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ANALYTICAL SOLUTION TO HEAT AND MASS TRANSFER OF MHD FLOW OF VISCO-ELASTIC FLUID OVER AN EXPONENTIALLY STRETCHING SHEET THROUGH POROUS MEDIUM

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Abstract:

In this paper, we analyze the heat and mass transfer of MHD visco-elastic flow through porous medium over an exponentially stretching sheet. The governing boundary layer equations are transformed into ordinary differential equations by using suitable similarity transformations. The resulting equations are solved by using analytic technique Homotopy Analysis Method (HAM). The convergence of obtained series solutions is explicitly discussed. The results are represented by the graphs for various physical parameters like magnetic parameter, Grashof number, modified Grashof number, Prandtl number and Schmidt number for velocity, temperature and concentration. The effect of thermal diffusion, diffusion of species is to increase the velocity profile and to reduce the temperature and concentration profiles. Results are compared with available results in literature and are seen in good agreement.

Key Words: MHD, Visco-elastic fluid, porous medium, exponentially stretching sheet, HAM.

Introduction:

The boundary layer flow over a stretching sheet has wide range of practical applications in the field of industrial and engineering applications such as manufacturing of plastic sheets, paper production, glass blowing, etc. Initially, the boundary layer behavior on continuous solid surfaces is investigated by Sakiadis [1]. L.J.Crane [2] extended the work of [1] whose velocity is proportional to the distance from the slit which occurs in drawing of plastic films. With this motivation P.S.Gupta and A.S.Gupta [3] analyzed the similarity solutions for the heat and mass transfer of a boundary layer over a stretching sheet subject to suction or blowing. K.R. Rajgopal et.al, [4] studied the flow of an incompressible second order fluid past a stretching sheet which bears a problem of the continuous extrusion of a polymer sheet from a die. Magneto hydrodynamic flows have considerably a wide range of theoretical and practical applications in designing cooling systems with liquid metals. In recent years, the study of magneto hydrodynamic flow of visco-elastic fluid has gained lot of interest by many researchers due to its physiological applications. All the above studies concerns in absence of magnetic field. Subsequently, A.Chakrabarti and A.S.Gupta [5] studied the flow and heat transfer characteristics of an electrically conducting fluid permeated by uniform transverse magnetic field over a stretching sheet. H.I. Anderson [6] examined the viscoelastic fluid flow past a stretching sheet in presence of a transverse magnetic field. Ming-I Char [7] obtained the solutions for the heat and mass transfer in a hydrodynamic flow of the visco-elastic fluid over a stretching surface. In many Industrial applications the process of heat and mass transfer takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. E.M.A.Elabasbeshy [8] investigated the heat and mass transfer along a vertical plate under the effects of thermal and species diffusion in presence of magnetic field.

However, all the above work carried out without considering porous medium. In recent years, viscoelastic fluid flow through porous medium has gained importance by many researchers. The cooling of strips can be controlled by drawing them through porous media. Vajravelu [9] analyzed the flow and heat transfer characteristics of viscous fluid in a saturated porous medium over an impermeable stretching sheet. M. Subhas Abel et.al, [10] investigated the problem of heat and mass transfer of Walter's liquid B embedded in porous medium over a stretching sheet. The influence of reaction rate on the transfer of chemically reactive species in the viscoelastic fluid flow immersed in porous medium is considered by K.V.Prasad et.al, [11]. Later, Rafael Cortell [12] studied the flow and mass diffusion of an electrically conducting second grade fluid in porous medium over a stretching surface with applied magnetic field.

Shijun Liao proposed a powerful analytic tool Homotopy Analysis Method (HAM) for non-linear problems [13]. The most significant feature of this method is that the region of convergence can be controlled and adjusted when compared to other methods. The Homotopy Analysis Method is further improved and systematically described through a typical non linear problem for viscous boundary layer flow by Shijun Liao [14]. M. Sajid and T. Hayat

[15] applied Homotopy Analysis method to discuss the MHD viscous flow due to a shrinking sheet. Behrouz Raftari, et.al, [16] analyzed the flow and heat transfer characteristics of a magneto hydrodynamic visco-elastic fluid with a stretching wall by using Homotopy Analysis Method.

In the above stretching sheet problems, the solutions obtained are not unique. W.C. Troy et.al, [17] derived unique solution of the problem containing exponential terms of similarity variable. E.Magyari and B.Keller [18] examined the mass and heat transfer of viscous fluid over an exponential stretching surface. Sujith Kumar Khan et.al, [19] examined the viscoelastic fluid boundary layer flow and heat transfer over an exponentially stretching sheet. E. Sanjayanand, et.al, [20] examined heat and mass transfer of visco-elastic fluid flow over an exponentially stretching sheet for various physical parameters. Anuar Ishak [21] studied the effect of radiation on magneto hydrodynamic boundary layer flow of a viscous fluid over an exponentially stretching sheet. Recently, Hymavathi Talla and B.Akkaya [22] studied the heat and mass transfer characteristics of visco-elastic fluid flow over an exponentially stretching sheet in porous medium by using Homotopy Analysis Method.

In the present work, we employ Homotopy Analysis Method to investigate MHD visco-elastic fluid flow and heat and mass transfer through porous medium over an exponentially stretching sheet. The governing equations are transformed into ordinary differential equations by using suitable similarity transformations. Further resultant equations are solved analytically using Homotopy Analysis Method (HAM) [13]. The effect of various parameters like visco-elastic parameter, porosity parameter, magnetic parameter, Grashof number etc. on flow, heat and mass transfer are discussed and presented through the graphs.

Mathematical Formulation:

Consider a steady two-dimensional heat and mass transfer flow of an electrically conducting visco-elastic fluid over an exponentially stretching sheet in porous medium. The flow confined to $y>0$ and is generated due to linear stretching of adjacent boundary sheet, caused by the simultaneous application of two equal and opposite forces along x-axis. The x-axis is taken along the stretching surface in the direction of motion and y-axis is perpendicular to it. We assume that the flow is exposed under the influence of a transverse uniform magnetic field of strength B_0. In addition to this we also considered the chemically reactive species in the field of flow. Neglecting Soret and Dufour effects, the governing equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\upsilon}{k^*}u-\frac{\sigma B_0{}^2 u}{\rho}+g\beta_T(T-T_\infty)+g\beta_c(C-C_\infty)$$

(2)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \qquad (4)$$

Where u and v are the velocities in x and y directions respectively, T and C are temperature and concentration of chemical species of the fluid, $\upsilon=\frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the density of the fluid, μ is the dynamic viscosity , k_0 is the visco elastic parameter, k^* is the permeable coefficient of porous medium, σ is electrical conductivity of the fluid, g is the acceleration due to gravity, C_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, β_T is the volumetric coefficient of thermal expansion and β_c is the volumetric concentration coefficient, D is the mass diffusivity with the boundary conditions:

$$\begin{array}{ll} u=U_w(x)=U_0e^{x/l}\ ,v=0,\ T=T_w=T_\infty+T_0e^{x/\ 2l}\ \ C=C_w=C_\infty+C_0e^{x/\ 2l} & at\ \ y=0 \\ u=0,\ u_y=0,\ T=T_\infty\ ,C=C_\infty & as\ \ y\to\infty \end{array} \qquad (5)$$

Where U_o, l are the reference velocity and the reference length respectively. T_∞, C_∞ are the temperature and concentration far away from the stretching sheet.

Equation of continuity (1) is identically satisfied if we choose the stream function ψ such that

$$u=\frac{\partial\psi}{\partial y},\ \ v=-\frac{\partial\psi}{\partial x}, \qquad (6)$$

The momentum energy and concentration equations can be transformed into the corresponding ordinary differential equations by introducing the similarity transformations:

$$\eta = y\sqrt{\frac{U_0}{2\upsilon l}}\; e^{x/2l} \tag{7}$$

$$\psi(x,y) = \sqrt{2\upsilon l U_0}\, f(x.\eta)e^{x/2l}, \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\;, \qquad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{8}$$

Where η is the similarity variable and f is the dimensionless stream function considering $f(x,\eta) = f(\eta)$.

The momentum and energy equations are transformed to

$$f''' - 2f'^2 + ff'' - k_1\left\{3f'f''' - \frac{1}{2}ff^{iv} - \frac{3}{2}f''^2\right\} - (M + k_2)f' + G_r\theta + G_c\phi = 0 \tag{9}$$

$$\theta'' = \Pr(f'\theta - f\theta') \tag{10}$$

$$\phi'' = Sc(\phi f' - f\phi') \tag{11}$$

With the boundary conditions

$$\begin{array}{lllll} f = 0, & f' = 1, & \theta = 1 & \phi = 1 & at\ \eta = 0 \\ f' = 0, & f'' = 0, & \theta = 0 & \phi = 0 & as\ \eta \to \infty \end{array} \tag{12}$$

Where $k_1 = \frac{k_0 U_w}{\upsilon l}$ is the dimensionless visco-elastic parameter, $k_2 = \frac{2\upsilon l}{k^* U_w}$ is the porosity parameter, $M = \frac{2\sigma B_0^2 l}{\rho U_0 e^{x/l}}$ is the magnetic parameter, $\Pr = \frac{\mu C_p}{k_\infty}$ is the Prandtl number, $Sc = \frac{\upsilon}{D}$ is the Schmidt number and $G_r = \frac{2g\beta_T(T - T_\infty)}{U_w^2}$, $G_c = \frac{2g\beta_c(C - C_\infty)}{U_w^2}$.

Homotopy Analysis Solution:

In this section, we employ HAM to solve the equation (9) to (11) subject to the boundary conditions (12). Let us choose the initial guesses f_0, θ_0, ϕ_0 as

$$f_0(\eta) = 1 - e^{-\eta} \tag{13}$$

$$\theta_0(\eta) = e^{-\eta} \tag{14}$$

$$\phi_0(\eta) = e^{-\eta} \tag{15}$$

The linear operator is selected as

$$L(f) = f''' - f', \tag{16}$$

$$L(\theta) = \theta'' - \theta \quad (17)$$

$$L(\phi) = \phi'' - \phi \quad (18)$$

Which have the following property

$$L_f[C_1 + C_2 e^{\eta} + C_3 e^{-\eta}] = 0 \quad (19)$$

$$L_\theta[C_4 e^{\eta} + C_5 e^{-\eta}] = 0 \quad (20)$$

$$L_\phi[C_6 e^{\eta} + C_7 e^{-\eta}] = 0 \quad (21)$$

Where $C_i\ (i = 1,2,3,4,5,6,7)$ are the arbitrary constants.

If $p \in [0,1]$ is the embedding parameter, $\hbar_1, \hbar_2\ and\ \hbar_3$ are the non-zero auxiliary parameters and $H_1(\eta), H_2(\eta)\ and\ H_3(\eta)$ are the auxiliary functions, then we construct the following zeroth- order deformation equation:

$$(1-p)L(\tilde{f}(\eta;p) - f_0(\eta)) = p\hbar_1 H_1(\eta) N_1[\tilde{f}(\eta;p), \tilde{\theta}(\eta;p), \tilde{\phi}(\eta;p)] \quad (22)$$

$$(1-p)L(\tilde{\theta}(\eta;p) - \theta_0(\eta)) = p\hbar_2 H_2(\eta) N_2[\tilde{f}(\eta;p), \tilde{\theta}(\eta;p)] \quad (23)$$

$$(1-p)L(\tilde{\phi}(\eta;p) - \phi_0(\eta)) = p\hbar_3 H_3(\eta) N_3[\tilde{f}(\eta;p), \tilde{\phi}(\eta;p)] \quad (24)$$

Subject to the boundary conditions

$$\tilde{f}(0;p) = 0, \qquad \tilde{f}'(0;p) = 1 \qquad \tilde{f}'(\infty;p) = 0 \qquad \tilde{f}''(\infty;p) = 0$$

$$\theta(0;p) = 1, \qquad \theta'(\infty;p) = 0$$

$$\phi(0;p) = 1, \qquad \phi'(\infty;p) = 0 \quad (25)$$

We define non linear operator as

$$N_1(\tilde{f}(\eta;p)) = \frac{\partial^3 f}{\partial \eta^3} - 2\left(\frac{\partial f}{\partial \eta}\right)^2 + \frac{\partial^2 f}{\partial \eta^2} f - k_1\left(3\frac{\partial f}{\partial \eta}\frac{\partial^3 f}{\partial \eta^3} - \frac{1}{2} f \frac{\partial^4 f}{\partial \eta^4} - \frac{3}{2}\left(\frac{\partial^2 f}{\partial \eta^2}\right)^2\right) - (k_2 + M)\frac{\partial f}{\partial \eta} + G_r\theta + G_c\phi$$

$$(26)$$

$$N_2(\tilde{f}(\eta;p), \tilde{\theta}(\eta;p)) = \frac{\partial^2 \theta}{\partial \eta^2} + \Pr\left(f\frac{\partial \theta}{\partial \eta} - \frac{\partial f}{\partial \eta}\theta\right) \quad (27)$$

$$N_3(\tilde{f}(\eta;p),\tilde{\phi}(\eta;p)) = \frac{\partial^2\phi}{\partial\eta^2} + Sc\left(f\frac{\partial\phi}{\partial\eta} - \frac{\partial f}{\partial\eta}\phi\right) \quad (28)$$

For $p=0$ *and* $p=1$, we have

$$f(\eta,0) = f_0(\eta), \qquad f(\eta,1) = f(\eta)$$

$$\theta(\eta,p) = \theta_0(\eta), \qquad \theta(\eta,1) = \theta(\eta)$$

$$\phi(\eta,p) = \phi_0(\eta), \qquad \phi(\eta,1) = \phi(\eta) \quad (29)$$

Thus as p increases from 0 to 1, $\tilde{f}(\eta;p)$ varies from $f(\eta;0)$ *to* $f(\eta)$, $\tilde{\theta}(\eta;p)$ varies from $\theta(\eta;0)$ *to* $\theta_0(\eta)$ and $\tilde{\phi}(\eta;p)$ varies from $\phi(\eta;0)$ *to* $\phi_0(\eta)$

Now, expanding $\tilde{f}(\eta,p)$, $\tilde{\theta}(\eta,p)$, $\tilde{\phi}(\eta,p)$ using Taylor's Theorem with respect to p we have

$$\tilde{f}(\eta,p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m \quad (30)$$

$$\tilde{\theta}(\eta,p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)p^m \quad (31)$$

$$\tilde{\phi}(\eta,p) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta)p^m \quad (32)$$

Where $f_m(\eta) = \frac{1}{m!}\frac{\partial^m \tilde{f}(\eta,p)}{\partial p^m}\bigg|_{p=0}$ (33)

$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \tilde{\theta}(\eta,p)}{\partial p^m}\bigg|_{p=0} \quad (34)$$

$$\phi_m(\eta) = \frac{1}{m!}\frac{\partial^m \tilde{\phi}(\eta,p)}{\partial p^m}\bigg|_{p=0} \quad (35)$$

If the initial approximations, auxiliary linear operators and non zero auxiliary parameters are chosen in such a way that the series (30) to (32) converges at p=1, and thus

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \quad (36)$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta) \tag{37}$$

$$\phi(\eta)=\phi_0(\eta)+\sum_{m=1}^{\infty}\phi_m(\eta) \tag{38}$$

Differentiating equation (22) to (24) m times with respect to p, setting p=0 and finally dividing with m!, we get the mth order deformation equations are as follows:

$$L_f[f_m(\eta)-\chi_m f_{m-1}(\eta)]=\hbar_1 H_1(\eta)R_m^f(\eta), \tag{39}$$

$$L_\theta[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)]=\hbar_2 H_2(\eta)R_m^\theta(\eta), \tag{40}$$

$$L_\phi[\phi_m(\eta)-\chi_m\phi_{m-1}(\eta)]=\hbar_3 H_3(\eta)R_m^\phi(\eta), \tag{41}$$

Subject to the boundary conditions

$$f_m(0)=0,\quad f_m'(0)=0,\quad f_m'(\infty)=0,\quad f_m''(\infty)=0$$

$$\theta_m(0)=0,\quad \theta_m(\infty)=0$$

$$\phi_m(0)=0,\quad \phi_m(\infty)=0 \tag{42}$$

$$R_m^f(\eta)=f_{m-1}'''+\sum_{i=0}^{m-1}f_i f_{m-1-i}''-2\sum_{i=0}^{m-1}f_i'f_{m-1-i}'-K_1\{3\sum_{i=0}^{m-1}f_{m-1-i}'f_i'''-\frac{1}{2}\sum_{i=0}^{m-1}f_{m-1-i}f_i^{iv}-\frac{3}{2}\sum_{i=0}^{m-1}f_{m-1-i}''f_i''\}$$
$$-(k_2+M)f_{m-1}'+G_r\theta_{m-1}+G_c\phi_{m-1} \tag{43}$$

$$R_m^\theta(\eta)=\theta_{m-1}''+\Pr\sum_{i=0}^{m-1}(f_i\theta_{m-1-i}'-\theta_i f_{m-1-i}') \tag{44}$$

$$R_m^\phi(\eta)=\phi_{m-1}''+Sc\sum_{i=0}^{m-1}(f_i\phi_{m-1-i}'-\phi_i f_{m-1-i}') \tag{45}$$

$$\chi_m=\begin{cases}0, m\le 1\\ 1, m>1\end{cases} \tag{46}$$

We choose the auxiliary function as

$$H_1(\eta)=1\qquad H_2(\eta)=1\quad H_3(\eta)=1 \tag{47}$$

If we let $f_m^*(\eta)$, $\theta_m^*(\eta)$, $\phi_m^*(\eta)$ as the special functions of mth order deformation equations , then the general solutions are given by

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^{-\eta} + C_3 e^{\eta} \quad (48)$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 e^{-\eta} + C_5 e^{\eta} \quad (49)$$

$$\phi_m(\eta) = \phi_m^*(\eta) + C_6 e^{-\eta} + C_7 e^{\eta} \quad (50)$$

Where the integral constants C_i (i =1, 2, 3, 4, 5, 6, 7) are determined using the boundary conditions (42).Now it is easy to solve the linear non-homogeneous equation (39) - (41) using MATHEMATICA software one after the other by considering m = 1, 2, 3…

Convergence of HAM:

The convergence of the series solutions obtained by HAM are strongly depends on the values of the non-zero auxiliary parameters $\hbar_i$ (i=1, 2, 3) which can be adjusted to control the convergence of the solutions. Therefore, for the convergence of the solution, the $\hbar_i$ curves are plotted for 20th order approximations in fig.1, fig.2 and fig.3 for different physical parameters. The valid ranges of $\hbar_i$ are $-1.1 \le \hbar_1 \le -0.22,\ -1.75 \le \hbar_2 \le -0.19,\ -1.93 \le \hbar_3 \le -0.16$ respectively. The convergence of HAM solution for different orders of approximations is given in Table.1.

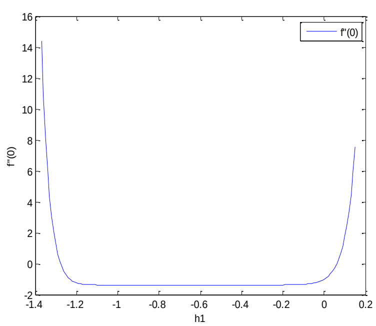

Fig.1 ħ1 curve for f ’´(η)

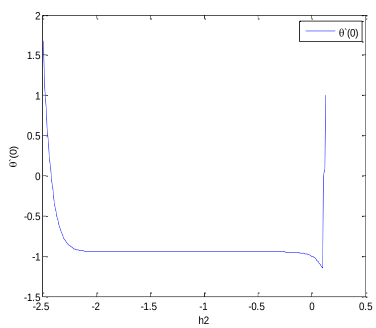

Fig.2 ħ2 curve for θ´(η)

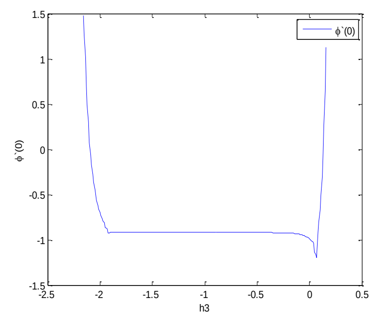

Fig.3 ħ3 curve for φ´(η)

Table 1. Convergence of HAM solution for different orders of approximations when..

$$k_1 = 0.1,\ k_2 = 0.1,\ Gr = 0.1, Gc = 0.1, \Pr = 1,\ Sc = 0.96, M = 0.1.$$

Order	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
5	1.37478	0.945019	0.920428
10	1.37486	0.945114	0.920326

15	1.37493	0.945145	0.920277
20	1.37498	0.945146	0.920265
25	1.37498	0.945147	0.920264
30	1.37498	0.945147	0.920264
35	1.37498	0.945147	0.920264
40	1.37498	0.945147	0.920264

Results and Discussion:

Analytical solutions are discussed for the velocity, temperature and concentration profiles under various values of visco-elastic parameter (k_1), porosity parameter (k_2), Magnetic parameter (M), Grashof Number (G_r) and modified Grashof Number (G_c), Prandtl number (P_r) and Schmidt number (S_c). Graphically the results are illustrated by plotting the figures.

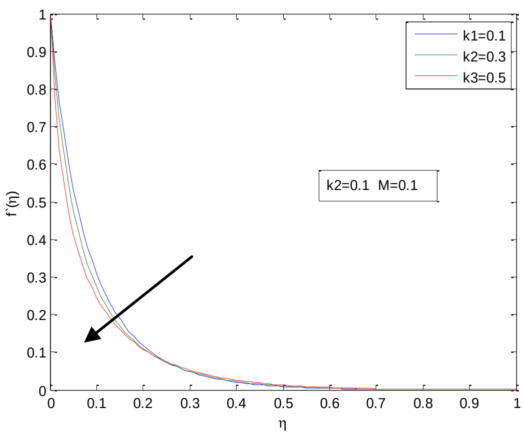

Fig.4 velocity f′ (η) for different values of k1

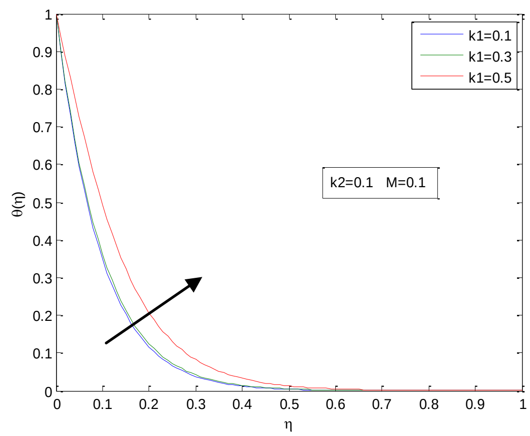

Fig.5 temperature θ (η) for different values of k_1

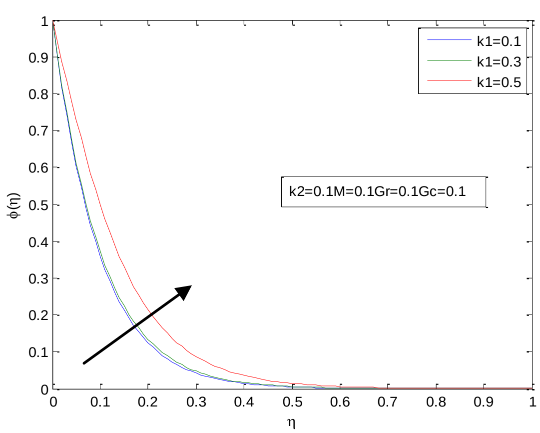

Fig.6 concentration φ (η) for different values of k_1

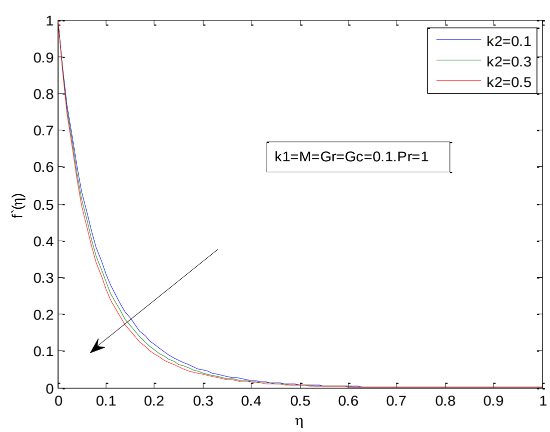

Fig.7 velocity f′ (η) for different values of k_2

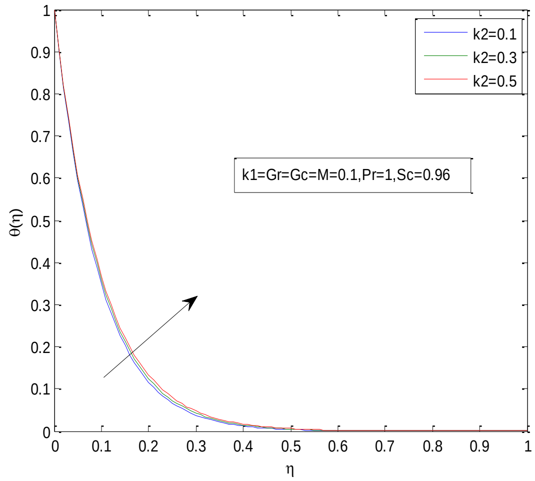

Fig.8 temperature θ (η) for different values of k_2

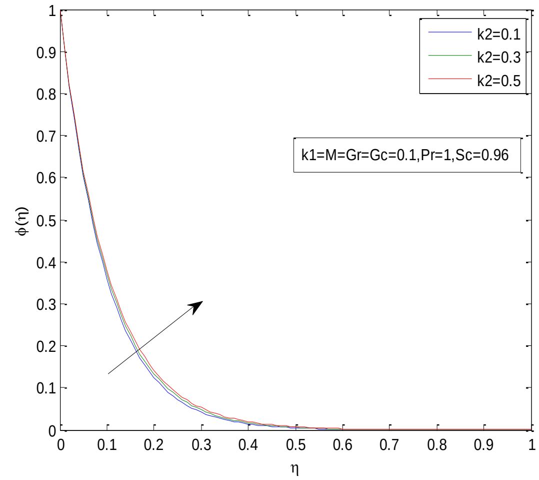

Fig.9 concentration φ (η) for different values of k_2

Fig.4-6 illustrates the effect of visco-elastic parameter k_1 on velocity, temperature and concentration profiles. From the figure it is clear that increasing the visco-elastic- parameter (k_1) declines the velocity and enhances the temperature and concentration.

Fig.7-9 illustrates the effect of porosity parameter k_2 on velocity, temperature and concentration profiles. From the figure it is clear that increasing in the porosity parameter (k_2) declines the velocity and enhances temperature and concentration. By increasing the porous parameter, the holes of the porous medium widens and a resistive force acts opposite to the flow which causes fall in velocity profiles.

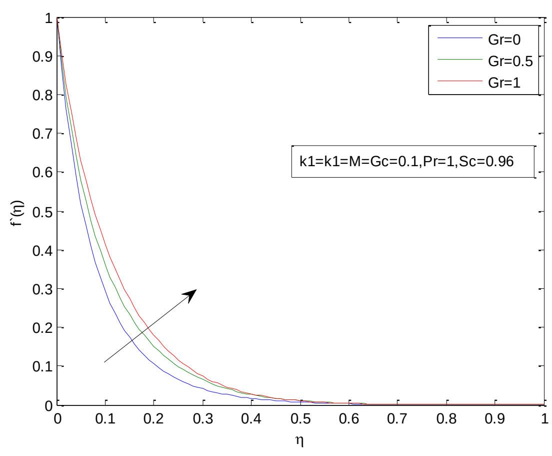

Fig. 10 velocity f′ (η) for different values of Gr

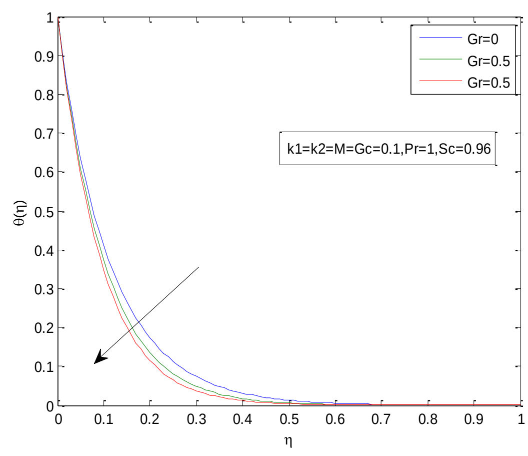

Fig.11 temperature θ (η) for different values of Gr

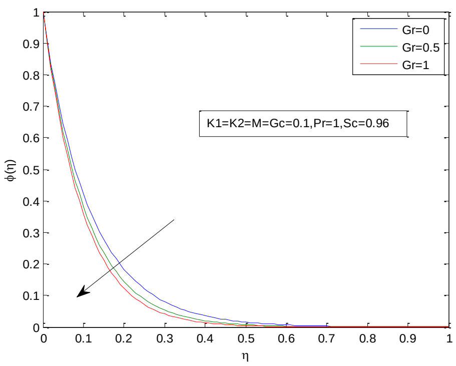

Fig12 concentration φ (η) for different values of Gr

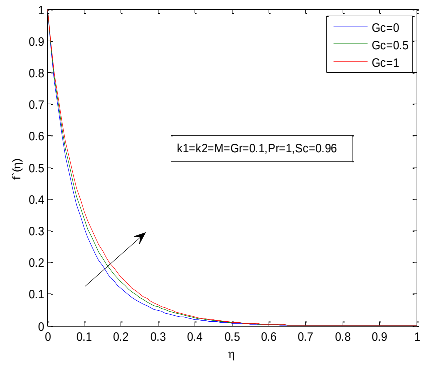

Fig. 13 velocity f′ (η) for different values of Gc

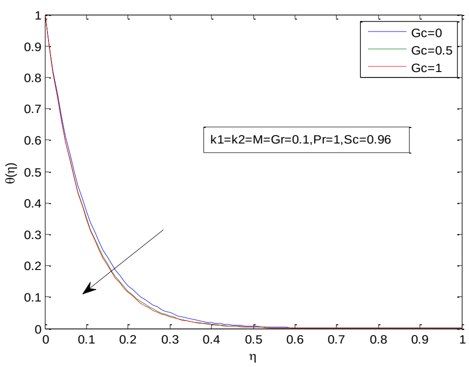

Fig.14 temperature θ (η) for different values of Gc

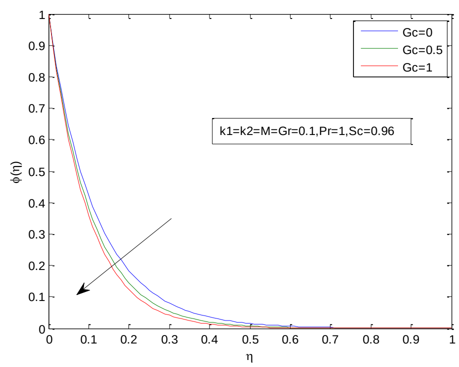

Fig15 concentration φ (η) for different values of Gc

Fig.10 -12 illustrates the behavior of velocity, temperature and concentration for different values of Grashof number Gr . It is seen that the velocity increases by increasing the Grashof number Gr where as temperature and concentration decreases.

Fig.13-15 shows the velocity, temperature and concentration profiles for different values of modified Grashof number Gc. It is noticed that by increasing the modified Grashof number Gc, there is increase in velocity and decrease in temperature and concentration.

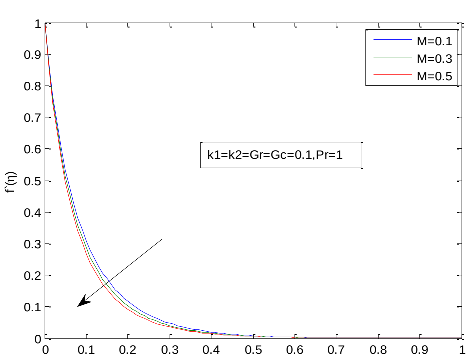

Fig. 16 velocity f′ (η) for different values of M

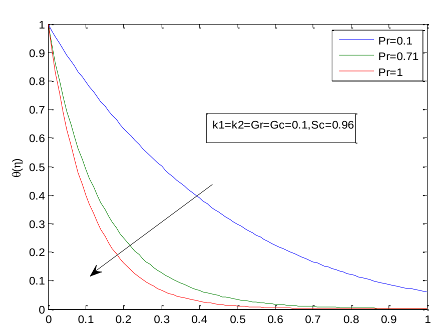

Fig.17 temperature θ (η) for different values of Pr

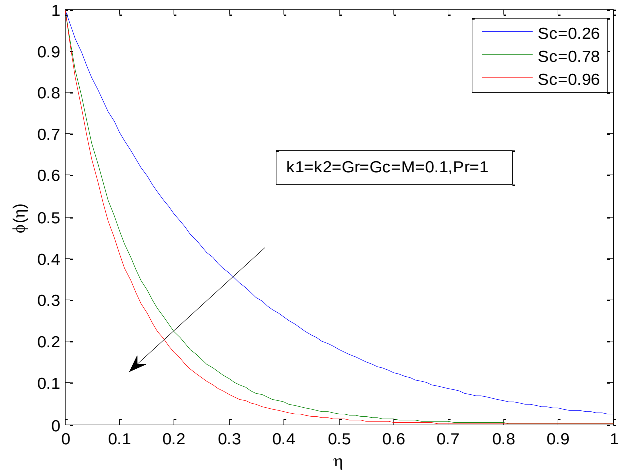

Fig.18 concentration φ(η) for different values of Sc

Fig.16 depicts the effect of magnetic parameter M on velocity. It shows that the rate of transport is considerably reduced with increasing values of M. It indicates that the transverse magnetic field opposes the transport phenomena.

The effect of Prandtl number Pr on temperature is shown in Fig.17. It is observed that the temperature decreases with increase in Prandtl number Pr. It means the thermal boundary layer thickness decreases with increasing values of Prandtl number.

The effect of Schmidt number Sc on concentration is shown in Fig.18. It is observed that the concentration decreases with increase in Schmidt number Sc. This is due to thinning of concentration boundary layer with the introduction of chemical species diffusion.

Table.2 enlists the comparison of θ′(0) for different values of Prandtl number Pr and are compared with the available results in the literature.

Table2. Comparison of $-\theta'(0)$ **for different values of** Pr **when** $k_1 = k_2 = Gr = Gc = M = 0.0$.

Pr	Bidin and Nazar	HAM
1.0	0.9547	0.954784
2.0	1.4714	1.471460
3.0	1.8691	1.869067

Conclusions:

In this paper, we employ Homotopy Analysis Method to heat and mass transfer of MHD visco-elastic fluid flow over an exponentially stretching sheet.

The solutions obtained by Homotopy Analysis Method are good agreement with existing results in literature. The results are summarized as follows.

(1) An increase in Grashof Number and modified Grashof number increases the velocity and reduces the heat and mass transfer rate.

(2) A raise in magnetic parameter reduces the velocity of the fluid.

(3) The effect of porous parameter and visco-elastic parameter is to increase heat and mass transfer rate of boundary layer region.

(4) The effect of increasing the values of Prandtl number and Schmidt number is to reduce the heat and mass transfer rate.

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