ISSN: 0975-766X
CODEN: IJPTFI
Available Online through **Research Article**
www.ijptonline.com

RATIO EDGE ANTIMAGIC LABELING OF CERTAIN CLASS OF GRAPH

S.Ganesh and J.Jayapriya
Department of Mathematics, Sathyabama University, Chennai-600119, Tamilnadu, India.
Email: priyanandam_1975@rediffmail.com

Received on: 11-02-2017 *Accepted on: 25-03-2017*

Abstract

In this paper we investigate the existence of ratio antimagic labeling of certain classes of graphs.

Keywords: Graph, Labeling, Antimagic, wheel graph, Comb graph.

1. Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs are becoming an increasingly useful family of Mathematical Models from a broad range of applications. The graph labeling problem that appears in graph theory has a fast development recently. This problem was first introduced by Alex Rosa in 1967. Since Rosa's article, many different types of graph labeling problems have been defined around this. An enormous body of literature has grown around the subject in about 1500 papers. They gave birth to families of graphs with attractive names such as graceful, Harmonious, felicitous, elegant, cordial, magic antimagic, bimagic and prime labeling etc. A useful survey to know about the numerous graph labeling methods is the one by J.A. Gallian recently [2012][2]. All graphs considered here are finite simple and undirected. Max-min edge antimagic labeling was first introduced by J.Jayapriya and D.Muruganandam in the year 2012, seeking application in Welding Technology [2, 3]. In the year 2013 [4] Jayapriya showed the existences of Max-min edge antimagic labeling for the graphs path, cycle, star, etc. Max-min edge antimagic labeling was renamed as ratio edge antimagic labeling [5].

In this paper the existence of ratio edge antimagic labeling of some class of graphs are shown.

2. Main Result:

Definition 2.1. Let $G\ (V, E)$ be a simple graph with p vertices and q edges. A bijection $f : V(G) \rightarrow \{1,2,\ldots,p\}$, is said to be ratio edge antimagic labeling if for all edges uv in E, the edge weights

$\lambda(uv) = \dfrac{\max\{f(u), f(v)\}}{\min\{f(u), f(v)\}}$ are distinct.

Theorem 2.2: C_n^+ admits ratio edge antimagic labeling.

Proof: The graph C_n^+ is obtained by adding pendent edge to each vertices of C_n where n is the length of the cycle. Let the vertices be $V=\{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$.

Let $f: V \rightarrow \{1, 2, \dots, 2n\}$ such that

$f(v_i) = 2i-1; 1 \le i \le n, f(u_i) = 2i; 1 \le i \le n,$

and the edge set $E = \{v_i v_{i+1}: 1 \le i \le n-1\} \cup \{v_1 v_n\} \cup \{v_i u_i: 1 \le i \le n\}$.

The edge weights are calculated as follows.

For $1 \le i \le n-1$, $\lambda(v_i v_{i+1}) = \frac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}} = (2i+1)/(2i-1)$.

When $i \ne j$, for $1 \le i, j \le n-1$, clearly $\lambda(v_i v_{i+1}) \ne \lambda(v_j v_{j+1})$

If $\lambda(v_i v_{i+1}) = \lambda(v_j v_{j+1})$, then

$(2i+1)/(2i-1) = (2j+1)/(2j-1)$

leads to i = j, which is a contradiction and therefore all edge labels are distinct.

Also $\lambda(v_1 v_n) = \frac{\max\{f(v_1), f(v_n)\}}{\min\{f(v_{1_i}), f(v_n)\}} = 2n-1$

and for $1 \le i \le n$, $\lambda(v_i u_i) = \frac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}} = (2i)/(2i-1)$.

Similarly for $1 \le i, j \le n$, when $i \ne j$, $\lambda(v_i u_i) \ne \lambda(v_j u_j)$.

If $\lambda(v_i u_i) = \lambda(v_j u_j)$ then $(2i)/(2i-1) = (2j)/(2j-1)$.

This implies $i = j$, which is a contradiction and therefore all edge labels are distinct.

Hence C_n^+ admits ratio edge antimagic labeling.

Theorem 2.3: The wheel graph admits ratio edge anti magic labeling.

Proof: The wheel graph has n vertices and $2(n-1)$ edges. Let the vertices be $V=\{v_1, v_2, \dots, v_n\}$. Let $f: V \rightarrow \{1, 2, \dots n\}$ such that $f(v_1) = 1$, $f(v_i) = i; 2 \le i \le n-1$, and the edge set $E = \{v_1 v_i: 2 \le i \le n-1\} \cup \{v_i v_{i+1}: 2 \le i \le n-1\} \cup \{v_2 v_n\}$.

The edge weights are calculated as follows.

For $2 \le i \le n-1$, $\lambda(v_1 v_i) = \frac{\max\{f(v_1), f(v_i)\}}{\min\{f(v_1), f(v_i)\}} = i$.

For $2 \le i, j \le n-1$, $\lambda(v_1 v_i) \ne \lambda(v_1 v_j)$ and $i \ne j$.

If $\lambda(v_1 v_i) = \lambda(v_1 v_j)$, then this implies

$i = j$, which is a contradiction and therefore all edge labels are distinct.

For $2 \le i \le n-1$, $\lambda(v_i v_{i+1}) = \frac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}} = (i+1)/(i)$.

When $i \neq j$, for $2 \le i, j \le n-1$, $\lambda(v_i v_{i+1}) \neq (v_j v_{j+1})$.

If $\lambda(v_i v_{i+1}) = (v_j v_{j+1})$, then $(i+1)/(i) = (j+1)/(j)$.

This leads to i = j, which is a contradiction and therefore all edge labels are distinct.

Also $\lambda(v_2 v_n) = \frac{\max\{f(v_2), f(v_n)\}}{\min\{f(v_2), f(v_n)\}} = n/2$.

For $2 \le i \le n-1$, $\lambda(v_1 v_i) \neq \lambda(v_i v_{i+1})$

If $\lambda(v_1 v_i) = \lambda(v_i v_{i+1})$, then $i^2 = i+1$ which is a contradiction and therefore all edge labels are distinct.

Hence the wheel graph admits ratio edge antimagic labeling.

Theorem 2.4: The comb graph admits ratio edge antimagic labeling.

Proof: The comb graph is obtained by adding pendent edge to each of the vertices of P_n .Let the vertices be $V = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$.

Let $f: V \rightarrow \{1, 2, \ldots, 2n\}$ such that $f(v_i) = 2i-1; 1 \le i \le n$ and $f(u_i) = 2i; 1 \le i \le n$.

Let the edge set be $E = \{v_i u_i;\ 1 \le i \le n\} \cup \{v_i v_{i+1}\ 1 \le i \le n-1\}$. The edge weights are calculated as follows.

For $1 \le i \le n$, $\lambda(u_i v_i) = \frac{\max\{f(u_i), f(v_i)\}}{\min\{f(u_i), f(v_i)\}} = 2i/(2i-1)$.

When $i \neq j$, for $1 \le i, j \le n$, clearly $\lambda(u_i vi) \neq \lambda(u_j v_j)$.

If $\lambda(u_i v_i) = \lambda(u_j v_j)$ then

2i/ (2i-1) = 2j/ (2j-1).

This implies $i = j$, which is a contradiction and therefore all edge labels are distinct.

For $1 \le i \le n-1$, $\lambda(v_i v_{i+1}) = \frac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}} = 2i+1/(2i-1)$.

When $i \neq j$, *for* $1 \le i, j \le n$, clearly $\lambda(v_i v_{i+1}) \neq \lambda(v_j v_{j+1})$.

If $\lambda(v_i v_{i+1}) = \lambda(v_j v_{j+1})$, then $(2i+1)/(2i-1) = (2j+1)/(2j-1)$.

This implies $i = j$, which is a contradiction and therefore all edge labels are distinct.

Now, for $1 \le i \le n-1$, $\lambda(u_i v_i) \neq \lambda(v_i v_{i+1})$.

If $\lambda(u_i v_i) = \lambda(v_i v_{i+1})$,then $2i/(2i-1) = (2i+1)/(2i-1)$.

This implies $2i = 2i+1$ which is a contradiction and therefore all edge labels are distinct. Hence the comb graph admits ratio edge antimagic labeling.

Theoremb2.5: The one vertex union of any cycle is ratio edge antimagic labeling.

Proof: Let G = $C_{n,t}$, $\forall$ n ≥ 2, t ≥ 1, be a graph of one vertex union of any cycle C_n *with* $t(n-1)+1$ vertices and nt edges. Let the vertices be $V=\{u, v_i;\ 1\le i\le t(n-1)\}$ and the edge set $E=\{u\, v_i;\ 1\le i\le t(n-1)\} \cup \{v_i\, v_{i+1};\ 1\le i\le t(n-1)\}$.

Let $f: V\rightarrow\{1,2,\ldots, t(n-1)+1\}$ such that $f(u)=1$, $f(v_i) = i+1 : 2\le i\le t(n-1)$. The edge weights are calculated as follows.

For $1\le i\le t(n-1)$, $\lambda(v_i\, v_{i+1}) = \frac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}} = (i+2)/(i+1)$.

For $1\le i, j\le t(n-1)$, clearly $\lambda(v_i\, v_{i+1}) \ne \lambda(v_j\, v_{j+1})$ and $i\ne j$.

If $\lambda(v_i\, v_{i+1}) = \lambda(v_j\, v_{j+1})$, then $i = j$, which is a contradiction and therefore all edge labels are distinct.

Now for $1\le i\le t(n-1)$, $\lambda(uv_{i+1}) = \frac{\max\{f(u), f(v_{i+1})\}}{\min\{f(u), f(v_{i+1})\}} =(i+1)/1=i+1$.

When $i\ne j$, for $1\le i\le t(n-1)$, Clearly $\lambda(uv_{i+1}) \ne \lambda(uv_{j+1})$

If $\lambda(uv_{i+1}) = \lambda(uv_{j+1})$, then $i=j$, which is a contradiction and therefore all edge labels are distinct.

Also for $1\le i\le t(n-1)$, clearly $\lambda(v_i\, v_{i+1}) \ne \lambda(uv_{i+1})$.

If $\lambda(v_i\, v_{i+1}) = \lambda(uv_{i+1})$, then $(i+2)/(i+1) = i+1$.

This implies $i^2 = -i+1$, which is a contradiction and therefore all edge labels are distinct. Hence the one vertex union of any cycle is ratio edge antimagic.

3. Conclusion: Here in this article the ratio edge antimagic labeling of C_n^+ ,comb, wheel graph, one vertex union of any cycle is shown.

References

1. J.A. Gallian 2012, A Dynamic Survey of graph labeling, Electr. J. combin. 19, 2012.
2. J. Jayapriya, D. Muruganandam, K.Thirusangu and Susil Lal Das, Analyzing Welding Process using Labeling Techniques , European Journal of Scientific Research Vol 79,Issue 2, june 2012 no. 253-257.
3. J. Jayapriya, D. Muruganandam and K. Thirusangu, Parametrical analysis of friction stir welding of dissimilar aluminum AA7075 and AA2024 using graph labeling, Indian Journal of Science and Technology, Vol. 5, No. 8 (2012), pp. 3223–3226 .
4. J.Jayapriya, K.Thirusangu, Max-min edge magic and antimagic labeling, European Scientific Journal, Vol. 3, No. 3 (2013), pp. 253–259.
5. Jayapriya J. and Thirusangu K., Ratio edge antimagic labeling for subdivision graphs, Applied Mathematical Sciences, Vol. 7, No. 117 (2013), pp. 5839–5845.