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METHOD FOR APPROXIMATING THE WORKSPACE OF THE PARALLEL ROBOT

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Abstract:

This paper presents various methods for the determination of the workspace of parallel robots. Different types of workspaces are defined, and algorithms for calculating them are presented. We propose a universal method of approximating the working space of a robot by reducing to finding the solution set of system of nonlinear inequalities. The method obtains a set of multidimensional boxes approximating the working space of the robot with the given accuracy. As a result of proposed algorithm obtained the outer and inner approximations, which defined as a set of parallelepipeds. This method is useful for visualization and further processing to produce a variety of shells workspace. Application of the method is illustrated by solving this problem for the realistic planar robot with parallel structure.

Keywords: parallel robots, robot workspace, systems of non-linear inequalities.

Introduction.

Along with the traditional serial kinematic schemes of the robot, that allows independent variation of the coordinates of nodes, in recent years a new class of robots with parallel kinematics attracted attention. In parallel robots changing in one node may require simultaneous coordinated changes in others. Parallel robots have advantages w.r.t.t. traditional ones: simple and robust mechanics, high precision of movements, low mass etc.

Finding the workspace of a parallel robot (a set of possible positions of the working tool) is one of the most important problems in robotics. There are two approaches to this problem developed so fat: geometric methods [1-4] and discretization [5-7]. Geometric approaches are quite efficient but can be applied to relatively simple robots. Approaches based on discretization can be applied to more wide class of parallel robots. But methods of this sort are quite sophisticated and require careful modelling analysis of the robot’s trajectory.

In this paper we propose the new universal method of approximating the working space of a parallel robot by reducing it to finding the solution set of system of nonlinear inequalities. For a given ε the method obtains a set of multidimensional boxes approximating the working space of the robot. We also provide an example of application of the proposed method to working space approximation for the realistic robot.

1. Workspace limits, representation and type

Parallel manipulators motions can be restricted by different factors: mechanical limits on passive joints, self-collision between the elements of the robot, limitations due to the actuators and singularity varieties that may split the workspace into separate components. The main problem with the workspace representation of parallel robots is that the limitations on the d.o.f. are all usually coupled. Hence for robots having more than 3 d.o.f. there will be no possible graphical illustration of the robot workspace. This is not usually the case with serial robots. For example, the workspace of a 6 d.o.f. serial robot with a concurrent axis wrist may be represented by the 3D volume that may be reached by the center of the wrist - this illustrates the translations, and by the surface that may be reached by the extremity of the end-effector (which illustrates 2 degrees of freedom in rotation). The 3D volume depends only on the motion capability of the first three actuated joints, while the orientation uses only the last three joints. A graphical representation of the workspace of parallel robots will be possible only for 3 d.o.f robots. For robots with $n > 3$ d.o.f., workspace representation will be possible only if we fix $n - 3$pose parameters. According to which types of parameters are fixed or to the constraint we impose on the parameter, we will obtain different typesof workspace.

The most usual types of workspace are:

− constant orientation workspace or translation workspace: all possible locations of the operating point C of the robot that can be reached with a given orientation 213

− orientation workspace: all the possible orientations that can be reached while C is in a fixed location

− maximal workspace or reachable workspace: all the locations of C that may be reached with at least one orientation of the platform.

− inclusive orientation workspace: all the locations of C that may be reached with at least one orientation among a set defined by ranges on the orientation angles. The maximal workspace is a particular case of inclusive orientation workspace for which the ranges for the orientation angles are $[0, 2\pi]$

− Total orientation workspace: all the locations of C that may be reached with all the orientations among a set defined by ranges on the orientation angles

− Dextrous workspace: all the locations of C for which all orientations are possible. The dextrous workspace is a particular case of total orientation workspace, the ranges for the rotation angles being [0, 2π].

− Reduced total orientation workspace: all the locations of C that may be reached with a subset of the orientation angles that may have any value in defined ranges, while the others may have arbitrary values. Such a workspace may be important for applications that do not involve all the d.o.f. of the robot. For example, if a 6 d.o.f. robot is used as a 5-axis machine-tool, the φorientation parameter is not important. In this chapter we will focus on workspaces that are limited only by geometrical constraints. But other constraints may limit the useful robot workspace: for instance we have already seen that a threshold on the value of a singularity index may play a role in limiting the workspace of therobot [8]; Kumar [9] introduces the notion of controllably dexterous workspace as the subset of the dextrous workspace which does not containany singular configuration. The workspaces we will calculate in this chapter will always include such specialized workspaces. Even if we assume a perfect knowledge of the robot geometry, calculation of the workspaces is in general a complex task. We will thus suppose that this geometry is perfectly known, but we must remember that manufacturing errors may have an influence on the workspace.

The most usual orientation parameters are the standard Euler angles ψ, θ, φand the pitch, yaw, roll angles. A problem with the Euler angles is that they exhibit a representation singularity for θ= 0 as in this configuration the rotation angle is ψ+ φ. To solve this problem, Bonev[10] propose using modified Euler angles, the tilt and torsion angles, describing the orientation by a first rotation of angle φaround the z axis, followed by a rotation of angle θaround y, and then a rotation of angle ψ, the torsion, around the new z axis. Bonev then discussed the best way to represent an orientation workspace. He considered 3 possibilities:

− a coordinate system whose axes represent the three Euler angles

− a spherical coordinate system, where φ, θare the azimuth and zenith angles and ψis the ray length

− a cylindrical coordinate system, where φ, θare the circular coordinates and ψis the z coordinate

Bonev concluded that the last choice offers the best orientation workspace representation. As an alternative Pernkopf proposed the use of the Euler parameters which may offer a better visibility for some motions [11].

2. Workspace calculation methods

Various approaches may be used to calculate the workspace of a parallelrobot. We will summarize these methods in the next sections.

Geometrical approach. The purpose of this method is to determine geometrically the boundary of the robot workspace. The principle is to deduce from the constraints on each leg a geometrical object Wlthat describes all the possible locations of **X** that satisfy the leg constraints. One such object is obtained for each leg and the robot workspace is constituted of the intersection of all Wl. Consider for example a chain of a 6−UPS robot, and assume that the only constraint is that the length of the leg has to lie in the range [ρmin, ρmax]: the extremity B of the chain is constrained to lie in a volume which is comprised between the spheres with center A and radii ρmin, ρmax(figure 1). If we further assume that the orientation of the end-effector is constant, then each leg constrains C to lie in a similar region Wil whose center is obtained by translating Ai by the constant vector BiC. The works pace of the robot is then obtained as the intersection of the six Wil .We may also consider a PRRS chain in which the prismatic actuator is connected by a universal joint to a link of length l: the volume V that may be reached by B is constituted of part of a cylinder with radius l and height ρmax−ρmin, topped by two hemispheres with radius l (figure 1). If the orientation is constant we can get Wil from V by a simple translation.

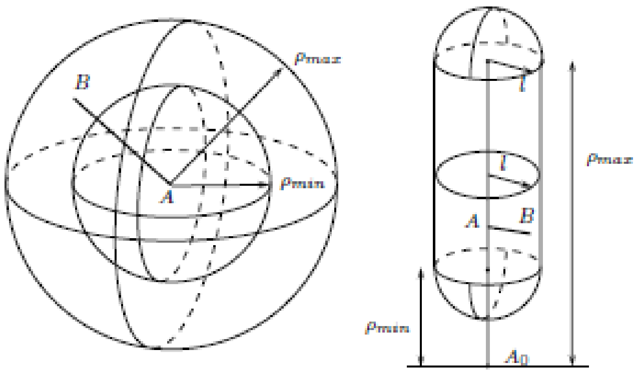

Figure 1.The volumes that can be reached by the B point of the chain of a parallel robot.

The choice of the task space may be important to simplify the calculation. For example Husty[12], shows that homogeneous coordinates with an appropriate kinematic mapping may be appropriate, since they allow a simple representation of the constraints for a planar 3−RPR robot. This approach is usually restricted to 3D workspace and is able to deal mostly with the constraints on the joint coordinates, although we will see that joint limits and interference constraints may be taken into account in some cases.

The main interest of the geometrical approach is that it is usually very fast and accurate, and provides a minimal representation of the workspace which may be used to calculate efficiently some characteristics of the workspace, such as its volume. Its drawbacks are that it must be tailored to the considered robot, it may be difficult to take all constraints into account, and the minimal representation of the workspace may not be the most appropriate for tasks such as motion planning. A possible simplified approach is to compute only slices of the workspace, and to approximate the section of Wl by polygons. This approach requires a good computational geometry library that is able to execute Boolean operations, whether intersection, union, or difference, on polygons with arbitrarily many edges.

Discretisation Method. Numerous papers dealing with workspace calculation use methods based on the discretisation of the pose parameters, in order to determine the workspace boundary. In this discretisation approach, the workspace is covered by a regular grid, either cartesian or polar, of nodes. Each node is then tested to see whether it belongs to the workspace. The boundary of the workspace is constituted of the set of valid nodes, where at least one close neighbor does not belong to the workspace. The advantage of this method is that it allows one to take into account all constraints. But this approach has many drawbacks:

− the accuracy of the boundary depends on the sampling step that is used to create the grid, the computation time grows exponentially with the sampling step, so that there is a limit on the accuracy.

− problems occur when the workspace possesses voids.

− the boundary representation may involve a large number of nodes

− the boundary is used for different operations such as determination of the workspace volume, inclusion of a trajectory in the workspace, etc. When performed on a boundary represented by a discrete set of poses, these operations are computer intensive. To avoid this drawback, Chablat (79) proposed storing the workspace representation as an octree structure that allows faster motion planning and volume calculation. Still, getting the structure is computer and memory intensive. A web page allows us to use this method to calculate on-line the workspace of 6−UPS and 6−PUS robots.

Numerical method. Another approach to workspace calculation was suggested by Jo [14].Taking the constraints on the joint coordinates into consideration, he transformed the inequalities that are imposed by these constraints into equalities by introducing extra variables. He then considered the generalized coordinates(vector **X**), the joint coordinates (vector **Θ**) and the variables (vector**w**) that are introduced by the transformation of the inequalities into equalities. Let **q** be the vector that is constituted of all of these unknowns. The structure of the mechanism leads to constraint equations on the components of **q** which may be written in implicit form as Φ(**q**) = 0. Let JΦbe the jacobian of the system, i.e. the matrix:

$$J_{\Phi}=\left(\left(\frac{\partial \Phi}{\partial X},\frac{\partial \Phi}{\partial \Theta},\frac{\partial \Phi}{\partial w}\right)\right) \qquad (1)$$

The workspace boundary is obtained as the set of vectors **q**, such that for agiven **X**, there will be not be a unique set of vectors **Θ**,**w**. In other words,the rank of the matrix:

$$\left(\left(\frac{\partial \Phi}{\partial \Theta}, \frac{\partial \Phi}{\partial w}\right)\right) \quad (2)$$

is lower than its dimension. A numerical procedure is then used to calculatethe pose of the platform where this condition is satisfied. We note, however, that Jo illustrated this approach only for the simple case of the calculation of the constant orientation workspace of a 6−UPS robot. The introduction of other constraints limiting the workspace would lead to a jacobian so large as to render the procedure quite difficult to manage. Adkins [15] and Haugh[16] manage to find a point on the boundary, and use a numerical continuation method to follow the boundary. Although this approach is general, Adkins and Haugh restrict their calculation for a constant orientation workspace, as the general problem will be very complicated. Butthese authors were able to predict the singularity barriers that may splitthe workspace into different aspects (an aspect is a maximal singularity-free component of the workspace). Instead of using a continuation method to follow the boundary, some authors have proposed formulating that pointas a constrained optimization problem [17].Another method is based on the principle that, for a pose on the boundaryof the workspace, the velocity vector of the moving platform cannot have a component along the normal of the boundary. Agrawal [18] and Kumar[19,20] use this method for manipulators with only revolute joints, andapplied it to various planar robots to calculate the maximal workspace and the dextrous workspace. The main drawbacks of this method are that prismaticactuators cannot be considered, and it is quite difficult to introduce the notions of mechanical limits and of interference between links. An efficient method based on interval analysis is described in the interval analysis appendix. It has the advantages of being able to deal will almostany constraint and any number of d.o.f., and has proved to be efficient in computing the most difficult case of 6D workspaces of 6−UPS robots. However it provides only an approximation to the workspace (but up to anarbitrary accuracy and with an error bound on the error) and is relatively computer intensive. Finally let us mention two special cases: micro-robots and wire robots. Arai [21] considers the micro-robot case; he supposes that the actuatormotions are small enough so that he may consider the inverse jacobian to be constant. Moreover, as the orientation changes are small, he considers only the part $J_t^{-1}$of the inverse kinematic jacobian corresponding to the translatory motions, and writes for each link j

$$\Delta^2\Theta_a^j = \Delta X^T J_t^{-T} J_t^{-1} \Delta X \quad (3)$$

This relation means that, for each link and for a fixed maximal$\Delta\theta_a^j$, thecenter of the platform is inside a zone which is bounded by a quadric, andthe workspace is the intersection of these quadrics.

3. Mathematical formulation

Assume that the working space $X \subseteq R^n$,$n = 2,3$ of a parallel robot is defined by a system of inequalities

$$\begin{cases} g_j(x) \leq 0, j = 1, \dots, m, \\ a_i \leq x_i \leq b_i, i = 1, \dots, n. \end{cases} \quad (4)$$

From (4) follows that $X \subseteq P$, where $P = \{x : a_i \leq x_i \leq b_i\}, i \in \overline{1,n}$— n-dimensional parallelepiped (box). It is assumed that $g_j(\cdot) : R^n \to R, j \in 1,m$ satisfy Lipschitz condition on the set P with a constant L, i.e. for any $x_1, x_2 \in P$:

$$| |g_i(x_1) - g_i(x_2)| \leq L|x_1 - x_2|, i = 1, \dots, n \quad (5)$$

To solve the problem (4) means to somehow approximate the set $X \subseteq R^n$ of its solutions.

Cover is the set of n-dimensional boxes Pi, i = 1,...,k, such that

$$P \subseteq \cup_{i \in \overline{1,k}} P_i \quad (6)$$

where each Pi, $i \in \overline{1,k}$ satisfies at least one of the three conditions:

$g_j(x) > 0$ for some $j \in \overline{1,m}$ and all $x \in P_i$, (7)

$g_j(x) \leq 0$ and all $j \in \overline{1,m}$ and all $x \in P_i$, (8)

(7) and (8) are not valid and $d(P_i) \leq \delta$, (9)

where δ is apredefined positive value, which determines the accuracy of the approximation.

Denote by $I \subseteq \overline{1,k}$ a set of indices that Pi on condition $i \in I$ satisfies the relation (8) or (9). In the sequel we call the collection of these parallelepipeds the approximation family, and let Q denote their union: $Q = \cup_{i \in I} P_i$.

Theorem. Suppose that boxes P_i, i = 1,...,k form a cover. Then

$$X \subseteq Q \subseteq X_\varepsilon, \quad (10)$$

where $\varepsilon = \delta L$ and $X_\varepsilon = \{x \in P : g_j(x) \leq \varepsilon\}$. In the sequel $X_\varepsilon = \{x \in P : g_j(x) \leq \varepsilon\}$ will be called ε-feasible set. In [9] it is shown that under the assumptions (5) the set X_ε tends (in the Hausdorff metric) to X when ε tends to zero. According to (8) in the limit of Q will also aim to X.

4. The method of the cover construction

The method is based on ideas proposed in [21]. A the first step conditions (7) - (9) are checked or the original n-dimensional box P. Ifat least one of them is satisfied, the box is added to the cover. If the box is satisfied the conditions (8), (9) it is added to the approximation family. Otherwise, it is divided along the longest edge into two equal boxes. Then the described process is repeated for each of the obtained parallelepipeds. The process ends after all created boxes were eliminated.

The algorithm requires checking conditions (7) and (8). It is assumed that there are procedures for determining the global maximum and minimum of functions gj(x),j = 1,...,m on the n-dimensional box:

$m_j(P_i) = min_{x \in P_j} g_j(x)$,$m_j(P_i) = max_{x \in P_j} g_j(x)$ (11) Then, if$m_j(P_i) > 0$ for at least one j ∈ 1,...,m, then the condition (7) holds. If for allj ∈ 1,...,mif holds that$M_j(P_i) \leq 0$, then the condition (8)is true.

The proposed method can be applied to solve the problem of determining the workspace of the parallel robot described in [3].The problem is formulated as follows: for the parallel planar robot, shown schematically in figure 2, it is necessary to estimate with some accuracy the set X ⊆ R2 of possible position of the point Z . Two robot rod fixed in points 1,2via mechanisms permitting to change the length l 1, l 2 and ensuring the unhindered rotation of the rods around the fixing points.

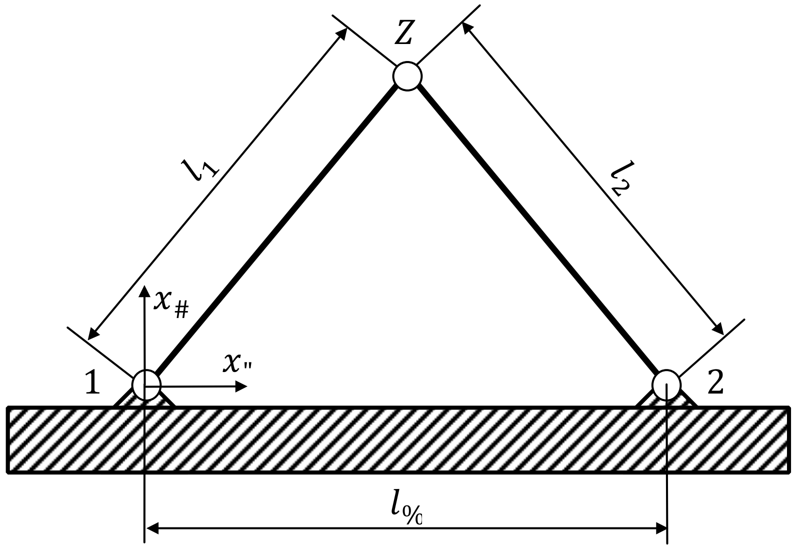

Figure 2.The scheme of a planar parallel robot.

Approximating the working area of the robot can be reduced to solving the system of inequalities (4) where function gj(x) are defined as follows:

$$g_2(x_1, x_2) = (l_1^{min})^2 - x_1^2 - x_2^2$$

$$g_1(x_1, x_2) = x_1^2 + x_2^2 - (l_1^{max})^2$$

$$g_3(x_1, x_2) = (x_1 - l_0)^2 + x_2^2 - (l_2^{max})^2$$

$$g_4(x_1, x_2) = (l_2^{min})^2 - (x_1 - l_0)^2 - x_2^2$$

where x_1,x_2 are coordinates of the working tool of the robot,l_1^{min} andl_1^{max}, l_2^{min} and l_2^{max}—structurally possible minimum and maximum length of the rod l_1 and l_2, with l_0 forming a triangle $l_0 l_1 l_2$. For the functions $g_1(x)$, ...,$g_4(x)$ in (10) problem (8) can be solved analytically.

The length of the horizontal edges of the source rectangle P may not exceed $l_2^{max} + l_0 + l_1^{max}$, and vertical — $\min(l_1^{max}, l_2^{max})$, lower left corner is located at a point$(-l_2^{max}, 0)$. The following robot parameters were chosen for the experiment: $l_1^{min} = l_2^{min} = l^{min} = 8$, $l_1^{max} = l_2^{max} = l^{max} = 12$, $l_0 = 5$. Calculation performed on a personal computer that with a quad-core Intel i7 processor 2.2 GHz and 16 GB of RAM. The algorithm was implemented in Python using the ete [4] library. Figure 4 shows three approximating families for$\delta_1 = 0.5$ (fig. 4a), $\delta_2 = 0.25$ (fig. 4b) и$\delta_3 = 0.07$ (fig. 4c) and analytically obtained workspace (fig. 4d). Experimental results show that changing the parameter δ controls the accuracy of the approximation. As expected increasing accuracy also increases the computation time, which for the considered three values of $\delta$was 0.9, 2.4 and 6.6 seconds respectively.

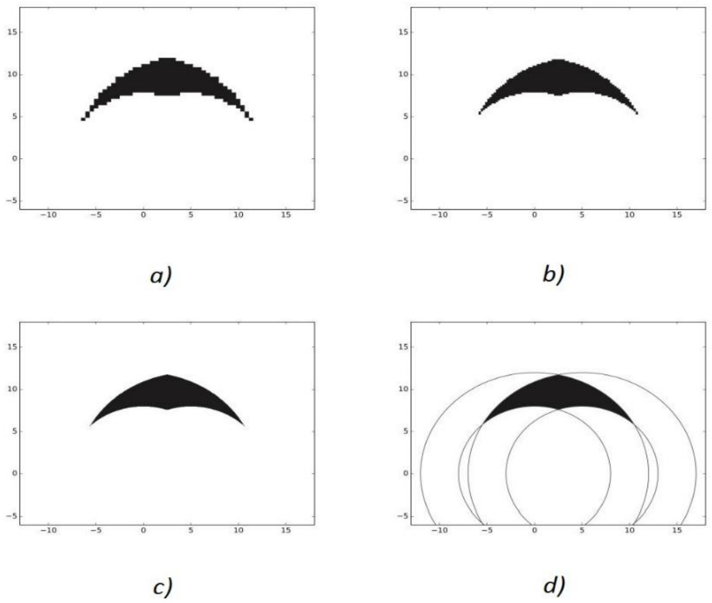

Figure 3.Three approximating family and analytically built workspace.

$$\Delta(A, B) = \max_{a \in A} \min_{b \in B} ||a - b||, A = \cup_{i \in M} P_i, B = \cup_{j \in N} Q_j$$

$$\Delta(A, B) = \max_{i \in M} \rho(P_i, B)$$

$$\Delta(P_i, B) = \max_{a \in P_i} \min_{b \in B} ||a - b||$$

$$\Delta(P_i, B) = \max_{a \in P_i} \min_{j \in N} \min_{b \in Q_j} ||a - b||$$

$$\Delta(P_i, B) = \min_{j \in N} \max_{a \in P_i} \min_{b \in Q_j} ||a - b|| = \min_{j \in N} \rho(P_i, Q_j)$$

$$\rho(P_i, Q) = \max_{p \in P_i} \min_{q \in Q} ||p - q|| = \min_{q \in Q} \max_{p \in P_i} ||p - q||$$

5. Conclusion

The article describes a method for constructing approximation of working space of the parallel robot defined by a system of nonlinear inequalities. The proposed approach is based on the non-uniform covering method. Application of the method is illustrated by solving this problem for therealistic planar robot with parallel structure. In the future, we plan to apply this approach to the problem of constructing working sets of three-dimensional robots of hexapod and tripod types.

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References

1. R. Clavel, 1991. Conception d'un robot parallelerapide a 4 degres de liberte. Ph.D. Thesis, EPFL, Lausanne, (925).
2. Di Gregorio R. and R.Zanforlin, 2003. Workspace analytic determination of two similar translational parallel manipulators. Robotica, 21(5):555–566.
3. Rybak L. A., Y.A.Mamaev, D.I.Malyshev, L.G.Virabyan, 2016.A software module for implementing a predetermined path of movement of the output link of the robot hexapod, for 3D printer products. Bulletin of Belgorod State Technological University named after V.G. Shukhov. (8): 155-164.
4. Bulca F., J.Angeles, and P.J.Zsombor-Murray, 1999. On the workspace determination of spherical serial and platform mechanisms. Mechanism and Machine Theory, 34(3):497–512.
5. Chablat D. andWenger P., 1998.Moveability and collision analysis for fullyparallel manipulators. In 12th RoManSy, pp: 61-68.
6. Gosselin C. and J.Angeles,1990.Kinematic inversion of parallel manipulators in the presence of incompletely specified tasks. ASME J. of Mechanical Design, 112(4):494–500.
7. Merlet J-P., 2001.A parser for the interval evaluation of analytical functions and its applications to engineering problems. J. Symbolic Computation, 31(4):475–486.
8. Chablat D., P.Wenger, F.Majou, and J-P Merlet,2004. An interval analysis based study for the design and the comparison of three-degreesof-freedom parallel kinematic machine. Int. J. of Robotics Research, 23(6):615–624.
9. Kumar V., 1992. Characterization of workspaces of parallel manipulators. ASME J. of Mechanical Design, 114(3):368–375.
10. Bonev I.A. and J. Ryu, 2001.A new approach to orientation workspaceanalysis of 6 dof parallel manipulator. Mechanism and Machine Theory,36(1):15–28.
11. Pernkopf F, Husty ML (2006) Workspace analysis of Stewart-Gough-type parallel manipulators. J MechEngSci 220(7):1019–1032.

12. Husty M.L.,1996.On the workspace of planar three-legged platforms. In World Automation Congress, volume 3, pp: 339–344.
13. Chablat D. andP.Wenger,1998.Moveability and collision analysis for fullyparallelmanipulators. In 12th RoManSy, pp: 61–68, Paris.
14. Jo D.Y. and E.J.Haug, 1989. Workspace analysis of closed loop mechanisms with unilateral constraints. In ASME Design AutomationConf., pp: 53–60.
15. Adkins F.A. and E.J.Haug, 1997. Operational envelope of a spatial Stewart platform. ASME J. of Mechanical Design, 119(2):330–332.
16. Haugh E.J., F.A.Adkins, and C.M.Luh, 1998. Operational envelopes for working bodies of mechanisms and manipulators. ASME J. of Mechanical Design, 120(1):84-91.
17. Snyman J.A., L.J.Du Plessis, and J. Duffy, 2000. An optimisation approach to the determination of the boundaries of manipulator workspaces. ASME J. of Mechanical Design, 122(4):447–456.
18. Agrawal S.K., 1992. Workspace boundaries of in-parallel manipulator systems. Int. J. of Robotics and Automation, 7(2):94–99.
19. Kumar V.,1992. Characterization of workspaces of parallel manipulators. ASME J. of Mechanical Design, 114(3):368–375.
20. Rybak, L.A.,2014.The method of structural synthesis of parallel mechanisms with the set of the trajectory of output link on the plane [Text] / L.A. Rybak, C. Gosselin, A.V. Chichvarin, A.V. Grinek. Advances in Environmental Biology, 8 (4): 855-861.
21. Arai T., R.Larsonneur, and Y.M.Jaya, 1993. Calibration and basic motionof a micro-hand module. In Int. Conf. on Indus. Electronics, Controland Instrumentation (IECON), pp: 1660–1665.