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RECONSTRUCTION AND RESTORATION OF SUPER RESOLUTION IMAGE BYWAVELETS

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Abstract:

It is proposed to reconstruct and restore high resolution image from several low resolution images of an object by wavelet method. This method is applicable to biological object as well as space objects. Super resolution is the process of enhancing resolution of image slightly beyond different limit by extrapolating high frequency details. There are three steps for attaining super resolution namely registration, interpolation and restoration. In registration one of the low resolution image is chosen as standard and for other images the details of motion of the camera with respect to object are furnished in terms of three parameter translation along horizontal axis, translation along vertical axis and rotation angle. Besides registration for attaining high resolution wavelet based interpolation is affected. Final step of super resolution is restoration. Denoising is implemented by thresholding. The resulting super resolved image is restored by inverse imaging operation.

Keywords: Diffraction limited resolution, super resolution, directional interpolation, Markov tree model, denoising by thresholding.

Introduction

Super resolution [1] is the process of recovering high resolution image from several low resolution images of the same scene at different sub pixel shifts. In super resolution by image reconstruction and restoration, we obtain a resolution slightly beyond diffraction limit. So super resolution is the technique of extrapolation of band limited image.

One of the most important steps in designing super resolution image is the registration [2]. In registration we select a particular low resolution image as standard. The motion details of other low resolution images are furnished in terms of three parameters horizontal shift h along X axis, vertical shift 'l' along Y axis and rotation angle ϕ about Z axis. If x, y denotes the position coordinates before motion (x', y') after the motion, they are related as follows

$$x' = x cos\phi - y sin\phi + h \qquad (1)$$

$$y' = x sin\phi + y cos\phi + l \qquad (2)$$

If $f(x', y')$ denotes the image function after arbitrary motion, then

$$f(x', y') = f(x cos\phi - y sin\phi + h + x sin\phi + y cos\phi + l) \qquad (3)$$

and expanding $cos\phi$ and $sin\phi$ in terms of Maclacurinseries

$$cos\phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} + \cdots, \qquad (4)$$

$$sin\phi = \phi - \frac{\phi^3}{3!} + \cdots, \qquad (5)$$

And retaining powers of ϕ, up to square term

$$f(x', y') = f(x\left(1 - \frac{\phi^2}{2}\right) - y(\phi) + h + x\phi + y\left(1 - \frac{\phi^2}{2} + l\right) \qquad (6)$$

Expanding $f(x', y')$by Taylor's series, retaining only first order terms

$$f(x', y') = f(x, y) + \frac{\partial f}{\partial x}\left(h - x\frac{\phi^2}{2} - y\phi\right) + \left(l + x\phi - y\frac{\phi^2}{2}\right)\frac{\partial f}{\partial y} \qquad (7)$$

The error$E = |f(x', y') - f(x, y)|$

Assuming all possible values of h, l and ϕ, square of the error is given by

$$E^2 = \sum |(h - y\phi - \frac{x\phi^2}{2})\frac{\partial f}{\partial x} + (l + x\phi - y\phi^2)\frac{\partial f}{\partial y}|^2 \qquad (8)$$

Σ means summation over the overlapping regions of $f(x, y) and\ f(x', y')$ and minimum error implies that

$$\frac{\partial E}{\partial h} = 0, \frac{\partial E}{\partial l} = 0, and\ \frac{\partial E}{\partial \phi} = 0 \qquad (9)$$

The relative displacement of undersampled images is estimated with interpixel accuracy. Usually LR images are (4×4) pixel frames.HR images are (8×8) pixel frames. Thus approximate motion details are obtained and this can be incorporated in each low resolution image.

Interpolation based on wavelet- Directional interpolation

It is observed that spatial frequency information in a particular frequency range can be processed independent of information in any other frequency range. In most of natural images, there will be correlation among neighboring pixel, but not in specific direction. This principle is made use of in wavelet method of interpolation.

The image function in one dimension can be decomposed into an approximate one at a particular resolution and a series of fluctuation at higher scales of resolution. The image function is thus expanded,

$$f(x) = \sum_{k} C_{j_0,k}\, \phi_{j_0,k} + \sum_{j_0 \le j \le J} \sum_{k} d_{j,k}\, \psi_{j,k} \qquad (10)$$

$C_{j_0,k}$ and $d_{j,k}$ are scaling function coefficient and wavelet coefficients.

The one dimensional transform can be extended to two dimensions. In two dimensions, we arrive at 1) a two dimensional scaling function

$$\phi(x,y) = \phi(x)\phi(y) \qquad (11a)$$

2) three wavelets a) horizontal wavelet

$$\psi^H(x,y) = \psi(x)\phi(y) \qquad (11b)$$

b) Vertical wavelet

$$\psi^V(x,y) = \phi(x)\psi(y) \qquad (11c)$$

and diagonal wavelet

$$\psi^D(x,y) = \psi(x)\psi(y) \qquad (11d)$$

These wavelet measure intensity variation along different orientations namely horizontal, vertical and diagonal including NE-SW and NW-SE. In directional interpolation, intensity variations in these orientations are interpolated and then reassembled for reconstructing super resolution images. The directional interpolation scheme is excellent for analysis of texture part of the image where intensity variation is of smaller extent.

For edge enhancement or getting sharper image adaptive directional interpolation can be implemented. Once pixel density is more at the edges of the image, a weight factor is used to mark denser pixel density along the edges or ridges. Then interpolation along different orientation is affected. This interpolation is called adaptive directional interpolation. As a result of adaptive directional interpolation images with clarity and sharp edges are obtained.

For getting sharper boundary of the images super resolution images can be reconstructed using Hidden Markov tree model. Using statistical analysis, image function f(x) can be realized as a random field with joint probability density function f(x). We can model local image pixels in spatial domain as Markov random field model. The probability density function consists of two Gaussian states S and L. S corresponds to a zero mean low variance Gaussian function and L corresponds to a zero mean high variance Gaussian function. So that combined probability density function is expressed as

$$f(\omega_L) = {P_i}^s g(\omega{:}\,0, {\sigma_{s_i}}^2) + {P_i}^L g(\omega{:}\,0, {\sigma_L}^2)$$

$$\text{with } {P_i}^s = 1 - {P_i}^L \; with \quad {\sigma_L}^2 > {\sigma_s}^2 \qquad (12a)$$

${P_i}^s$is the probability density function for state S , ${P_i}^L$ probability density function for state L, σ_s is the root mean square variance for state S. In wavelet transform wavelet coefficient measures the differences at different scales of resolution. The portions of image which do not vary significantly from scale to scale (smooth region) will be expressed by few coefficients at coarse scale. The portions of images that do vary from scale to scale are regions around edges and are represented by large coefficients at each scale of resolution in the wavelet transform. We assume that wavelet coefficients corresponding state S have peaky and heavy tailed marginal distribution and magnitudes of wavelet coefficients decay exponentially across scales. Treating probability density function of state S is exactly similar to that of Markov tree model, probability density function at finest scales vary according to the relation

$$P({\omega_s}^k) = \frac{1}{\sigma_{c,m}\sqrt{2\pi}} \frac{({\omega_c}^k - {\mu_{j,m}}^k)}{2{\sigma_{s,m}}^k} \qquad (13)$$

where${\omega_c}^k$ are wavelet coefficient at the finest scale ${\sigma_{c,m}}^k$ and ${\mu_{c,m}}^k$ are obtained from a standard set for implementing the method, we create a hidden Markov tree model of image statistics of a similar image(training set). This model will consist of a transition probability with a certain mean and variance. We collect information about the changes occurring in the training set wavelet coefficient from parent to child. We determine the wavelet transform of the image to be reconstructed. Use the appropriate equation and wavelet coefficient at higher scales of resolution using probability density function variation. Finding the significant wavelet coefficient at higher scales of resolution, super resolution images can be reconstructed by Markov tree model.

Restoration of image by Wavelet method

When an object, which is equal to the captured image of 100% resolution, is imaged by an LR imaging device, the resulting image function y is related to object function x by the relation

$$\vec{y} = A\vec{x} + \vec{n} \qquad (14)$$

where$\vec{n}$ is the noise function, A represents the transforming operation from HR image to LR image.$\vec{x}$ represents $N^2 \times 1$ column matrix. A is $M^2 \times N^2$matrix and A is the product of (1) down sampling matrix B caused by increasing the size of pixel, (2) H blurring matrix associated with the imaging system and in general it can be represented by point spread function,(3) warping matrix S and this represents the spatial shifts between LR image and the standard image. $\vec{n}$is the noise factor caused by imaging operation.

The general method of super resolution reconstruction is combining all the LR images and finding HR image x from them. First step in restoration or attaining high resolution image is denoising. For denoising signals less than a minimum value or threshold is ignored. Another method is minimizing the difference between LR image y and transformed object. For that consider the residual represented by

$$R = \sum_{k=0}^{\sim} |(y_k - (DHS)_{kl} x_l)|^2 \qquad (15)$$

Minimizing R, we can remove noise factor. Neglecting noise factor, LR image is related to high resolution image as

$$y = Ax$$

where A is transformation matrix. Usually A is a null matrix. By regularization method A is made singular and

$$x = A^{-1}y \qquad (16)$$

whereA^{-1} is the inverse operation. Several experiments can be carried out to test the performance of suggested regularized algorithm for restoration of image. By denoising operation of thresholding, the image can be made sharper. Thresholding in fact introduces ringing effect. This ringing effect can be minimized by cyclic spinning which make use of periodic time invariance of wavelet transform. Noisy signal is shifted in time and then shifted back. By zeroing out wavelet coefficient less than threshold, noises are eliminated and significant wavelet transforms are retained. Thus by this cycle spinning operation, we get sharp images. Our algorithm for super resolution reconstruction and restoration by wavelet method we get sharp images of high resolution. This method is suitable for biological image rapid variation all along the ridges and edges of the object. This method is applicable to images photographed from satellites.

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