f_{j-1}) \\ (a_2)_j &= (a_1)_j - 2.0 \\ (a_3)_j &= \frac{\beta h_j}{4(\beta+1)}(q_j + q_{j-1}) \\ (a_4)_j &= (a_3)_j \\ (a_5)_j &= -\frac{\beta h_j}{(\beta+1)}((p_j + p_{j-1}) + M) \end{aligned}\right\} \qquad (29)$$

$$(a_6)_j = (a_5)_j$$

$$\left.\begin{aligned}
(b_1)_j &= 1 + \frac{\Pr h_j}{4}(f_j + f_{j-1}) \\
(b_2)_j &= (b_1)_j - 2.0 \\
(b_3)_j &= \frac{\Pr h_j}{2}(t_j + t_{j-1}) \\
(b_4)_j &= (b_3)_j \\
(b_5)_j &= -\frac{\Pr h_j}{2}(g_j + g_{j-1}) \\
(b_6)_j &= (b_5)_j
\end{aligned}\right\} \quad (30)$$

$$(b_7)_j = -\frac{\Pr h_j}{2}(p_j + p_{j-1})$$

$$(b_8)_j = (b_7)_j$$

and

$$\left.\begin{aligned}
(r_1)_j &= f_{j-1} - f_j + \frac{h_j}{2}\left(p_j + p_{j-1}\right) \\
(r_2)_j &= p_{j-1} - p_j + \frac{h_j}{2}\left(q_j + q_{j-1}\right) \\
(r_3)_j &= g_{j-1} - g_j + \frac{h_j}{2}\left(t_j + t_{j-1}\right) \\
(r_4)_j &= q_{j-1} - q_j - \frac{\beta h_j}{4(\beta+1)}\left(f_j + f_{j-1}\right)\left(q_j + q_{j-1}\right) + \frac{\beta h_j}{2(\beta+1)}\left(p_j + p_{j-1}\right)^2 + M\frac{\beta h_j}{2(\beta+1)}\left(p_j + p_{j-1}\right)
\end{aligned}\right\} \quad (31)$$

$$(r_5)_j = t_{j-1} - t_j - \frac{\Pr h_j}{4}\left(f_j + f_{j-1}\right)\left(t_j + t_{j-1}\right) + \frac{\Pr h_j}{4}\left(p_j + p_{j-1}\right)\left(g_j + g_{j-1}\right)$$

Taking j=1,2,3…

The system of equations becomes

$$\left.\begin{aligned}&[A_1][\delta_1]+[C_1][\delta_2]=[r_1]\\&[B_2][\delta_1]+[A_2][\delta_2]+[C_2][\delta_3] = [r_2]\\&.....[B_{J-1}][\delta_1]+[A_{J-1}][\delta_2]+[C_{J-1}][\delta_3] = [r_{J-1}]\\&[B_J][\delta_{J-1}]+[A_J][\delta_J]= [r_J]\end{aligned}\right\} \tag{32}$$

Where

$$A_1=\begin{bmatrix}0&0&1&0&0\\d&0&0&d&0\\0&d&0&0&d\\(a_2)_1&0&(a_3)_1&(a_1)_1&0\\0&(b_2)_1&(b_3)_1&0&(b_1)_1\end{bmatrix}\quad A_j=\begin{bmatrix}d&0&1&0&0\\-1&0&0&d&0\\0&-1&0&0&d\\(a_6)_j&0&(a_3)_j&(a_1)_j&0\\(b_6)_j&(b_8)_j&(b_3)_j&0&(b_1)_j\end{bmatrix}$$

$$B_j=\begin{bmatrix}0&0&-1&0&0\\0&0&0&d&0\\0&0&0&0&d\\0&0&(a_4)_j&(a_2)_j&0\\0&0&(b_4)_j&0&(b_2)_j\end{bmatrix}\quad C_j=\begin{bmatrix}d&0&0&0&0\\1&0&0&0&0\\0&1&0&0&0\\(a_5)_j&0&0&0&0\\(b_5)_j&(b_7)_j&0&0&0\end{bmatrix} \tag{33}$$

2.4 The Block Elimination Method

The linearized differential equations of the system has a block diagonal structure. This can be written in tri diagonal matrix form as

$$\begin{bmatrix}[A_1]&[C_1]&&&\\ [B_2]&[A_2]&[C_2]&&\\ &&\vdots&&\\ &&[B_{J-1}]&[A_{J-1}]&[C_{J-1}]\\ &&&[B_J]&[A_J]\end{bmatrix}\begin{bmatrix}[\delta_1]\\ [\delta_2]\\ \vdots\\ [\delta_{J-1}]\\ [\delta_J]\end{bmatrix}=\begin{bmatrix}[r_1]\\ [r_2]\\ \vdots\\ [r_{J-1}]\\ [r_J]\end{bmatrix} \tag{34}$$

This is of the form $A\ \delta = r$ (35)

To solve the above system

Write [A] = [L] [U] (36)

Where

$$L = \begin{bmatrix} [\alpha_1] & & & \\ [\beta_2] & [\alpha_2] & & \\ & & \ddots & \\ & & [\alpha_{j-1}] & \\ & & [\beta_J] & [\alpha_J] \end{bmatrix} \text{ and } U = \begin{bmatrix} [I] & [\Gamma_1] & & & \\ & [I_2] & [\Gamma_2] & & \\ & & \ddots & & \\ & & & [I] & [\Gamma_{J-1}] \\ & & & & [I] \end{bmatrix} \quad (37)$$

Where [I] is the identity matrix

$[\alpha_i][\Gamma_i]$ are determined by the following equations

$[\alpha_1] = [A_1]$

$[A_1][\Gamma_1] = [C_1]$

$[\alpha_j] = [A_j]-[B_j][\Gamma_{j-1}]$ j=2,3,. ....,J

$[\alpha_j][\Gamma_j] = [C_j]$ j=2,3,..........,J-1

Substituting (36) in (35)

LUδ = r

Let U δ = W

Then LW = r

$$\text{Where } W=\begin{bmatrix} [w_1] \\ [w_2] \\ \vdots \\ [w_{j-1}] \\ [w_J] \end{bmatrix}$$

Now $[\alpha_1]$ $[w_1]$ = $[r_1]$

$[\alpha_j]$ $[w_J]$ = $[r_J]-[B_J][W_{j-1}]$ for $2 \le j \le J$

Once the elements of W are found, substitute in Lδ=W and solve for δ

$[\delta_J] = [W_J]$

$[\delta_J] = [W_J]-[\Gamma_J][\delta_{J+1}]$, $1 \le j \le J-1$

These calculations are repeated until some convergence criterion is satisfied and we stop the calculations when $|\delta g_0^{(i)}| \le \varepsilon$, where ε is very small prescribed value taken to be ε = 0.0001.

3. Results and discussion

The velocity and temperature profiles are plotted graphically using MATLAB for various values of Casson parameter, Magnetic parameter, Suction parameter, Prandtl number. It is observed that the velocity is found to be decreasing with increase in Casson parameter as shown in figure2 in presence of suction / blowing.

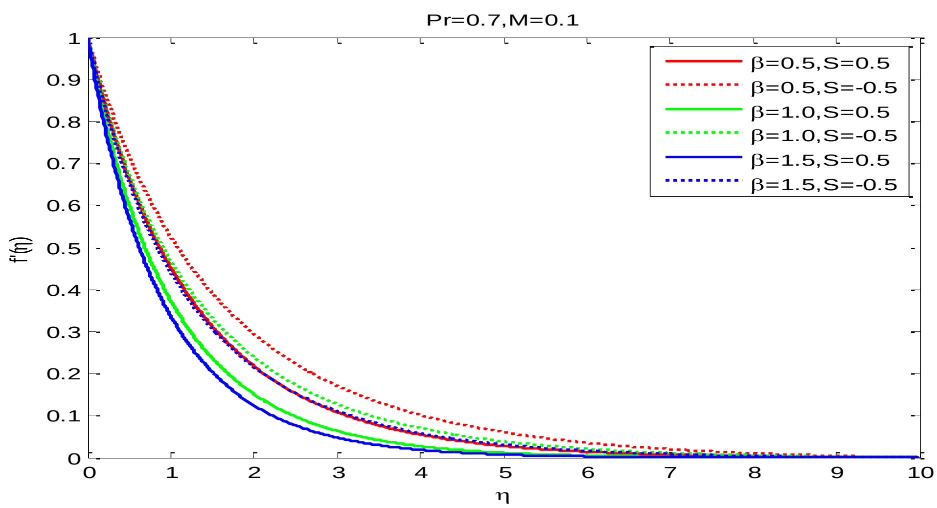

Fig.**2. Velocity profiles with Casson Parameter β in presence of suction/blowing**

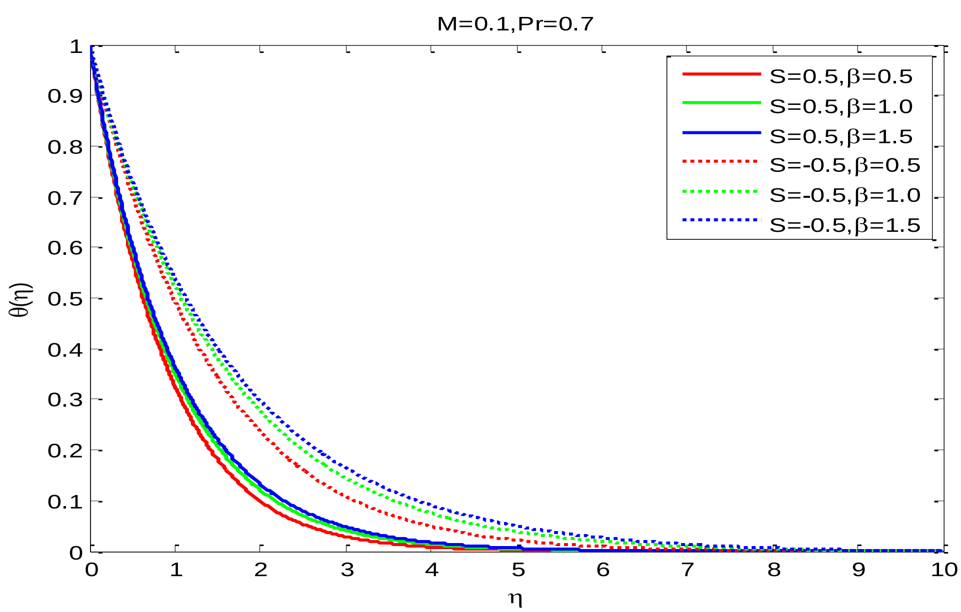

Fig.3. Temperature profiles with Casson Parameter β in presence of suction/blowing

Figure3 shows that the temperature is found to be increasing with increase in Casson Parameter in presence of Suction / blowing.

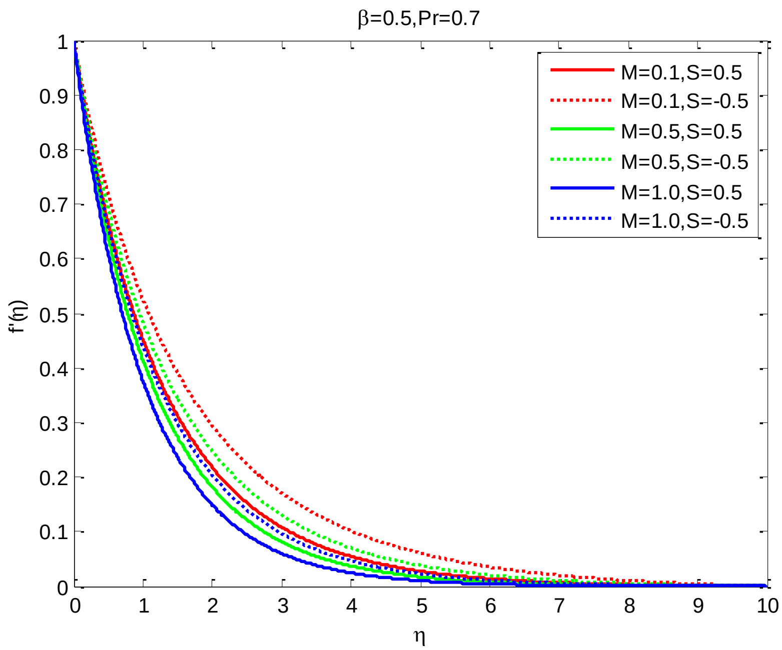

Fig.4. Velocity profiles with Magnetic Parameter in presence of suction/blowing.

The velocity is found to be decreasing with increase in Magnetic parameter as shown in figure 4 in presence of suction / blowing.

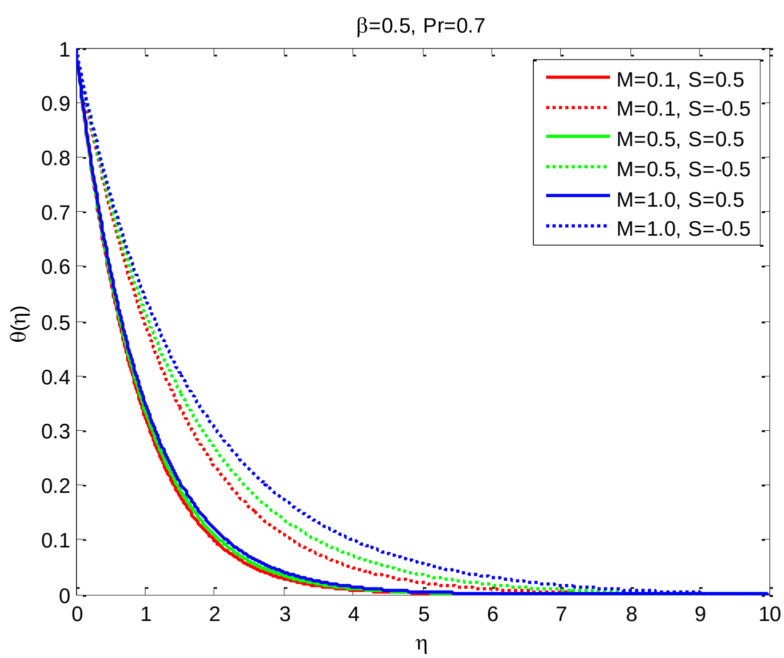

Fig.5. Temperature profiles with Magnetic Parameter in presence of suction/blowing.

The temperature is found to be increasing with increase in Magnetic parameter as shown in figure4 in presence of suction / blowing.

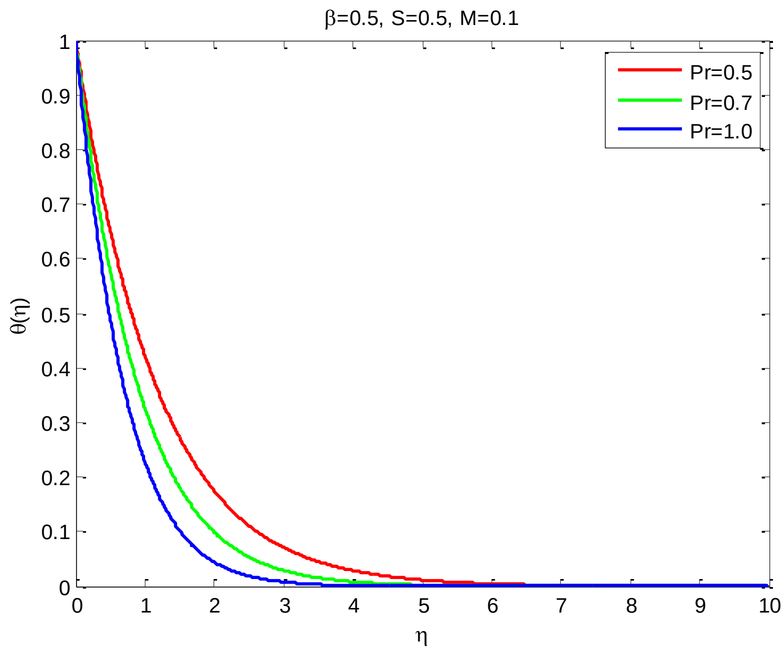

Fig.**6. Temperature profiles with Prandtl number.**

Fig.6 represents the effect of Prandtl number Pr on temperature profiles in presence of suction and observed that as Prandtl number increases, temperature decreases i.e., increase in Prandtl number reduces the thermal boundary layer thickness.

Conclusions:

The present work investigates the numerical solution for steady boundary layer flow and heat transfer analysis for a MHD Casson fluid over an exponentially stretching surface. It is observed that

1. Velocity is found to be decreasing with increase in casson parameter whereas temperature is found to be increasing in presence of suction/blowing.

2. Velocity is found to be decreasing with increase in magnetic parameter whereas temperature is found to be increasing in presence of suction/blowing.

3. Temperature is found to be decreasing with increase in Prandtl number.

4. With increase in Pr, temperature decreases and in turn reduces the thermal boundary layer thickness.

Conflict of Interest: We declared that this review does not have any conflict of interest.

References

1. Sakiadis B.C. Boundary layer bahaviour on continous solid surface: I boundary layer equation for two dimensional and axisymmetric flow. – AIChE J., vol.7,1961, pp.26-28.
2. Crane, L. J., Flow Past a Stretching Plate,ZAMP, 21,1970, pp. 645-647.
3. Magyari, E., and Keller, B., Heat and Mass Transfer in the Boundary Layers on an Exponentially Stretching Continuous Surface, J. Phys. D, 32,1999 pp.577-585.
4. Elbashbeshy, E. M. A.,Heat Transfer Over an Exponentially Stretching Continuous Surface With Suction,Arc. Mech., 53.2001, pp. 643-651.
5. Sajid, M., and Hayat, T., Influence of Thermal Radiation on the Boundary Layer Flow Due to an Exponentially Stretching Sheet,Int. Commun. Heat Mass Transfer. 35,2008, pp. 347-356.
6. Bidin, B., and Nazar, R., Numerical Solution of the Boundary Layer Flow Over an Exponentially Stretching Sheet With Thermal Radiation, Euro.J. Sei. Resc, 33(4), 2009, pp. 710-717.
7. T. Hymavathi, W.Sridhar, Numerical Solution To Flow and Heat Transfer of a Casson Fluid over an exponentially permeable Stretching Surface, *International Journal of Pharmacy & Technology, 9(4), 2017, 31088-31102.*
8. Swathi Mukhopadhyay, Casson Fluid Flow and Heat Transfer at an Exponentially Stretching Permeable Surface, J.Applied Mechanics,ASME v.80,2013,054502(1-9).
9. Swathi Mukhopadhyay, MHD boundary layer flow and heat transfer over an exponentially stretching sheet embded in a thermally stratified medium, Alexandria Engg.J.52,(2013),259-265
10. Swathi Mukhopadhyay, Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation, Ain Shams Eng.J.(2012)
11. T.Cebeci, P.Bradshaw, Physical and computational aspects of convective heat transfer, springer-verlag, Newyork, 1988.
12. Dash, R. K., Mehta, K. N., and Jayaraman, G., Casson Fluid Flow in a Pipe Filled With a Homogeneous Porous Medium, Int. J. Eng. Sei., **34(10), 1996,**pp. 1145-1156.

13. Eldabe, N. T. M.. and Salwa, M. G. E., Heat Transfer of MHD Non- Newtonian Casson Fluid Flow Between Two Rotating Cylinders, J. Phys. Soc.Jpn., 64, 1995,pp. 41-64.
14. Boyd, J., Buick, J. M., and Green, S., 2007, Analysis of the Casson and Carreau-Yasuda Non-Newtonian Blood Models in Steady and Oscillatory Flow Using the Lattice Boltzmann Method,Phys. Fluids, **19,** pp. 93-103.
15. T.Hymavathi, W.Sridhar, Numerical solution to mass transfer on MHD flow of Casson fluid with suction and chemical reaction, Int. Journal of Chemical Sciences14(4)-2016,2183-2197.
16. Mustafa, M., Hayat, T., Pop, I. and Hendi, A. A,,Stagnation-point flow and heat transfer of a Casson fluid towards a stretching sheet. Z. Naturforsch., 67a, p.70 (2012).
17. Bachok, Ishak ,Nazar flow and heat transfer over an unsteady stretching sheet in a micropolar fluid with prescribed heat flux,Meccanica,46(5),2011,pp935-942.

Corresponding Author:

T.Hymavathi*,

Email: *talla.hymavathianur@gmail.com*