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Research Article

STATISTICAL ANALYSIS OF COLLECTIVE PHENOMENA IN COMPLEX SYSTEMS

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Abstract

The description of complex system evolution with the unique properties different from the sub-system properties is an important task of modern science. The solution of this problem is complicated not only by the absence of full information about the processes implemented in complex systems, but also by very diverse nature of the interaction between components. In this case, simplification is achieved in those approaches which allow “to abstract” from a studied system organization. Thus, the description of distributed system evolution is performed based on the extraction of information from the signals generated by them. This paper presents the possibilities of statistical analysis methods: the memory function formalism (MFF) and flicker-noise spectroscopy (FNS) during the study of synchronization phenomena.

Using the example of a man's magnetoencephalogram signal analysis (MEG) and radio emission of quasars examined the effects of coordination between the constituent parts of these systems are analyzed. MFF analysis of MEG signals mutual dynamics among healthy subjects led to the conclusion about the processes, compensating the abnormal collective activity of neurons in response to the third-party effects which are not observed in the case of photosensitive epilepsy. The effects of frequency-phase synchronization in the signals of quasar emission at different frequencies discovered within FNS contributed to the establishment of qualitatively distinct mechanisms concerning their disk accretion.

The obtained results determine the possible ways of collective phenomena study in complex systems, as well as a key role of coordination effects in their evolution.

Keywords: evolution of complex systems, collective phenomena, statistical analysis methods, synchronization effects, cross-correlations

1. Introduction. Collective phenomena in complex systems

The complex systems occupy a special place in the diversity of the world objects which were the subjects of scientific research [1]. They have a set of unique properties, primarily non-equilibrium, nonstationarity and nonlinearity, which are fundamentally different from other objects. The first property means that a complex system is far from equilibrium with the environment. The second property means that the system evolves, that is, it changes the nature of its dynamics continuously. The third property states that the processes within the system and the response to external factors are nonlinear ones. These properties, on the one hand, provide an extraordinary "flexibility" to complex systems, and make the task of their description quite a difficult one on the other.

Besides the description of complex object evolution is complicated by the presence of a large number of interacting components. On the one hand, the components generate its own dynamics, and on the other they are in a close interaction with the environment and act as an integral whole. Then collective processes occurring during the interaction or the redistribution of relations become a key factor in a complex system stability.

There is a considerable number of approaches to describe the processes in complex systems. First of all, we should highlight the concepts of phase and frequency synchronization [2–5], which are based on the detection of the characteristic frequencies and the analysis of phase signal difference produced by the parts of a complex system. In practice, the oscillation frequencies and phases different from the harmonic ones, can be set in the "analytic signal" model using Fourier, Hilbert transformations [2–4] or the wavelet transformations [5]. In addition to the phase, applied The stochastic synchronization models are used in addition to phase ones, the essence of which is in the comparative analysis of the attractor topological structure, describing the dynamics of nonlinear interacting oscillators [6], "generalized" synchronization [7], as well as the synchronization of time scales [8].

The description of complex system mutual dynamics is also carried out in cross-correlation terms, i.e., in the probabilistic relationships between two sequences of random variables – the time records of dynamic variables. The random matrix method is widely used in financial system study [9], the non trended cross-correlation analysis is used to study power dependencies in the cross-correlations between the non-stationary time series [9–11]. Besides, there is the group of methods in which the cross-correlation functions are used directly [12–17].

In this paper we will present the possibilities of cross-correlation and synchronization effect description in complex systems on the basis of statistical analysis methods: memory function formalism (MFF) and flicker-noise spectroscopy (FNS).

2.1. Cross-Correlation Description with in Memory Function Formalism

In a more general sense the statistical memory effects describe a complex and a hidden nature of correlation origin, distribution and attenuation. From a physical point of view, the time scales of the processes associated with the correlations and memory effects cannot be arbitrary ones. The quantitative comparison of these processes, as well as the description of the relationships between demands the introduction of function and statistical memory measure which characterizes the rate of random variable change at different levels of statistical description.

The memory functions formalism [14, 18, 19] – the theoretical approach to the study of auto and cross correlations generated by complex systems with discrete time. The method consists in the fact that the studied signals are represented as the sequence of $\{x_j\}$, $\{y_j\}$ values for two random variables X, Y:

$$X = \{x(T), x(T+\tau), x(T+2\tau), ..., x(T+(N-1)\tau)\},$$

$$Y = \{y(T), y(T+\tau), y(T+2\tau), ..., y(T+(N-1)\tau)\},$$

where T – the initial moment of time, $(N-1)\tau$ – signal record time, τ – time sampling step.

The average values $\langle X\rangle, \langle Y\rangle$, the fluctuations $\delta x_j, \delta y_j$ and the dispersions σ_x, σ_y are calculated for specified series:

$$\langle X\rangle = \frac{1}{N}\sum_{j=0}^{N-1} x(T+j\tau),\ x_j = x(T+j\tau),\ \delta x_j = x_j - \langle X\rangle,\ \sigma_x^2 = \frac{1}{N}\sum_{j=0}^{N-1}\delta x_j^2;$$

$$\langle Y\rangle = \frac{1}{N}\sum_{j=0}^{N-1} y(T+j\tau),\ y_j = y(T+j\tau),\ \delta y_j = y_j - \langle Y\rangle,\ \sigma_y^2 = \frac{1}{N}\sum_{j=0}^{N-1}\delta y_j^2.$$

Let's put down the expression for the temporary cross-correlation function (CCF) in the following form:

$$c(t) = \frac{1}{(N-m)\sigma_x\sigma_y}\sum_{j=0}^{N-m-1}\delta x(T+j\tau)\delta y(T+(j+m)\tau),$$

$$t = m\tau,\ 1 \le m \le N-1.$$

The memory functions $M_{n-1}^{XY}(t)$ of a higher order, as well as the expressions for kinetic λ_n^{XY} and relaxation Λ_n^{XY} parameters are calculated on the basis of CCF [14]:

$$M_{n-1}^{XY}(t) = \frac{\left\langle \mathbf{W}_{n-1}^X \{1 + i\tau\hat{L}_{22}\}^m \mathbf{W}_{n-1}^Y\right\rangle}{\left\langle \mathbf{W}_n^X \mathbf{W}_n^Y\right\rangle},\ \lambda_n^{XY} = \frac{\left\langle \mathbf{W}_{n-1}^X \hat{L}\mathbf{W}_{n-1}^Y\right\rangle}{\left\langle \mathbf{W}_{n-1}^X \mathbf{W}_{n-1}^Y\right\rangle},\ \Lambda_n^{XY} = i\frac{\left\langle \mathbf{W}_n^X \mathbf{W}_n^Y\right\rangle}{\left\langle \mathbf{W}_{n-1}^X \mathbf{W}_{n-1}^Y\right\rangle}.$$

Here $\mathbf{W}_i^X$, $\mathbf{W}_i^Y$ are the orthogonal dynamic variables produced on the basis of state vectors $\mathbf{A}_k^0(0)$, $\mathbf{B}_k^0(0)$ using the recurrence relations:

$$\mathbf{W}_0^X = \mathbf{A}_k^0(0),\ \mathbf{W}_1^X = (i\hat{L} - \lambda_1^{XY})\mathbf{W}_0^X,\ \mathbf{W}_2^X = (i\hat{L} - \lambda_2^{XY})\mathbf{W}_1^X - \Lambda_1^{XY}\mathbf{W}_0^X - ...,$$

$$\mathbf{W}_0^X = \mathbf{B}_k^0(0),\ \mathbf{W}_1^Y = (i\hat{L} - \lambda_1^{XY})\mathbf{W}_0^Y,\ \mathbf{W}_2^Y = (i\hat{L} - \lambda_2^{XY})\mathbf{W}_1^Y - \Lambda_1^{XY}\mathbf{W}_0^Y - \ldots,$$

$$\mathbf{A}_k^0 = \mathbf{A}_k^0(0) = \{\delta x_0, \delta x_1, \ldots, \delta x_{k-1}\},$$

$$\mathbf{B}_k^0 = \mathbf{B}_k^0(0) = \{\delta y_0, \delta y_1, \ldots, \delta y_{k-1}\},$$

$\hat{L}$ – Liouville quasi-operator.

The power spectra of memory functions are used for frequency-phase synchronization analysis:

$$\mu_0^{XY}(\nu) = \left|\Delta t \sum_{j=0}^{N-1} c(t_j)\cos 2\pi\nu t_j\right|^2,\ \ldots,\ \mu_i^{XY}(\nu) = \left|\Delta t \sum_{j=0}^{N-1} M_i^{XY}(t_j)\cos 2\pi\nu t_j\right|^2.$$

2.2. The Use of Memory Function Formalism to Live System Evolution Research

Then we will be present the results of the frequency-phase synchronization study in the activity of a man's cerebral cortex specific areas with photosensitive epilepsy (PSE) and healthy subjects. The signals [20, 21] are the magnetoelectric responses caused by the flickering light stimuli. This disease is the form of epilepsy in which seizures are triggered by the flickering light of high intensity. The record of MEG signals was performed by Neuromag-122 (Neuromag Ltd. Finland) device using 61-SQUID sensor with the sampling frequency of 500 Hz. The group of tested subjects included 9 healthy men (at the age of 22–27 years) and a PSE patient (a 12 year old teenager). Red and blue stimulus was projected on a special screen 80 times during two seconds at a three seconds interval. The results of all attempts were averaged. Let's use the signals recorded by the sensors from the left- (number 37) and the right- (number 60) temporal lobe of the cerebral cortex as an example. Fig. 1 shows the power spectra $\mu_0^X(\nu), \mu_0^Y(\nu)$ of autocorrelation functions and CCF $\mu_0^{XY}(\nu)$ for the signals of one of the healthy subjects (the basic expressions for the calculation of autocorrelation functions and their spectra can be found, for example, in [18, 19]). The comparative analysis of the amplitude rises within the specified spectra (see, for example, the peak 1 and 2) allows to set the frequencies at which the synchronization between the cerebral cortex signals is realized.

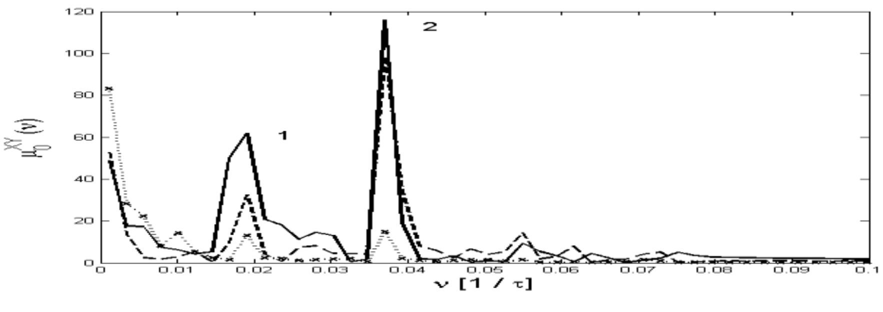

Fig. 1: The power spectra of auto-correlation functions (dashed lines) among two signals in comparison with the cross-correlation function power spectrum (solid line)

CCF power spectra calculated for a healthy person tested signal (Fig. 2, a) and a patient (Fig. 3, a) have sufficiently clear fractal structure. One may detect differences using the following relaxation levels (Fig. 2, b–d, Fig. 3, b–d), – the spectra of cross- correlation memory functions. The power spectra of an original CCF and the memory functions for a healthy person tested signals are characterized by a distinct low-frequency dynamics with the presence of large peaks in the range of up to 50 Hz, which correspond to physiological rhythms. These rhythms reflect the complex psycho-physiological processes of the brain activity.

A particular pattern is observed in CCF spectra and the memory functions for a PSE patient signal (Fig. 3). The dominant process is the process with the frequency of about 50 Hz. However, there are additional quasi periodic processes at higher frequencies.

These results suggest the presence of processes, compensating the development of abnormally high collective activity of neurons in the spatially separated regions of the brain cortex among healthy subjects in response to light exposures.

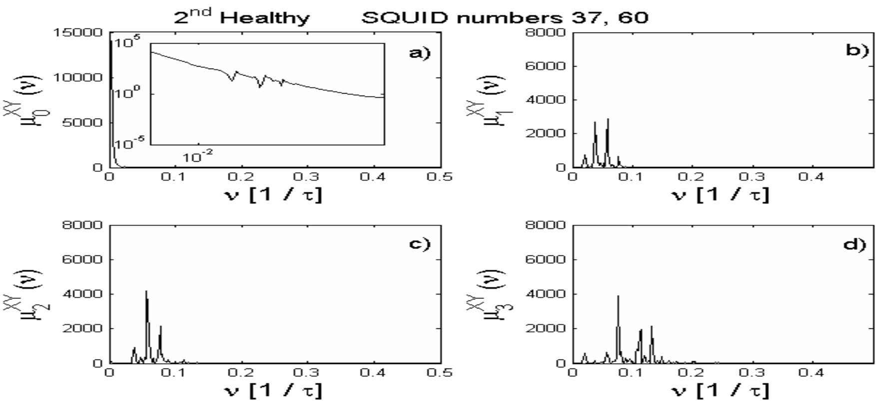

Fig. 2: The power spectra of the cross-correlation function and memory functions for the mutual dynamics of neuro-magnetic responses among healthy subjects.

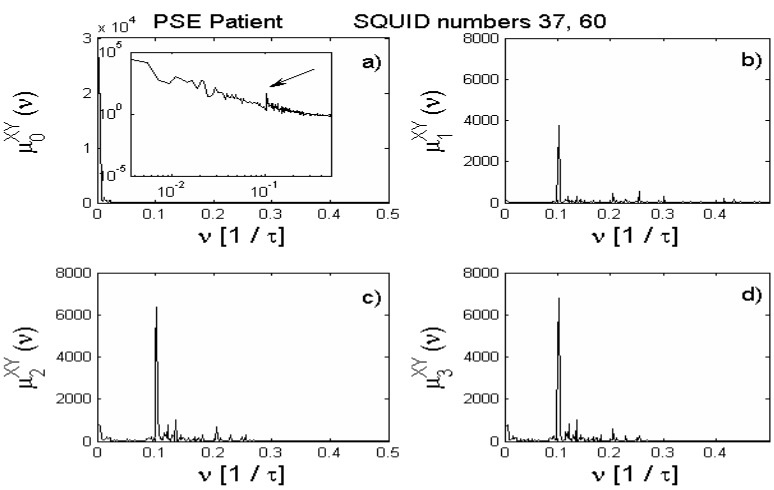

Fig. 3: The power spectra of the cross-correlation and memory functions for the stimulated MEG signals of the patient with photosensitive epilepsy.

3.1. The Description of Synchronization Effects within the Method of Flicker-Noise Spectroscopy

The prospect of flicker-noise spectroscopy application for signal study generated by the complex systems [16, 17, 22, 23] is caused by the introduction of information parameters characterizing the components of analyzed signals in different frequency ranges. The extracted information includes the specific low-frequency "resonances" for each signal, as well as the chaotic component parameters. The introduced FNS parameters are determined based on the basis of autocorrelation function "image" for statistical physics:

$$\psi(\tau)=\langle V(t)V(t+\tau)\rangle_{T-\tau}, \qquad \langle(\ldots)\rangle_{T-\tau}=\frac{1}{T-\tau}\int_0^{T-\tau}(\ldots)\mathrm{d}t,$$

where τ – time delay parameter, we assume that $0 \le \tau \le T_M$ ($T_M \le T/2$). In order to reveal the information contained in $\psi(\tau)$ (we assume that $\langle V(t)\rangle = 0$), it is more convenient to analyze some conversion functions, namely, cosine transformation $S(f)$ (f – frequency):

$$S(f)=2\int_0^{T_M}\langle V(t)V(t+t_1)\rangle_{T-\tau}\cos(2\pi f t_1)\mathrm{d}t_1$$

and differential moments (transitional "structural" Kolmogorov's function) $\Phi^{(2)}(\tau)$ of the 2-nd order:

$$\Phi^{(2)}(\tau)=\left\langle\left[V(t)-V(t+\tau)\right]^2\right\rangle_{T-\tau}.$$

The dependence character V(t) does not have any limitations, except for input average existence.

Two-parameter cross-correlation functions, introduced in the framework of FNS, allow to obtain the direct information about the dynamics of correlations between simultaneously measured signals – the dynamic variables $V_i(t)$ and $V_j(t)$, measured in a spatially separated points i and j of the studied system, or by the signals of different nature. The corresponding expression for "two-point" correlators or cross-correlators is represented as in [22]:

$$q_{ij}(\tau,\theta_{ij})=\left\langle\left[\frac{V_i(t)-V_i(t+\tau)}{\sqrt{2}\sigma_i}\right]\left[\frac{V_j(t+\theta_{ij})-V_j(t+\theta_{ij}+\tau)}{\sqrt{2}\sigma_j}\right]\right\rangle_{T-\tau-|\theta_{ij}|},$$

$$\sigma_i(\tau)=\left\{\left\langle\left[V_i(t)-V_i(t+\tau)\right]^2\right\rangle_{T-\tau-|\theta_{ij}|}\right\}^{\frac{1}{2}},$$

where τ – "delay time" (we assume that $\tau > 0$), θ_{ij} – the parameter of "time shift".

3.2. The Application Of Flicker-Noise Spectroscopy To Study The Evolution Of Astrophysical Objects

Let's study the effects of harmonization in the dynamics of the spectral radio emission flux density of quasars 0420m014, 2251p158 at the frequency of 2.7 GHz and 8.1 GHz within the above represented method [24]. According to the most common representations, quasars are the active nuclei of distant galaxies at the stage of development, in which a supermassive black hole absorbs matter from gas and dust disk. At the moving of falling matter layers the collective effects and resonance effects appear, which ultimately influence the quasar radiation dynamics.

Fig. 4 shows 3D cross-correlators for the radio emission of examined quasars at the frequencies of 2.7 and 8.1 GHz. The cross-correlation dependencies for the quasar 0420m014 signals demonstrate a clear oscillating structure reflecting the dominance of the dynamics of a certain frequency. The successive maximum values $q_{ij}(\tau, \theta_{ij})$, recorded on the Fig. 4, a at $\theta_{ij} > 0$, mean, that the signal at the frequency of 2.7 GHz is provided at regular intervals after the signal at the frequency of 8.1 GHz. At the same time, the successive maximum values $q_{ij}(\tau, \theta_{ij})$, recorded at $\theta_{ij} < 0$, mean, that the signal at the frequency of 8.1 GHz precedes the signal with the frequency of 2.7 GHz with the same periodicity.

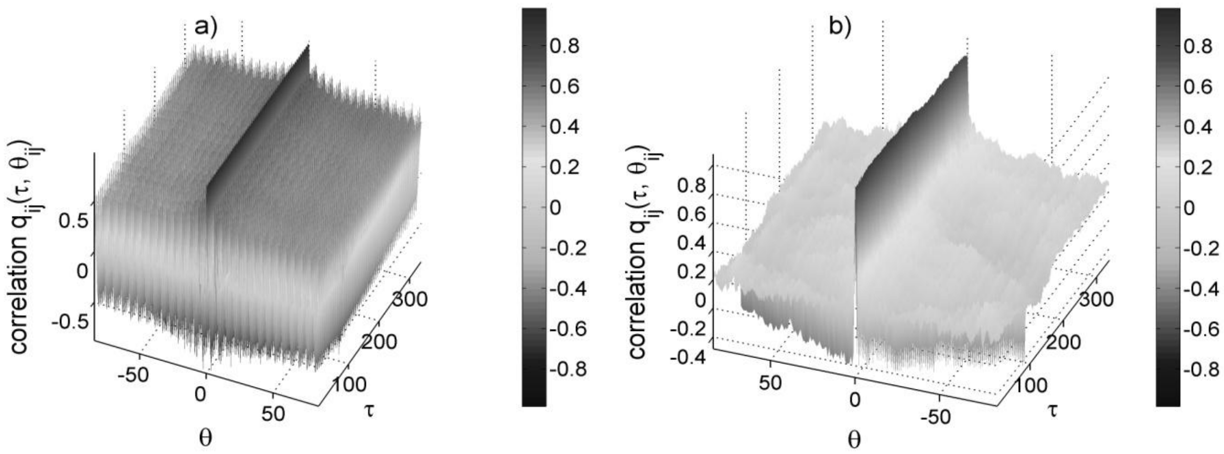

Fig. 4: 3D-dependencies of the cross-correlators $q_{ij}(\tau, \theta_{ij})$ for the radio emissions of quasars 0420m014 (a) and 2251p158 (b) at the frequencies of 2.7 GHz and 8.1 GHz

Depending on $q_{ij}(\tau, \theta_{ij})$ a low-frequency fuzzy large-scale structure is observed for quasar emission 2251p158 (Fig. 4, b). The absence of natural frequency developed set in the signal dynamics of the quasar 2251p158 leads to a lesser level of synchronization than it is observed in the quasar 0420m014. The significant influence of dynamical intermittency effects leads to the asymmetry of cross-correlator 3D-structure.

4. Conclusions

The concept of different types of synchronization was developed and the concept of cross-correlations is used in order to describe the evolution of the component objects. This paper analyzed the results of the author's methods application: the formalism of memory functions and flicker-noise spectroscopy to study the effects of coordination between a complex system components. The first method is based on the set of memory function power spectra to identify the sets of frequencies at which synchronization occurs. Using the presented method the characteristic frequencies were established, on which the synchronization of neuro-magnetic signals takes place for different areas of the cerebral cortex among healthy subjects. The violation or the suppression of these periodic processes was the basis for the detection of the diagnostic criteria in respect of pathological brain activity. The second method is based on the use of two-parameter cross-correlation functions, the structure of which reflects the collective effects manifested in a system. The effects of frequency-phase synchronization detected in the signals of quasar radioactivity at different frequencies, allowed to facilitate the establishment of different mechanisms for disk accretion.

5. Summary

Nature, the variety of structures and the nature of complex system dynamics make use the variety of approaches to their study. Besides, the distributed objects require the consideration of their interaction between the individual parts, which leads to the complication of applied mathematical approaches. The methods which allow "to disengage" from the internal structure of a studied system, make it possible to simplify the task of its evolution study. The use of such techniques allows to perform parameterization - the quantitative description of distributed system evolution based on the extraction of information from the signals produced by them.

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