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Research Article

ON DOMINATION NUMBER AND STAR CHROMATIC NUMBER OF CORONA GRAPHS

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Abstract

In this paper, we find the domination number and independent domination number of corona graphs $P_n \circ K_n$ and $P_n \circ C_n$. Further, we establish some relation between the domination number and star chromatic number of corona graphs $P_n \circ K_n$ and $P_n \circ C_n$.

Keywords: Domination number, Independent domination number, Star chromatic number, Corona graphs.

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1. Introduction

In this paper, we consider simple graphs that are finite, connected and undirected. A subset $D \subseteq V$ of a graph $G$is a dominating set if every vertex in $V - D$ is adjacent to atleast one vertex in D. A dominating set of G is a γ-set. The terms dominating set and domination number of a graph Gwere first defined by Ore in 1962, [7]. The domination number of G denotedby $\gamma(G)$, is the cardinality of a minimum dominating set of G. A dominating set $D$of a graph G is said to be an independent dominating set if no two vertices of D are adjacent.The independent domination number of a graph G denoted by$\gamma_i(G)$, is the cardinality of a minimum independent dominating set of G. The notion of star chromatic number was introduced by Branko Grünbaum in 1973, [3].

A star coloring [3] of a graph G is a proper vertex coloring in which every pathon four vertices uses at least three distinct colors. Equivalently, in a star coloring, theinduced subgraphs formed by the vertices of any two colors has connected componentsthat are star graphs. The star chromatic number $\chi_s(G)$ of G is the least number ofcolors needed to star color G.For other, basic graph theory definitions refer the reader to [1,4].

2. Preliminaries

We will use the following definition and lemmas of corona products to prove our main results.

Definition 2.1.[2]The corona of twographs G_1 and $G_2$is the graph $G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2, where the i^{th} vertex ofG_1 is adjacent to every vertex in the i^{th} copy ofG_2.

Lemma 2.2. [5]If P_n is a path with n vertices and K_n is a complete graph with n vertices then the star chromatic number of corona product of these graphs is $\chi_s(P_n \circ K_n) = n + 2$.

Lemma 2.3.[5]If P_n is a path with n vertices and C_n is a cycle with n vertices then the star chromatic number of corona product of these graphs is $\chi_s(P_n \circ C_n) = \begin{cases} 6 & if\ n = 5 \\ 5 & otherwise \end{cases}$.

3. Main Results

3.1. Domination number of $P_n \circ K_n$.

In this section, our aim is to find the domination number, independent domination number of corona graph$P_n \circ K_n$ and discuss the relation between the domination number and star chromatic number of corona graph$P_n \circ K_n$.

Theorem 3.1.1 For any$n \geq 2, \gamma(P_n \circ K_n) = n$.

Proof. Let$V(P_n) = \{v_1, v_2, \ldots, v_n\}$and $V(K_n) = \{u_1, u_2, \ldots, u_n\}$.Let$V(P_n \circ K_n) = \{v_i: 1 \leq i \leq n\} \cup \{u_{ij}: 1 \leq i \leq n;\ 1 \leq$ $j \leq n$. By the definition of corona graph, each vertex of Pn is adjacent to every vertex ofa copy of Kn.i.e., every vertex $v_i \in V(P_n)$ is adjacent to every vertex from the set$\{u_{ij}: 1 \leq i \leq n;\ 1 \leq j \leq n\}$. We prove our result by discussing the following cases:

***Case (i)*:** Suppose that $D \subseteq V(P_n)$.

Let D be a dominating set of $P_n \circ K_n$. The set of vertices $\{v_i: 1 \leq i \leq n\}$ of the path P_n is a dominating set D in corona graph $P_n \circ K_n$. i.e., $\{v_i: 1 \leq i \leq n\}$ has atleast one adjacent vertex in P_n and K_n. Therefore, $|P_n| = |D| = \gamma(P_n \circ K_n) = n$for any $n \geq 2$.

***Case (ii)*:** Suppose that $D \subseteq V(K_n)$.

Let D be a dominating set of $P_n \circ K_n$. The set of vertices $\{u_{i1}: 1 \leq i \leq n\}$ of the complete graph K_n is a dominating set D in corona graph $P_n \circ K_n$. Threfore, $|D| = \gamma(P_n \circ K_n) = n$for any $n \geq 2$.

***Case (iii)*:** Suppose that $D \subset V(P_n) \cup V(K_n)$.

Let D be a dominating set of $P_n \circ K_n$ such that $D = \left\{v_{2i-1}: 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor + 1\right\} \cup \{u_{2i\,1}: 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\}$ or $D = \left\{v_{2i-1\,2i-1}: 1 \le\right.$ $i \le n2+1 \cup v2i: 1 \le i \le n2$. Therefore, $D = \gamma Pn \circ Kn = n$.

Hence, $\gamma(P_n \circ K_n) = n$ for all $n \ge 2$.

Theorem 3.1.2 For any $n \ge 2, \gamma_i(P_n \circ K_n) = n$.

Proof. Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $V(K_n) = \{u_1, u_2, \dots, u_n\}$. Let $V(P_n \circ K_n) = \{v_i: 1 \le i \le n\} \cup \{u_{ij}: 1 \le i \le n;\ 1 \le$ $j \le n$. By the definition of corona graph, each vertex of Pn is adjacent to every vertex ofa copy of Kn i.e., every vertex $v_i \in V(P_n)$ is adjacent to every vertex from the set $\{u_{ij}: 1 \le i \le n;\ 1 \le j \le n\}$. We prove our result by discussing the following cases:

***Case (i)*:** Assume that $D \subseteq V(P_n)$.

Let D be a independent dominating set of $P_n \circ K_n$. The set of vertices $\{v_i: 1 \le i \le n\}$ of the path P_n is a dominating set D in corona graph $P_n \circ K_n$. i.e., $\{v_i: 1 \le i \le n\}$ has atleast one adjacent vertex in P_n, a contradiction to independent dominating set. Therefore our assumption is wrong and hence $D \nsubseteq V(P_n)$.

***Case (ii)*:** Suppose that $D \subseteq V(K_n)$.

Let D be an independent dominating set of $P_n \circ K_n$. The set of vertices $\{u_{i1}: 1 \le i \le n\}$ of the complete graph K_n is an independent dominating set D in corona graph $P_n \circ K_n$. By the definition of independent dominating set [6], every vertex in D has no adjacent vertex in D. Thus, $|D| = \gamma_i(P_n \circ K_n) = n$ for any $n \ge 2$.

***Case (iii)*:** Suppose that $D \subset V(P_n) \cup V(K_n)$.

Let D be an independent dominating set of $P_n \circ K_n$ such that $D = \left\{v_{2i-1}: 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor + 1\right\} \cup \{u_{2i\,1}: 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\}$ or $D = \left\{u_{2i-1\,2i-1}: 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor + 1\right\} \cup \{v_{2i}: 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\}$. Therefore, $|D| = \gamma_i(P_n \circ K_n) = n$. Hence, $\gamma_i(P_n \circ K_n) = n$ for all $n \ge 2$.

Remark 3.1.3

We note that the connected domination and total domination number of corona graph $P_n \circ K_n$ as

(i) $\gamma_c(P_n \circ K_n) = \gamma(P_n \circ K_n)$

(ii) $\gamma_t(P_n \circ K_n) = \gamma(P_n \circ K_n)$

Corollary 3.1.4 For any $n \ge 2, \gamma(P_n \circ K_n) + \chi_s(P_n \circ K_n) = 2n + 2$.

Proof. From Theorem 3.1.1, $\gamma(P_n \circ K_n) = n$ for $n \geq 2$ and Lemma 2.2, $\chi_s(P_n \circ K_n) = n + 2$ for $n \geq 2$ and so $\gamma(P_n \circ K_n) + \chi_s(P_n \circ K_n) = 2n+2$. Hence, $\gamma(P_n \circ K_n) + \chi_s(P_n \circ K_n) = 2n+2$.

3.2Domination number of $P_n \circ C_n$.

In this section, our aim is to find the domination number, independent domination number of $P_n \circ C_n$ and discuss the relation between the domination number and star chromatic number of $P_n \circ C_n$.

Theorem 3.2.1 For any$n \geq 3, \gamma(P_n \circ C_n) = n$.

Proof. LetV$(P_n) = \{v_1, v_2, \dots, v_n\}$and V$(C_n) = \{u_1, u_2, \dots, u_n\}$.LetV$(P_n \circ C_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n;\ 1 \leq j \leq n\}$. By the definition of corona graph, each vertex of Pn is adjacent to every vertex ofa copy of Cn. i.e., every vertex $v_i \in V(P_n)$ is adjacent to every vertex from the set$\{u_{ij} : 1 \leq i \leq n;\ 1 \leq j \leq n\}$. We prove our result by discussing the following cases:

***Case (i)*:**Suppose that $D \subseteq V(P_n)$.

Let D be a dominating set of $P_n \circ C_n$. The set of vertices $\{v_i : 1 \leq i \leq n\}$ of the path P_n is a dominating set D in corona graph $P_n \circ C_n$. i.e., $\{v_i : 1 \leq i \leq n\}$ has atleast one adjacent vertex in P_n and K_n. Therefore, $|P_n| = |D| = \gamma(P_n \circ C_n) = n$ for any $n \geq 3$.

***Case (ii)*:** Suppose that $D \subseteq V(C_n)$.

Let D be a dominating set of $P_n \circ C_n$. The set of vertices $\{u_{i1} : 1 \leq i \leq n\}$ of the cycle C_n is a dominating set D in corona graph $P_n \circ C_n$. Threfore, $|D| = \gamma(P_n \circ C_n) = n$ for any $n \geq 3$.

***Case (iii)*:** Suppose that $D \subset V(P_n) \cup V(C_n)$.

Let D be a dominating set of $P_n \circ K_n$ such that $D = \left\{v_{2i-1} : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor + 1\right\} \cup \{u_{2i\ 1} : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor\}$ or $D = \left\{v_{2i-1\ 2i-1} : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor + 1\right\} \cup \{v_{2i} : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor\}$. Therefore, $|D| = \gamma(P_n \circ C_n) = n$.

Hence, $\gamma(P_n \circ C_n) = n$ for all $n \geq 3$.

Theorem 3.2.2 For any$n \geq 3, \gamma_i(P_n \circ C_n) = n$.

Proof. Let V$(P_n) = \{v_1, v_2, \dots, v_n\}$ and V$(C_n) = \{u_1, u_2, \dots, u_n\}$. Let V$(P_n \circ C_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n;\ 1 \leq j \leq n\}$. By the definition of corona graph, each vertex of P_n is adjacent to every vertex ofa copy of C_n. i.e., every vertex

$v_i \in V(P_n)$ is adjacent to every vertex from the set $\{u_{ij}: 1 \le i \le n;\ 1 \le j \le n\}$. We prove our result by discussing the following cases:

***Case (i)*:**Assume that $D \subseteq V(P_n)$.

Let D be a independent dominating set of $P_n \circ C_n$. The set of vertices $\{v_i: 1 \le i \le n\}$ of the path P_n is a dominating set D in corona graph $P_n \circ C_n$. i.e., $\{v_i: 1 \le i \le n\}$ has atleast one adjacent vertex in P_n, a contradiction to independent dominating set. Therefore our assumption is wrong and hence $D \nsubseteq V(P_n)$.

***Case (ii)*:**Suppose that $D \subseteq V(C_n)$.

Let D be an independent dominating set of $P_n \circ C_n$. The set of vertices $\{u_{i1}: 1 \le i \le n\}$ of the complete graph C_n is an independent dominating set D in corona graph $P_n \circ C_n$. By the definition of independent dominating set [6], every vertex in D has no adjacent vertex in D. Thus, $|D| = \gamma_i(P_n \circ C_n) = n$ for any $n \ge 3$.

***Case (iii)*:**Suppose that$D \subset V(P_n) \cup V(C_n)$.

Let D be an independent dominating set of $P_n \circ C_n$ such that $D = \left\{v_{2i-1}: 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor + 1\right\} \cup \left\{u_{2i\,1}: 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor\right\}$ or $D = \left\{u_{2i-1\;2i-1}: 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor + 1\right\} \cup \left\{v_{2i}: 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor\right\}$. Therefore, $|D| = \gamma_i(P_n \circ C_n) = n$.Hence, $\gamma_i(P_n \circ C_n) = n$ for all $n \ge 3$.

Remark 3.2.3

We note that the connected domination and total domination number of corona graph $P_n \circ C_n$ as

(i) $\gamma_c(P_n \circ C_n) = \gamma(P_n \circ C_n)$

(ii) $\gamma_t(P_n \circ C_n) = \gamma(P_n \circ C_n)$

Corollary 3.2.4 For any$n \ge 3, \gamma(P_n \circ C_n) + \chi_s(P_n \circ C_n) = \begin{cases} n+6 & if\ n = 5 \\ n+5 & otherwise \end{cases}$

Proof. From Theorem 3.2.1, $\gamma(P_n \circ C_n) = n$for $n \ge 2$ and Lemma 2.3, $\chi_s(P_n \circ C_n) = \begin{cases} 6 & if\ n = 5 \\ 5 & otherwise \end{cases}$for $n \ge 3$ and so

$\gamma(P_n \circ C_n) + \chi_s(P_n \circ C_n) = \begin{cases} n+6 & if\ n = 5 \\ n+5 & otherwise \end{cases}$.

Hence, $\gamma(P_n \circ C_n) + \chi_s(P_n \circ C_n) = \begin{cases} n+6 & if\ n = 5 \\ n+5 & otherwise \end{cases}$.

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