ISSN: 0975-766X
CODEN: IJPTFI
Research Article

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THE EVOLUTION OF THE TRABECULAR BONE AT A CONSTANT COMBINED LOADING

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Received on 14-08-2016 *Accepted on 20-09-2016*

Abstract:

This paper deals with the mathematical model of restructured bone tissue under the influence of external forces. The mathematical model has been formed on the basis of Wolff law in the Cowin mathematical formulation. The paper considers a number of model problems: all-around compression with single-direction weakening, the problem of torsion and the problem of uniform compression with stress tensor rotation at a predetermined angle. We have calculated the relative change in the solid volume and the components of the structure tensor (tissue tensor), the solution was considered the convergence of asymptotic behavior of the strain tensor components to the value of -0.0015, which meant lazy zone. It was noted in the problem of all-around compression that the bone microstructure comes in homeostatic balance regardless of the size and sampling of the parameter of weakening. It was noted in the problem of torsion that an increase in the tangential components of the structure tensor results in significant variation of the behavior of the solid volume and the components of structure tensor. In the problem of stress tensor rotation, the components of structure tensor change first, which is consistent with Wolff law – the bone tissue is rearranged along the lines of principal stresses.

Keywords: biomechanics, orthotropic biological structures, structure tensor, evolutionary tissue restructuring.

Introduction

Bone tissue is a composite formation consisting of special homogeneous cells and performing a specific function. The tissue includes three components: cells, fibers and bone matrix. Bone substance in the diaphysis is mostly a dense bone, and only has a small amount of cancellous bone near the medullary canal. Depending on the location of bone plates, the dense bone is divided into three zones: ring-shaped plates, Haversian bone plates, and interosseous plates.

Bone is a complex matter, a complex anisotropic non-uniform vital material with elastic and viscous properties and good adaptive function. All the excellent properties of bones make up an indivisible unity with their functions.

The bone functions mainly have two sides, one of them is the formation of the skeletal system used to maintain the body and its normal shape, as well as to protect its internal organs. The skeleton is a part of the body, to which the muscles are attached and which provides the conditions for their contraction and body movements. The skeleton itself performs the adaptive function by changing sequentially its shape and structure. The second side of bone functions is that to maintain a balance of minerals in the human body by adjusting the concentration of Ca^{2+}, H^{+}, HPO_4^{+} in blood electrolyte, that is hematopoiesis, as well as the preservation and metabolism of calcium and phosphorus.

Physiological restructuring of the bone structure starts with occurrence of new functional conditions that change the load on a certain bone or a part of the skeleton [1-4]. This includes professional rearrangement and restructuring due to changes in static and dynamic state of the skeleton during inactivity, after amputations, traumatic strains, ankylosis, etc. New bone architectonics occurs in these cases in the result of the formation of new bone trabeculae and their arrangement according to new force lines, and in the result of resorption of old, non-effective bone trabeculae.

We have obtained a defining ratio, including the structure tensor, i.e., value able to reflect the internal structure of a cancellous bone. This ratio relates the stress-strain state in the cancellous bone tissue with its structure (the trabecular microstructure) and has the following form:

$$\tilde{\sigma} = (g_1 + g_2 e) tr\tilde{\varepsilon} \cdot E + (g_3 + g_4 e)\tilde{\varepsilon} + g_5(\tilde{\varepsilon}\tilde{K} + \tilde{K}\tilde{\varepsilon}) + g_6\left(tr(\tilde{K}\tilde{\varepsilon})\cdot E + tr\tilde{\varepsilon}\cdot\tilde{K}\right), \quad (1)$$

where $\tilde{K}$ – the structure tensor deviator $\tilde{H}$, normalized so that $tr\tilde{K} = 0$; e – change in the share of solid bone volume in the reference value v_0; $g_1 - g_6$ – constant values expressed in GPa. These constants were determined in [3-5] after a series of experiments with different samples of human or cattle cancellous bone.

Based on the Wolff law, we have also described the proposed mechanism of restructuring the trabecular microstructure, occurring as a result of adaptation processes, permanently taking place in living bone tissue. The law is based on the ratio: $\tilde{\sigma}\tilde{K} = \tilde{K}\tilde{\sigma}$

According to Wolff law, we have obtained evolutionary ratios for trabecular bone, describing the changes in the structure of trabecular bone under different loads. These kinetic equations can describe the change of the structure tensor deviator $\tilde{K}$ and the proportion of solid bone volume e. Namely:

$$\frac{d\tilde{K}}{dt} = (h_1 + h_3 e)\left(\tilde{\varepsilon} - tr\tilde{\varepsilon}\cdot\frac{E}{3} - \left(\tilde{\varepsilon}_0 - tr\tilde{\varepsilon}_0\cdot\frac{E}{3}\right)\right) + h_4 tr(\tilde{\varepsilon} - \tilde{\varepsilon}_0)\cdot\tilde{K} +$$
$$+ h_2\left(tr\left(\tilde{K}(\tilde{\varepsilon} - \tilde{\varepsilon}_0)\right)E - \frac{3}{2}\left(\tilde{K}(\tilde{\varepsilon} - \tilde{\varepsilon}_0) + (\tilde{\varepsilon} - \tilde{\varepsilon}_0)\tilde{K}\right)\right) \quad (2)$$

$$\frac{de}{dt}=(f_1+f_2e)(tr\tilde{\varepsilon}-tr\tilde{\varepsilon}_0)+f_3tr\left(\tilde{K}(\tilde{\varepsilon}-\tilde{\varepsilon}_0)\right), \quad (3)$$

where h_1-h_4 and f_1-f_3 – constant values expressed in day^{-1} and determined empirically so that the restructuring of the bone tissue could occur in a time corresponding to the natural reality (normally, adaptation of trabecular bone takes about 160 days). For ratios (1) - (3) we presented the statement of initial-boundary problem of the restructuring of the trabecular bone. This statement can be used in addressing the issues of the study of stress-strain state of the spongy bone tissue and its adaptation processes. However, the readjustment of the trabecular structure can be described only by the ratios (1) - (3) (taking into account the initial conditions). This is possible, assuming that the loads on the studied area (and thus its stress state) do not change continuously and do not cause permanent relevant adaptation processes in the trabecular bone. For example, it can be at a single change in the load, and thus change in the old stressed state $\tilde{\sigma}^0$ to the new one $\tilde{\sigma}^1$. In this case, the system of equations (1) – (3) in a rectangular Cartesian coordinate system, in general, can be transformed into a system including twelve scalar equations with twelve scalar unknowns.

Methods

Creating a mathematical model able to describe with reasonable certainty the restructuring process of the trabecular bone structure and easily solve the problem based thereon is a quite complicated task. In this regard, we shall consider the simplified statement of problems and their solutions in order to obtain results, on the basis of which we will be able to analyze the evolution of the bone tissue and come to appropriate conclusions.

Let us consider a physical ratio (1) between the stress-strain state in the trabecular bone and its structure, and the evolutionary ratios (2) - (3) obtained in accordance with Wolff law for trabecular bone, describing the changes in the structure of trabecular bone under different loads [6-8]. As a result of the calculations, we built the time variance graphs of porosity, strain tensor and structure.

$$\tilde{\sigma}=(g_1+g_2e)tr\tilde{\varepsilon}\cdot E+(g_3+g_4e)\tilde{\varepsilon}+g_5(\tilde{\varepsilon}\tilde{K}+\tilde{K}\tilde{\varepsilon})+g_6\left(tr(\tilde{K}\tilde{\varepsilon})\cdot E+tr\tilde{\varepsilon}\cdot\tilde{K}\right) \quad (1)$$

$$\frac{d\tilde{K}}{dt}=(h_1+h_3e)\left(\tilde{\varepsilon}-tr\tilde{\varepsilon}\cdot\frac{E}{3}-\left(\tilde{\varepsilon}_0-tr\tilde{\varepsilon}_0\cdot\frac{E}{3}\right)\right)+h_4tr(\tilde{\varepsilon}-\tilde{\varepsilon}_0)\cdot\tilde{K}+$$
$$+h_2\left(tr\left(\tilde{K}(\tilde{\varepsilon}-\tilde{\varepsilon}_0)\right)E-\frac{3}{2}\left(\tilde{K}(\tilde{\varepsilon}-\tilde{\varepsilon}_0)+(\tilde{\varepsilon}-\tilde{\varepsilon}_0)\tilde{K}\right)\right) \quad (2)$$

$$\frac{de}{dt}=(f_1+f_2e)(tr\tilde{\varepsilon}-tr\tilde{\varepsilon}_0)+f_3tr\left(\tilde{K}(\tilde{\varepsilon}-\tilde{\varepsilon}_0)\right) \quad (3)$$

where $\tilde{K}$ – the structure tensor deviator $\tilde{H}$, normalized so that $tr\tilde{K}=0$; e – change in the share of solid bone volume in the reference value v_0; g_1-g_6 – constant values expressed in GPa; h_1-h_4 and f_1-f_3 – constant values expressed in day^{-1} and determined empirically so that the restructuring of the bone tissue could occur in a time corresponding to the natural reality (normally, adaptation of trabecular bone takes about 160 days).

Results and discussion

We shall consider the solution of the three model problems: the case of non-uniform all-around compression; torsion; and stress tensor rotation. We shall introduce a general representation of constant values and tensors for all model tasks, describing the restructuring of the trabecular structure of the bone tissue, as well as the assumptions necessary to obtain the desired results. We shall take tensors K_0, e_0, S_0 in the following form for initial parameters:

$$\tilde{\sigma}_0=\begin{pmatrix} \sigma_0^{11} & \sigma_0^{12} & 0 \\ \sigma_0^{12} & \sigma_0^{22} & 0 \\ 0 & 0 & \sigma_0^{33} \end{pmatrix},\quad \tilde{K}_0=\begin{pmatrix} K_0^{11} & 0 & 0 \\ 0 & K_0^{22} & 0 \\ 0 & 0 & -K_0^{11}-K_0^{22} \end{pmatrix},\quad \tilde{\varepsilon}_0=\begin{pmatrix} \varepsilon_0^{11} & 0 & 0 \\ 0 & \varepsilon_0^{22} & 0 \\ 0 & 0 & \varepsilon_0^{33} \end{pmatrix},$$

as well as constant values:

$$g_1 = 154.9 ,\ g_2 = 1147 ,\ g_3 = 612.9 ,\ g_4 = 4536 ,\ g_5 = 2384 ,\ g_6 = 510.8 .$$

A computational experiment was conducted to determine the coefficients $h_i, f_j, i=\overline{1,4}, j=\overline{1,3}$, which considered that the bone adaptation process must take 160 days. The experiment showed that the coefficients for the evolutionary ratios (2) and (3) with the assumption of independence and the structural and volumetric adjustment (4):

$$\frac{d\tilde{K}}{dt}=\tilde{f}_1^{'}(\tilde{\varepsilon},\tilde{K}), \frac{de}{dt}=f_2^{'}(tr\tilde{\varepsilon},e) \tag{4}$$

must be as follows:

$$h_1=-5;\ h_2=10;\ h_3=0,\ h_4=0,\ f_1=-2,5;\ f_2=5;\ f_3=0$$

(these coefficients are expressed in day^{-1}). Coefficients $g_i, i=\overline{1,6}$ are expressed in GPa. Let us assume that the initial equilibrium state is described by the following stress tensor:

$$\tilde{\sigma}_0=\begin{pmatrix} -1,4024 & 0 & 0 \\ 0 & -1,4024 & 0 \\ 0 & 0 & -1,4024 \end{pmatrix}, \tag{5}$$

and the initial strain state will be set based on the theory of lazy zone (a certain range without rearrangement of the trabecular microstructure):

$$\tilde{\varepsilon}_0 = \begin{pmatrix} -0,0015 & 0 & 0 \\ 0 & -0,0015 & 0 \\ 0 & 0 & -0,0015 \end{pmatrix}.$$

The system of equations (1) (in the componential representation) allows determining numerically the initial parameters of the trabecular bone architecture and changes in the porosity, namely: $e_0 = -0,0179$,

$$\tilde{K}_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The fact that $\tilde{K}_0 = 0$, indicates that the structure tensor $\tilde{H}$ is spherical, and the initial microstructure of the trabecular bone tissue is isotropic.

Let us consider in detail the case of non-uniform all-around compression. In this problem, we shall consider the non-uniform all-around compression, characterized by a gradual change in the component value of the stress tensor responsible for the load applied along the axis Oz. We shall define a new stress state, by increasing the loading conditions along the axis 3:

$$\tilde{\sigma}_1 = \begin{pmatrix} -1,4024 & 0 & 0 \\ 0 & -1,4024 & 0 \\ 0 & 0 & -1,5472 \cdot d \end{pmatrix},$$

where $d \in [1,1.5]$, and solve numerically the system (1) - (3), by changing the parameter *d* in increments of 0.1. The results obtained after 160 days are shown in Fig. 1: in Figure 1 (a), changes in the solid volume $e(t)$, depending on the parameter d, occur upwards, while in Figure 1 (b) - downwards. We can see that the bone microstructure came again to homeostatic equilibrium:

$$\tilde{\varepsilon}_1 = \begin{pmatrix} -0,0015 & 0 & 0 \\ 0 & -0,0015 & 0 \\ 0 & 0 & -0,0015 \end{pmatrix}.$$

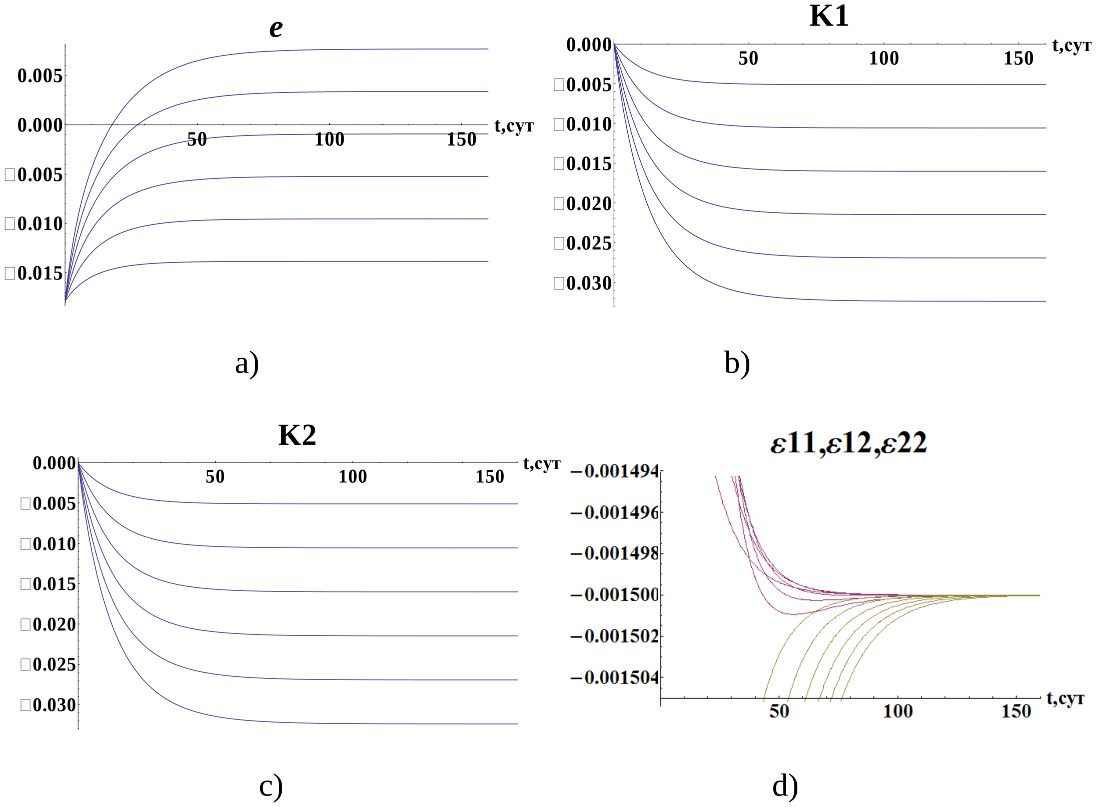


Fig. 1. Changes in the structure (the component of the structure tensor deviator K1(t) (b), K2(t) (c)) and density (the share of solid bone volume e(t)) of the trabecular microstructure of (a), a strain diagram (d)

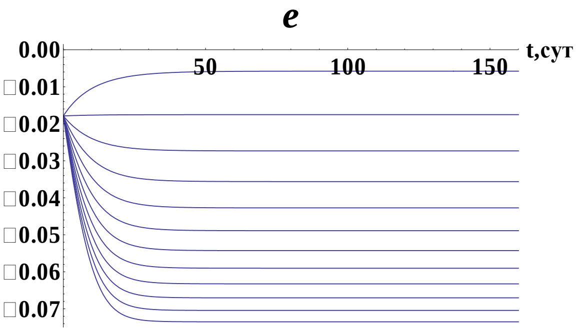
We shall consider the problem of torsion, where we define the new stress state, which characterizes the rotation, and add shear stresses. Subject to the fact that upon zeroing of normal stresses solution goes into singularity, we will gradually direct the diagonal components of the stress tensor to zero:

$$\tilde{\sigma}_1 = \begin{pmatrix} -1.5472/d & -1.5472 & 0 \\ -1.5472 & -1.5472/d & 0 \\ 0 & 0 & -1.5472/d \end{pmatrix},$$

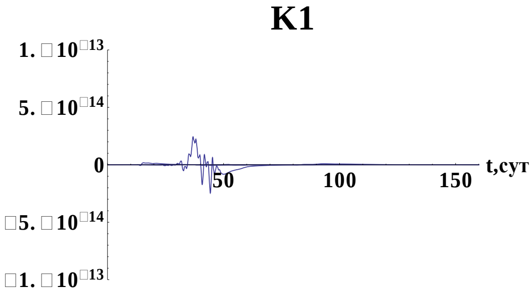
where $d \in [1;2,1]$ and varies in increments of 0.1. Numerical solution of the system (1)-(3) presents the following results:

We should note that the values of the structure tensor components are equal to zero, and the strain tensor becomes as follows over time:

$$\tilde{\varepsilon}_1 = \begin{pmatrix} -0,0015 & 0 & 0 \\ 0 & -0,0015 & 0 \\ 0 & 0 & -0,0015 \end{pmatrix}. \qquad (6)$$



a)



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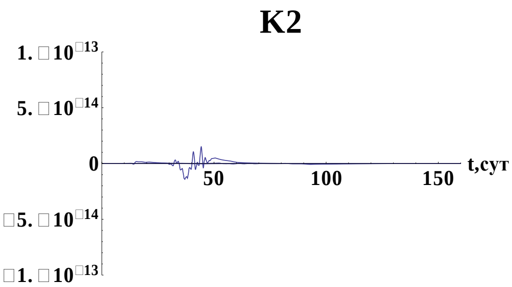

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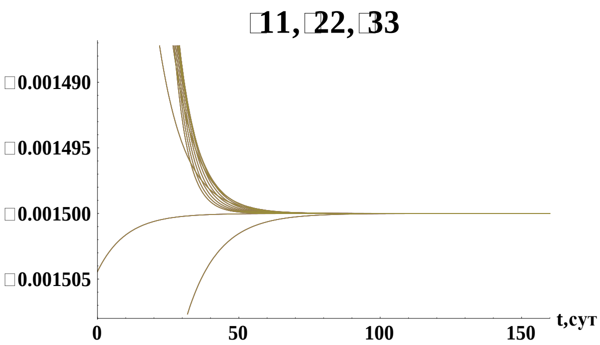
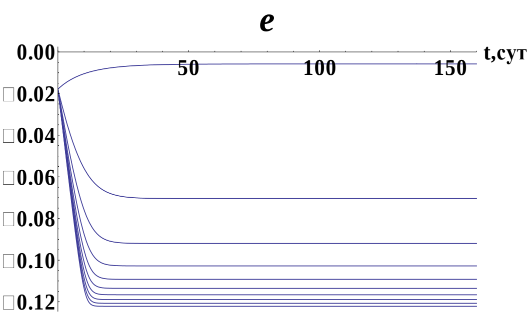


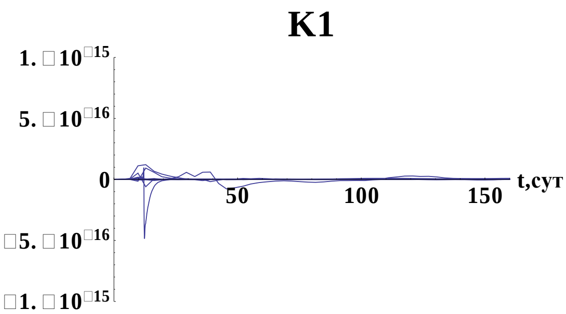
d)

Fig. 2. Changes in the structure (the component of the structure tensor deviator K1(t) (b), K2(t) (c)) and density (the share of solid bone volume e(t)) of the trabecular microstructure of (a). A strain diagram (d)

Further increase in the coefficient *d*, and, as a consequence, reduction in the normal stresses results in the following diagrams, which show that the change of porosity goes to a certain asymptote, the values of the structure tensor components are close to zero, and the strain tensor is still (6):



a)



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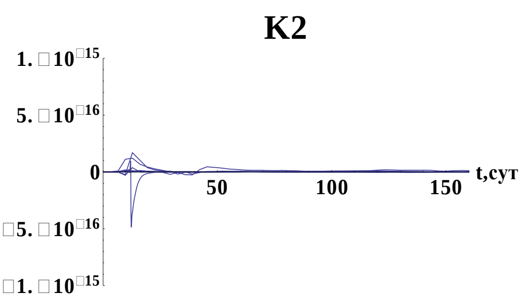
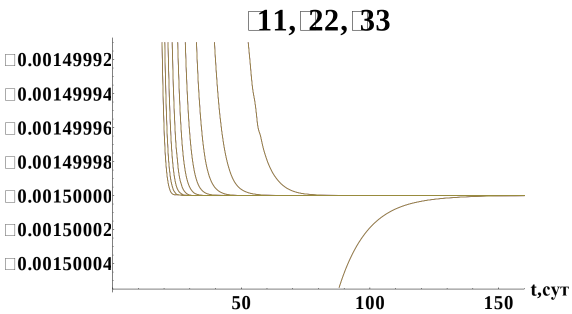




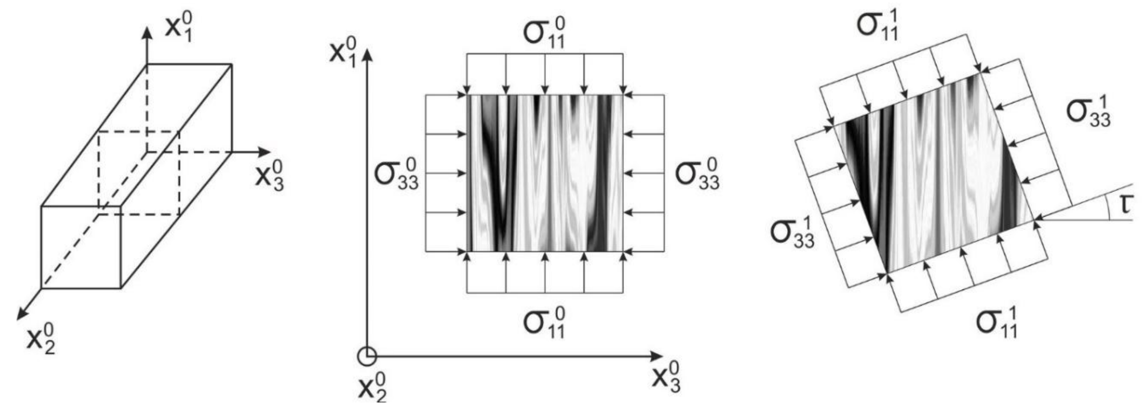
c) d)

Fig. 3. Changes in the structure (the component of the structure tensor deviator K1(t) (b), K2(t) (c)) and density (the share of solid bone volume e(t)) of the trabecular microstructure of (a). A strain diagram (d)

Let us consider the problem of the rotation of the stress tensor for the following stress states: a uniform and non-uniform all-around compression. We obtain appropriate ratios of changes in strain, structure and density of the trabecular microstructure to a gradual change in the rotation angle τ. Further, we introduce an orthogonal transformation $\tilde{Q}(\tau)$ from the main directions of the tensor $\tilde{\sigma}_1$ to the main directions of the tensor $\tilde{\sigma}_0$:

$$\tilde{Q}(\tau)=\begin{pmatrix} Cos\tau & 0 & -Sin\tau \\ 0 & 1 & 0 \\ Sin\tau & 0 & \mathrm{Cos}\,\tau \end{pmatrix},$$

where τ – an angle in the plane 1–3 between the main axes (coordinate system) $P(\tilde{\sigma}_1)$ and $P(\tilde{\sigma}_0)$ – see Fig. 4.



a *b* *c*

Fig. 4. Illustration of the rearrangement of the trabecular bone tissue structure from one state of physiological equilibrium to another: a – the studied area; b – a bone cross-section in the plane 1–3 prior to the load variation; c – load variation in the bone cross-section in the plane 1–3

Therefore, the new stress state $\tilde{\sigma}_1$ is defined as follows:

$$\tilde{\sigma}_1^{P(\tilde{\sigma}_0)}=\tilde{Q}(\tau)\cdot\tilde{\sigma}_1^{P(\tilde{\sigma}_1)}\cdot\tilde{Q}(\tau)^T. \quad (7)$$

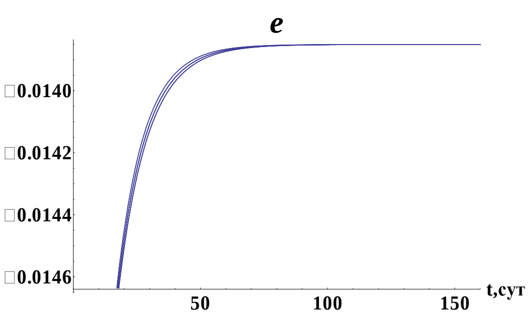
Obviously, if we consider the initial stress state in the form (5), we will find that $\tilde{\sigma}_1=\tilde{\sigma}_0$:

$$\tilde{\sigma}_1=\begin{pmatrix} -1,4024Cos^2\tau-1,4024Sin^2\tau & 0 & 0 \\ 0 & -1,4024 & 0 \\ 0 & 0 & -1,4024Cos^2\tau-1,4024Sin^2\tau \end{pmatrix}=\begin{pmatrix} -1,4024 & 0 & 0 \\ 0 & -1,4024 & 0 \\ 0 & 0 & -1,4024 \end{pmatrix}.$$

For this reason, we shall consider the case of non-uniform all-around compression, as in problem 1. In this case, the stress tensor $\tilde{\sigma}_1$ shall be:

$$\tilde{\sigma}_1=\begin{pmatrix} -1{,}4024\cdot Cos^2\tau-1{,}5472\cdot Sin^2\tau & 0 & 0{,}1448\cdot Cos\tau\cdot Sin\tau \\ 0 & -1{,}4024 & 0 \\ 0{,}1448\cdot Cos\tau\cdot Sin\tau & 0 & -1{,}5472\cdot Cos^2\tau-1{,}4024\cdot Sin^2\tau \end{pmatrix}.$$

We shall numerically solve the system (1) - (3), by changing the angle $\tau\in\left[0,\frac{\pi}{2}\right]$ in increments of $\frac{\pi}{2}$. We obtain the following results: $e_1=-0{,}0139$, the strain tensor again takes the form (6), and change in the structure tensor directly depends on the rotation angle. In Figure 5 (b), the component K_1 varies upwards. The component K_2 takes on value of -0.005.



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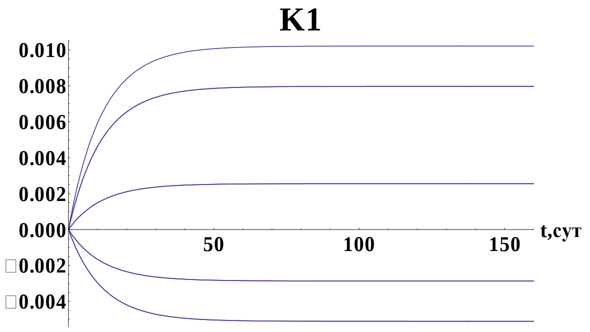

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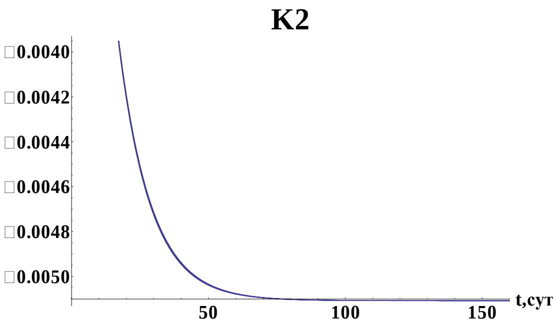

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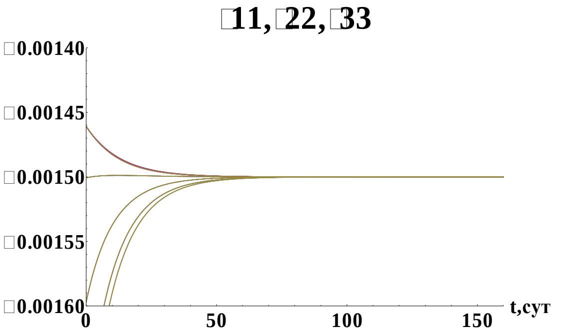

d)

Fig. 5. Changes in the structure (the component of the structure tensor deviator K1(t) (b), K2(t) (c)) and density (the share of solid bone volume e(t)) of the trabecular microstructure of (a). A strain diagram (d)

Conclusion

During computing, we determined the initial change in solid volume e_0 and the initial structure tensor $\tilde{K}_0$, built a numerical solution for actual structure tensor $\tilde{K}_1(\mathrm{t})$, the strains $\tilde{\varepsilon}_1(\mathrm{t})$, and changes of solid volume $e_1(t)$ in the interval $t \in [0,160]$ (day), and also obtained the corresponding diagrams. The problem 1 shows graphically that the bone microstructure comes in homeostatic balance regardless of the size and sampling of the parameter *d.* As for the problem of rotation, it should be noted that the variation in the solid volume depends directly on the parameter *d,* and changes abruptly at a value of *d*=1.1, namely, - the function diagram changes the direction of the convexity; the strain diagram shows that the bone microstructure also comes to the homeostatic equilibrium. In the problem 3, the component of the structure tensor deviator $K_1(t)$ is the most sensitive to changes in the parameter *d.* Similarly to problem 2, it changes its character upon reaching the value of *d*=1.1. The bone microstructure comes again to homeostatic equilibrium.

Summary

Based on the results, we can speak about the following character of bone restructuring: various loads considered in the model problems lead to changes in the solid volume of the bone substance, to the strains, leveling in magnitude over time, indicating the achievement of homeostatic equilibrium, and to the rotation of the main directions of the structure tensor components [9-11], which follows from the diagrams of variation in the components of structure tensor deviator.

Acknowledgements

The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University and the work was partially supported by the Russian Foundation for Basic Research within scientific projects No. 16-04-00772.

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