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NON-UNIFORM HEAT SOURCE/SINK EFFECT ON UNSTEADY LIQUID FILM FLOW OF MAGNETOHYDRODYNAMIC CASSON AND CARREAU FLUIDS

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Abstract

Numerical investigation is carried out to analyze the effect of non-uniform heat source/sink on magnetohydrodynamic (MHD) unsteady liquid film flow of non-Newtonian (Casson and Carreau) fluids past a stretching sheet. Similarity transformation is used to convert the partial differential equations to nonlinear ordinary differential equations. Further, the transformed equations are solved numerically by employing R-K based shooting technique. The effects of various controlling flow dynamical parameters on flow and heat transfer is examined and discussed in detail. It is found that flow and thermal boundary layers are non-uniform for Carreau and Casson fluids. Heat transfer performance of Carreau fluid is comparatively better than Casson fluid.

Keywords: MHD, thin film flow, Carreau fluid, Casson fluid, non-uniform heat source/sink.

1. Introduction

Boundary layer flow and heat transfer behavior of magnetohydrodynamic flows over a stretching surface is very important due to its practical applications in several engineering processes, shrinking wrapping, bundle wrapping, computer storage devices, hot rolling, and extrusion of sheet, gas turbine rotors, air cleaning machines, crystal growth processes. By keeping this in to view, in 1972 Carreau [1] presented a rheological model from molecular network theories. Himasekhar et al. [2] studied the effect of mixed convection on the flow over a vertical rotating cone. Akbar et al. [3] presented the dual solutions for magnetohydrodynamic stagnation flow of a Carreau fluid by considering the flow over a shrinking surface. Boundary layer analysis of Carreau fluid over a convectively heated stretching sheet is numerically studied by Hayat et al. [4]. Raju et al. [5, 6] studied the heat transfer behaviour of magnetohydrodynamic non-Newtonian flows by considering the flow over a stretching sheet and a rotating cone and found that heat transfer performance of the Carreau fluid is comparatively good in Carreau fluid.

Heat source/sink and Chemical reaction effects on mixed convection flow of Casson fluid over a stretching sheet was numerically studied by Hayat et al. [7]. Impact of homogeneous–heterogeneous reactions on viscoelastic flow with induced magnetic-field and nonlinear thermal radiation was illustrated by Animasaun et al. [8]. Effect of radiation and chemical reaction on non-Newtonian flow past a cone was numerically studied by Raju et al. [9]. Further, thermophoresis and Brownian moment effects on MHD Carreau fluid flow with thermal radiation was discussed by Sulochana et al. [10]. Cross diffusion effects on unsteady Casson fluid flow in the stagnation region was studied by Pushpalatha et al. [11]. Magnetohydrodynamic gyrotactic microorganisms contained non-Newtonian flow over a rotating cone and plate was discussed by Raju et al. [12]. Magnetohydrodynamic Casson flow past an exponentially shrinking sheet was studied by Nadeem et al. [13].

Non-Newtonian fluid flow, heat and mass transfer over a wedge with variable wall temperature and concentration was numerically investigated by Chamkha et al. [14]. Effect of non-uniform surface heat flux on non-Newtonian flow past a vertical plate was investigated by Hakim and Amin [15]. Babu and Sandeep [16] studied the MHD, Soret and Dufour effects on non-Newtonian flow over a non-uniform thickness stretching sheet. Modified kinematic viscosity model for Casson fluid flow within boundary layer formed on a surface at absolute zero was presented by Sandeep et al. [17]. Convective heat transfer in wall jet flow of Casson fluid with thermal radiation was illustrated by Sathish Kumar et al. [18]. Non-uniform heat source/sink effects on the MHD nanofluid flow over a stretching sheet embedded with dust particles was studied by Sulochana et al. [19]. Transpiration effects on free convective MHD flow over a vertical cone was investigated by Ravi Chandran and Ganapathirao [20]. Nadeem and Saleem [21] analytically studied the third grade fluid flow over a rotating vertical cone. Khan and Sulthan [22] analyzed the cross diffusion effects on double diffusive convection flow of non-Newtonian fluid caused by a cone. Mass flux effects on MHD Jeffrey flow were numerically studied by Abbasi et al. [23].

In this study, a numerical investigation is carried out to analyze the effect of non-uniform heat source/sink on magnetohydrodynamic (MHD) unsteady liquid film flow of Casson and Carreau fluids past a stretching sheet. Similarity transformation is used to convert the partial differential equations to nonlinear ordinary differential equations. Further, the transformed equations are solved numerically by employing R-K based shooting technique. The effects of various controlling flow dynamical parameters on flow and heat transfer is examined and discussed in detail.

2. Mathematical formulation

Two-dimensional, incompressible, electrically conducting, unsteady heat transfer of liquid film flow of Casson-Carreau fluids past a stretching surface is considered. The elastic sheet starts from a narrow slit, which is located at the origin of a coordinate system (x, y) as shown in Fig.1. Here *x*-axis is taken along the stretching surface with stretched velocity $u_w(x,t) = bx/(1-\alpha t)$, where b, α constants and y-axis is normal to it. The wall temperature is considered as $T_s(x,t) = T_0 - T_r(bx^2/2v_f)(1-\alpha t)^{-1.5}$, where T_0, T_r are the slit and reference temperatures and concentrations and v_f is the kinematic viscosity of the base fluid. A magnetic field of strength $B(t) = B_0(1-\alpha t)^{-0.5}$ is applied along the stretching sheet. Non-uniform heat source/sink is taken into account.

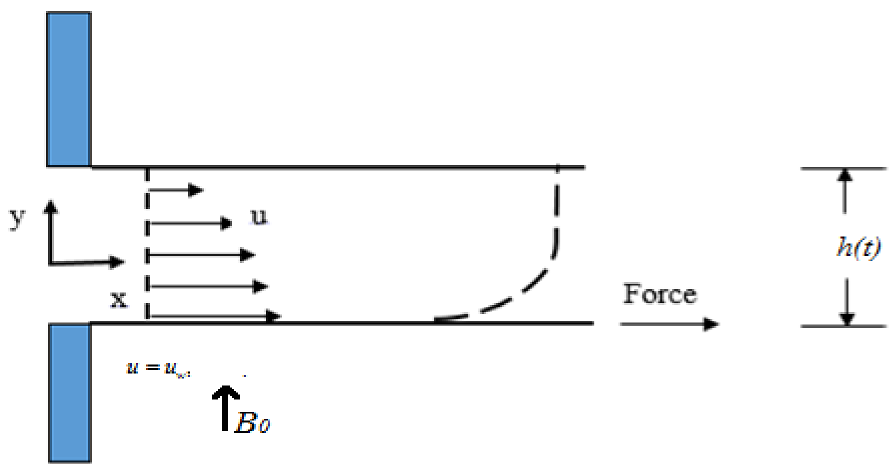

Fig.1 Physical model of the problem.

With the above assumptions, the governing conservation equations in unsteady state can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\left(\left(1+\delta^{-1}\right)\frac{\partial^2 u}{\partial y^2} + 1.5(n-1)\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}\right) - \sigma B^2(t)u, \tag{2}$$

$$(\rho c_p)\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + q''', \tag{3}$$

The boundary conditions for the present problem are

$$u = u_w, v = 0, T = T_s, \quad at\ y = 0,$$
$$u_y = 0, v = h_t, T_y = 0, \quad at\ y = h, \tag{4}$$

where u and v are the velocity components along x and y directions respectively, ρ is the fluid density, μ is dynamic viscosity, σ is the electrical conductivity, $B(t)$ is the applied magnetic field, T is the fluid temperature, k represent the thermal conductivity, ρc_p represent the heat capacitance, δ represent the Casson parameter, Γ is the time constant and n is the power-law index. The time and temperature dependent heat source/sink q''' is given by

$$q''' = \frac{k_f (T_s - T_0) u_w(x,t)}{x v_f}\left(A^* f' + B^* \frac{(T - T_0)}{(T_s - T_0)} \right), \tag{5}$$

where $A^* > 0$, $B^* > 0$ corresponds to internal heat generation and $A^* < 0$, $B^* < 0$ corresponds to internal heat absorption. To get inside analysis of the problem, we use following similarity transformation.

$$\eta = \frac{1}{\beta}\left(\frac{b}{v_f (1 - \alpha t)} \right)^{0.5} \text{y}, \psi = \beta \left(\frac{v_f b}{(1 - \alpha t)} \right)^{0.5} x f(\eta),$$
$$T_s = T_0 - T_r (b x^2 / 2 v_f)(1 - \alpha t)^{-1.5} \theta(\eta), \theta = \frac{(T - T_0)}{(T_s - T_0)}, \tag{6}$$

we define the stream function ψ as $u = \psi_y = bx(1 - \alpha t)^{-1} f'(\eta)$

and $v = -\psi_x = -\beta \sqrt{vb(1 - \alpha t)^{-1}} f(\eta)$.

with the help of Eqs. (5) and (6), the Eqs. (2) and (3) transformed as

$$\left(1 + \delta^{-1}\right) f''' + 1.5(n-1) We f''^2 f''' + \lambda \left(f f'' - f'^2 - S\left(f' + \frac{\eta}{2} f'' \right) \right) - M f' = 0, \tag{7}$$

$$\theta'' - \Pr \lambda \left(2 f' \theta - f \theta' + \frac{S}{2}\left(3\theta + \eta \theta'\right) \right) + \left(A^* f' + B^* \theta \right) = 0, \tag{8}$$

with the associated boundary conditions are

$$f(0) = 0, f'(0) = 1, \theta(0) = 1,$$
$$f(1) = S/2, f''(1) = 0, \theta'(1) = 0, \tag{9}$$

where primes denotes the differentiation with respect to η, the characterized Prandtl number, magnetic field parameter, unsteadiness parameter and Weissenberg number are given as

$$\Pr = \frac{v_f}{\alpha_f}, M = \frac{\sigma B_0^2}{b\rho_f}, S = \frac{\alpha}{b}, \mathrm{We} = \frac{\Gamma^2 x^2 b^3}{(1-\alpha t)^3}, \qquad (10)$$

where $\lambda = \beta^2$ is the dimensionless film thickness and β is an unknown constant defined by

$$\beta = \left(\frac{hb}{v_f}\right)(1-\alpha t)^{-0.5}, \qquad (11)$$

The physical quantities of engineering interest, the skin friction at the surface C_{fx} and local Nusselt number Nu_x are given by

$$\mathrm{Re}_x^{0.5} C_{fx} = \left[\left(1+\delta^{-1}\right) f''(0) + 1.5(n-1)We\left(f''(0)\right)^3\right], \mathrm{Re}_x^{-0.5} Nu_x = -\theta'(0), \qquad (12)$$

where $\mathrm{Re}_x = \frac{u_w x}{v_f}$ is the local Reynolds number.

3. Results and Discussion

The nonlinear ordinary differential equations (7) and (8) with the restrictions (9) are resolved numerically by using R-K based shooting process. In this paper, we have chosen the non-dimensional parameter values as $S = 0.5$, $Pr = 6.8$, $M = 1$, $We = 2$, $\lambda = 0.3$, $A^* = 0.5$, $B^* = 0.5$, $Sc = 0.6$. These values are maintained as invariable in this study unless the varied parameters as depicted in the figures. In this investigation, the solid line indicates the Casson fluid and dashed line indicates the Carreau fluid flow.

Figs. 2-5 depict the effect of magnetic field parameter on velocity, temperature, friction factor and local Nusselt number of Carreau and Casson flows. It is evident that rising values of magnetic field parameter boost the temperature profiles of both Casson and Carreau fluids. But reverse trend has been observed in velocity field, wall friction and heat transfer rate. Physically, increasing values of the magnetic field parameter develops the resistive force opposite to the flow field. Due to this reason we have noticed above trends. It is also observed that the influence of Lorentz force is high on Carreau flow when compared with Casson flow.

The influence of film thickness parameter on velocity, temperature, friction factor and local Nusselt number of Carreau and Casson flows is displayed in Figs. 6-9. It is clear that increasing values of film thickness parameter boosts the heat transfer rate, in particular heat transfer rate of Casson fluid is high when compared with Carreau fluid. We observed depreciation in momentum, thermal boundary layers along with skin friction coefficient for rising values of film thickness parameter. It is also evident that velocity boundary layer of Carreau fluid is highly affected by increasing the film thickness. The similar type of results has been observed for boosting values of unsteadiness parameter, which is displayed in Figs. 10-13. Generally, increasing the unsteadiness in the flow fluxgates the buoyancy forces acts on the flow. Due to this reason, we noticed above trends.

Figs. 14-17 illustrate the influence of space and temperature dependent heat source/sink parameters on temperature field and local Nusselt number of Carreau and Casson flows. It is observed that rising values of non-Uniform heat source/sink parameter enhance the thermal boundary layer thickness and declines the local Nusselt Number of both fluids. This may happen due to the fact that positive values of non-uniform heat source/sink parameters acts like heat source parameters. Validation of the present results with published results is depicted in Table.1.

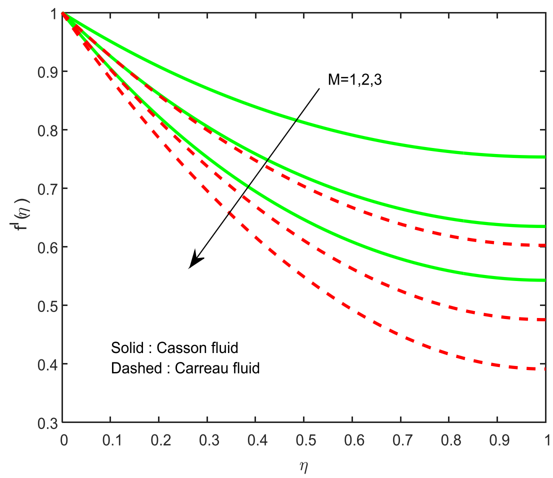

Fig.2 Magnetic field effect on velocity profiles.

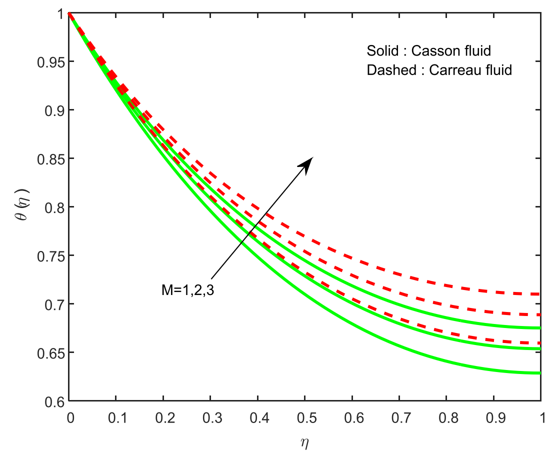

Fig.3 Magnetic field effect on temperature profiles.

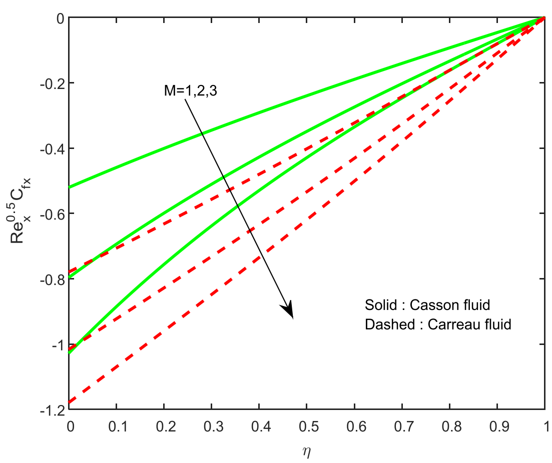

Fig.4 Magnetic field effect on friction factor.

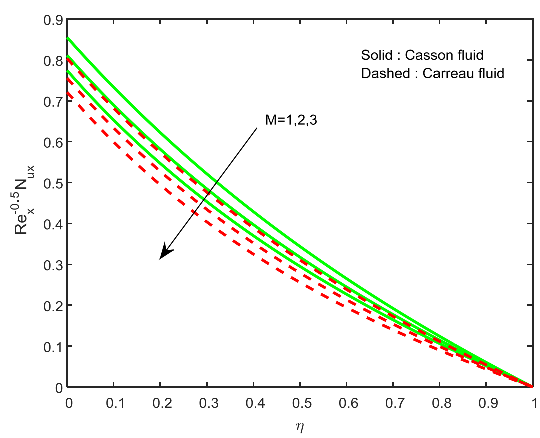

Fig.5 Magnetic field effect on local Nusselt number.

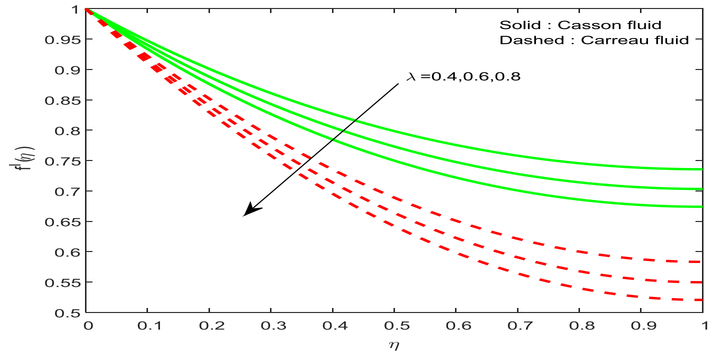

Fig.6 Effect of film thickness parameter on velocity profile.

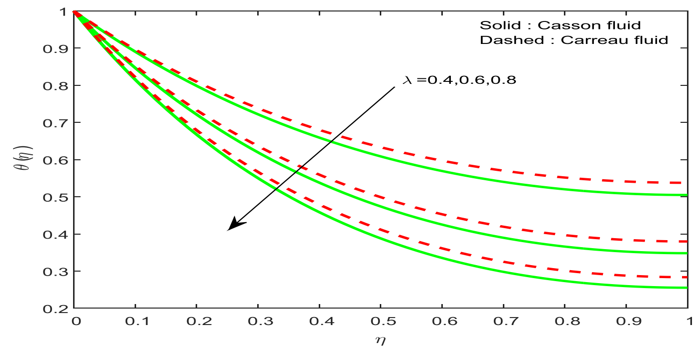

Fig.7 Effect of film thickness parameter on temperature profile.

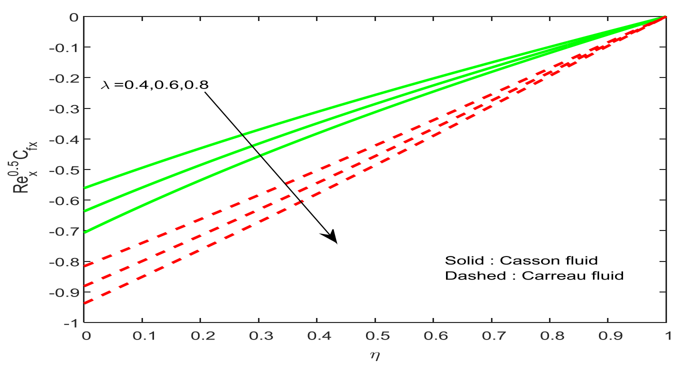

Fig.8 Effect of film thickness parameter on friction factor.

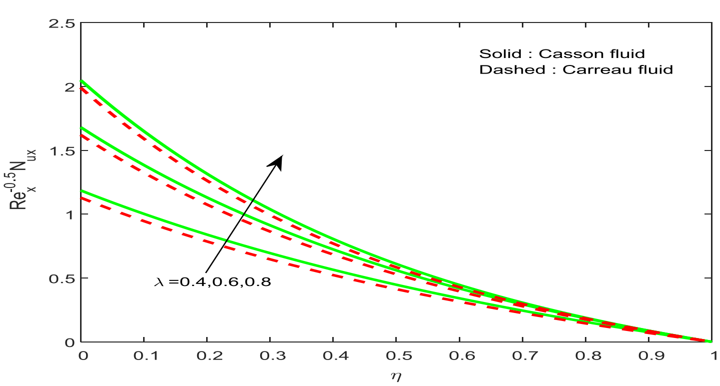

Fig.9 Effect of film thickness parameter on local Nusselt number.

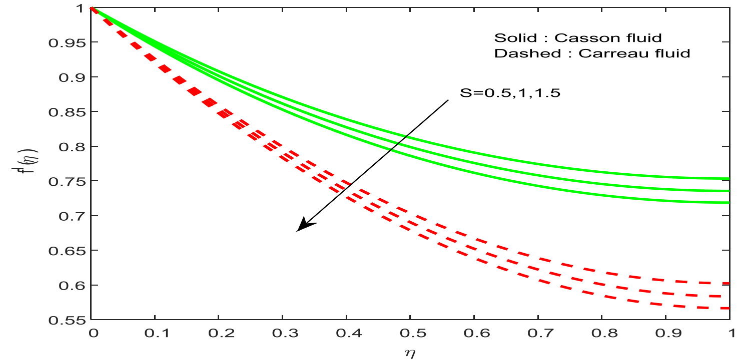

Fig.10 Effect of unsteadiness parameter on velocity profile.

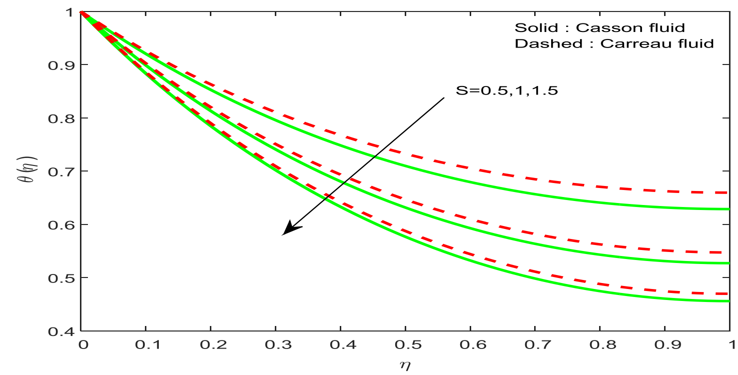

Fig.11 Effect of unsteadiness parameter on temperature profile.

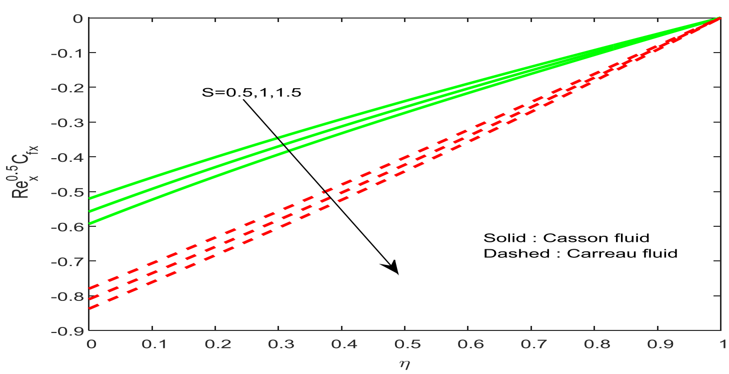

Fig.12 Effect of unsteadiness parameter on temperature profile.

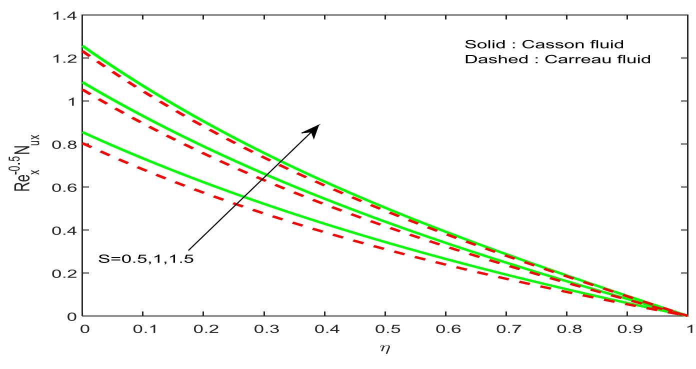

Fig.13 Effect of unsteadiness parameter on local Nusselt number.

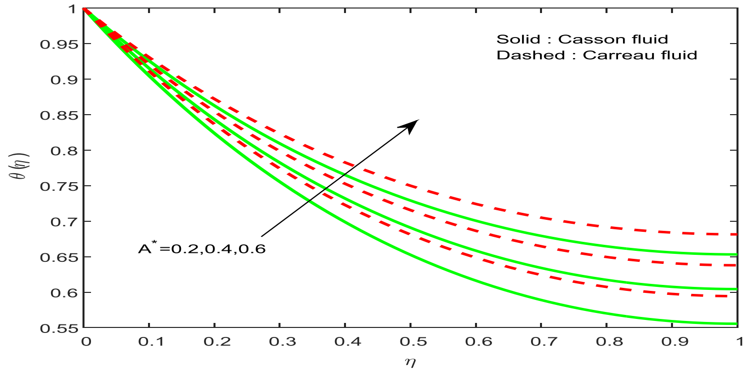

Fig.14 Effect of non-uniform heat source/sink parameter on temperature profile.

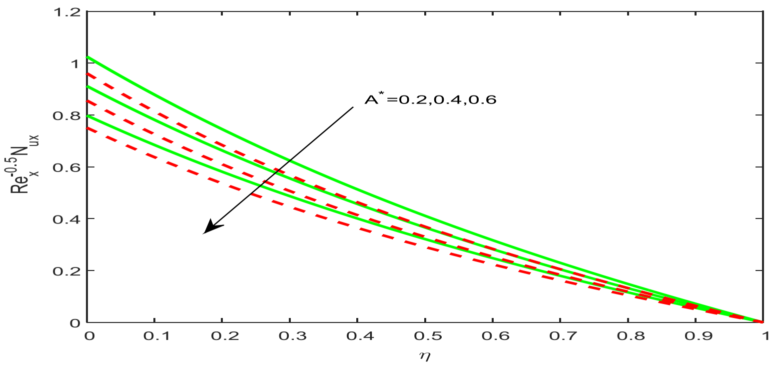

Fig.15 Effect of non-uniform heat source/sink parameter on local Nusselt number.

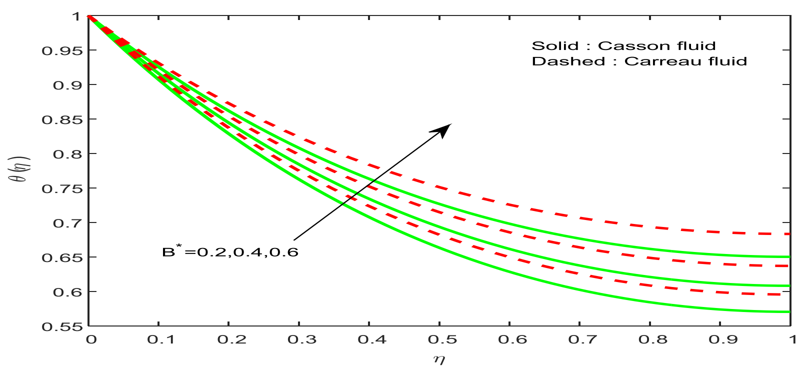

Fig.16 Effect of non-uniform heat source/sink parameter on temperature profile.

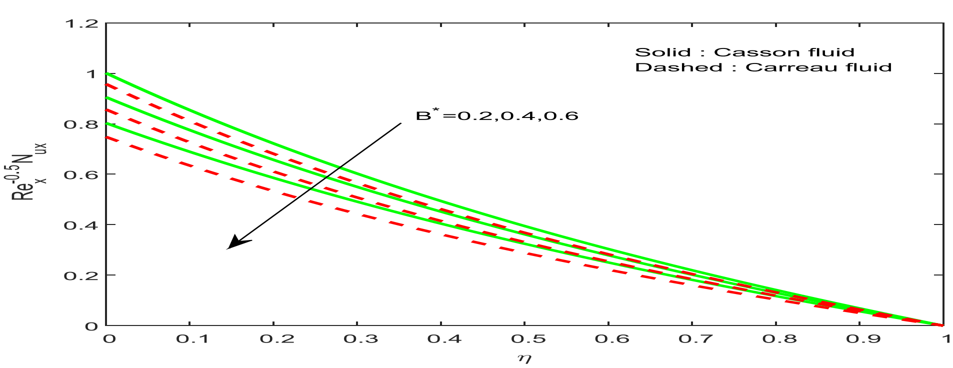

Fig.17 Effect of non-uniform heat source/sink parameter on local Nusselt number.

Table 1. Comparison of values $-\theta'(0)$ for different values of S when $M = 0, \delta \to \infty$ and $\Pr = n = 1$.

S	Xu et al. [34]	Present Results
1.0	2.677222162	2.677222
1.2	1.999591426	1.999591
1.4	1.447754361	1.447754
1.6	0.956697844	0.9566978
1.8	0.484536632	0.4845366

4. Conclusions

This study presents the numerical exploration of flow and heat transfer of magnetohydrodynamic non-Newtonian thin film flows over a stretching sheet in the presence of non-Uniform heat source/sink. Similarity transformation is used to convert the partial differential equations to nonlinear ordinary differential equations. Further, the transformed equations are solved numerically by employing R-K based shooting technique. The effects of various controlling flow dynamical parameters on flow and heat transfer is examined and discussed in detail. Numerical findings are as follows:

1. Carreau fluid is highly influenced by the magnetic field effect when compared with the Casson fluid.
2. Flow and thermal boundary layers of Casson and Carreau fluids are non-uniform.
3. Non-uniform heat source/sink regulates the thermal boundary layer.
4. Rising values of film thickness parameter encourages the heat transfer arte.
5. Velocity and flow fields are depreciates by rising values of unsteadiness parameter.

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