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ENERGY OF FUZZY GRAPHS: AREVIEW

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Abstract

After introducing and developing fuzzy set theory, a lot of studies have been done in this field and then a result appeared as anEnergy of Fuzzy Graph (Combination of graph theory, matrices and fuzzy set theory). This is now known as Energy of Fuzzy graph theory. In this article we review essential works on different types of energy of fuzzy graphs.

Keywords: Energy of fuzzy graph, Laplacian Energy of Fuzzy Graphs, Energy of an Intuitionistic Fuzzy Graph, Laplacian Energy of an Intuitionistic Fuzzy Graph.

1. Introduction

In 1965, L.A.Zadeh [11] introduced the concepts of a fuzzy subset of a set as a way for representing uncertainty. His idea has been applied to a wide range of scientific areas. A Fuzzy conceptis also introduced in graph theory. A fuzzy graph $G=(V,\sigma,\mu)$ is a non-empty set *V* together with a pair of functions $\sigma\rightarrow[0,1]$ and $\mu=V\times V\rightarrow[0,1]$ such that for all inV, σ is called the fuzzy vertex set of *G* and μ is called the fuzzy edge set of *G*.

In 1986, Atanassov [2] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In the definition of intuitionistic fuzzy set, Atanassov [3] added a new component, which determines the degree of non-membership of an element in a given set. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science,engineering, mathematics, medicine, chemistry, economics etc.. In 1975, Rosenfeld [7] introduced the concept of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and developed the structure of fuzzy graphs, obtaining analogues of several graph theoretical concepts. Atanassov [3] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs.

Several matrices can be associated to a fuzzy graph such as the adjacency matrix (denoted by *A*) or the Laplacian fuzzy matrix $L=D-A$ where *D* is the diagonal fuzzy matrix of degrees. Some structural properties can be deduced from their spectrum but in general we can't determine a fuzzy graph from its adjacency or Laplacian spectrum. A concept related to the spectrum of a fuzzy graph is that of energy. As its name suggests, it is inspired by energy in chemistry. In 2013, Anjali and Sunil Mathew [1] defined energy of fuzzy graphs. Energy of fuzzy graphs has many mathematical properties which are being investigated. Energy of different fuzzy graphs including regular, non-regular and intuitionistic fuzzy graphs is also under study.

2. Preliminaries

Definition 2.1[1] Let $\tilde{G} = (\sigma, \mu)$ be a fuzzy graph with p vertices and q edges .The adjacency matrix of $\tilde{G} = (\sigma, \mu)$ is a square matrix of order p whose $\tilde{A}(\tilde{G}) = [\tilde{a}_{ij}]$ where $\tilde{a}_{ij} = \mu(u_i, u_j)$ entry is as the strength of relation between the vertices u_i and u_j.

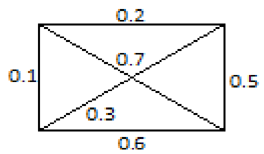

Figure 1. A fuzzygraph $\tilde{G}_1$

Example 2.2 Suppose $\tilde{G}_1$ is a fuzzy graph depicted in Figure 1.Adjacency matrix of the fuzzy graph $\tilde{G}_1$ is

$$\tilde{A} = \begin{bmatrix} 0 & 0.2 & 0.7 & 0.1 \\ 0.2 & 0 & 0.5 & 0.3 \\ 0.7 & 0.5 & 0 & 0.6 \\ 0.1 & 0.3 & 0.6 & 0 \end{bmatrix}$$

Definition 2.3 [1] Let $\tilde{G} = (V, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. The eigenvalues of *A* are called the eigenvalues of *G*.The spectrum of *A* is called the spectrum of *G* and is denoted by spec *G*.

Definition 2.4 [1] Energy of fuzzy graph $\tilde{G} = (V, \sigma, \mu)$ is defined as the sum of the absolute values of the eigenvalues *A*. i.e., if $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ are eigenvalues of *G* then energy of fuzzy graph $E(G) = \sum_{i=1}^{n} |\lambda_i|$.

Theorem 2.5[1] Let $\tilde{G} = (V, \sigma, \mu)$ be a fuzzy graph with $|V| = n$ and $\mu^* = \{e_1, e_2, e_3, ..., e_m\}$. If $m_i = (e_i)$ is the strength of the relation associated with the *i*th edge, then

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)|A|^{2/n}} \leq E(G) \leq \sqrt{2\left(\sum_{i=1}^{m} m_i^2\right) n}$$

Definition 2.6 Let u be a vertex of the fuzzy graph $\tilde{G} = (V, \sigma, \mu)$ .The degree of u is defined as

$$d_{\tilde{G}}(u) = \sum_{uv \in E(\tilde{G})} \mu(uv).$$

Definition2.7 Let $\tilde{G} = (\sigma, \mu)$ be a fuzzy graph on G= (V, E) of $d_{\tilde{G}}(v) = k$ for all $v \in V$, then $\tilde{G}$ is said to be a regular fuzzy graph of degree *k* or a *k*-regular fuzzy graph.

Definition 2.8 Let $\tilde{G} = (\sigma, \mu)$ be a fuzzy graph with *n* vertices and *m* edges. The degree matrix of $\tilde{G} = (\sigma, \mu)$ is a square matrix of order *n* whose $D(\tilde{G}) = [\tilde{d}_{ij}]$ and we have:

$$\tilde{d}_{ij} = \begin{cases} d_{\tilde{G}}(v_i) & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}.$$

Definition2.9[6]An intuitionistic fuzzy graph is defined as $G = (V, E, \mu, \gamma)$ where *V* is the set of vertices and *E* is the set of edges ,μ is a fuzzy membership function defined on $V \times V$ and γ is a fuzzy non membership function we define $\mu(v_i, v_j)$ by μ_{ij} and $\gamma(v_i, v_j)$ by γ_{ij} such that (i) $0 \le \mu_{ij} + \gamma_{ij} \le 1$ (ii) $0 \le \mu_{ij}, \gamma_{ij}, \pi_{ij} \le 1$ where $\pi_{ij} = 1 - \mu_{ij} - \gamma_{ij}$ .Hence $(V \times V, \mu, \gamma)$ is an Intuitionistic fuzzy graph.

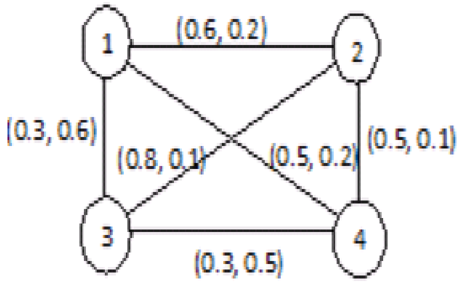

Figure 2: $\tilde{G}$, An intuitionistic fuzzy graph

Example 2.10

Definition 2.11 [6]An intuitionistic fuzzy adjacency matrix of an intuitionistic fuzzy graph is defined as the adjacency matrix of the corresponding intuitionistic fuzzy graph. That is for an intuitionistic fuzzy graph $G = (V, E, \mu, \gamma)$ an intuitionistic fuzzy adjacency matrix is defined by $A(IG) = [a_{ij}]$ where $a_{ij} = (\mu_{ij}, \gamma_{ij})$.Note that μ_{ij} represents the strength of the relationship between v_i, v_j and γ_{ij} the strength of non-relationshipbetween v_i and v_j.

Example 2.12 For an intuitionistic fuzzy graph in fig 2.the adjacency matrix is

$$A(IG) = \begin{bmatrix} 0 & (0.6,0.2) & (0.5,0.2) & (0.3,0.6) \\ (0.6,0.2) & 0 & (0.5,0.1) & (0.8,0.1) \\ (0.5,0.2) & (0.5,0.1) & 0 & (0.3,0.5) \\ (0.3,0.6) & (0.8,0.1) & (0.3,0.5) & 0 \end{bmatrix}$$

Definition 2.13 The adjacency matrix of an intuitionistic fuzzy graph can be written as two matrices one containing the entries as membership values and other containing membership values i.e., $A(I(G)) = [(\mu_{ij}),(\gamma_{ij})]$ where

$$A\ (\mu_{ij}) = \begin{bmatrix} 0 & 0.6 & 0.5 & 0.3 \\ 0.6 & 0 & 0.5 & 0.8 \\ 0.5 & 0.5 & 0 & 0.3 \\ 0.3 & 0.8 & 0.3 & 0 \end{bmatrix} \text{ and } A\ (\gamma_{ij}) = \begin{bmatrix} 0 & 0.2 & 0.2 & 0.6 \\ 0.2 & 0 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0 & 0..5 \\ 0.6 & 0.1 & 0.5 & 0 \end{bmatrix}$$

3. Types of Energy of Fuzzy Graphs

3.1 Laplacian Energy of Fuzzy Graphs [9]

Definition 3.1.1[6] Let $A(\tilde{G})$ be a adjacency matrix and $D(\tilde{G}) = [d_{ij}]$ be a degree matrix of $\tilde{G} = (\sigma, \mu)$. The matrix $L(\tilde{G}) = D(\tilde{G}) - A(\tilde{G})$ is defined as fuzzy Laplacian matrix of $\tilde{G}$.

Definition 3.1.2 Let $L(\tilde{G})$ be a fuzzy Laplacian matrix of $\tilde{G} = (\sigma, \mu)$. The Lapacian polynomial of G is the characteristic polynomial of its Laplacian matrix, $\phi(\tilde{G}, \tilde{\mu}) = \det(\tilde{\mu} I_n - L(\tilde{G}))$. The roots of $\phi(\tilde{G}, \tilde{\mu})$ is the fuzzy Laplacian eigenvalues of $\tilde{G}$.

Definition 3.1.3 Let $\tilde{G} = (\sigma, \mu)$ be a fuzzy graph with $|V| = n$ vertices and $\tilde{\mu}_1 \geq \tilde{\mu}_2 \geq \cdots \geq \tilde{\mu}_n$ be the Laplacian eigenvalues of $\tilde{G} = (\sigma, \mu)$. The Laplacian energy of fuzzy graph $\tilde{G} = (\sigma, \mu)$ is defined as

$$LE(\tilde{G}) = \left| \tilde{\mu}_i - \frac{2 \sum_{1 \leq i < j \leq n} \mu(u_i, u_j)}{n} \right|$$

3.2 Energy of an Intuitionistic Fuzzy Graph [6]

Definition 3.2.1 An intuitionistic fuzzy adjacency matrix of an intuitionistic fuzzy graph is defined as the adjacency matrix of the corresponding intuitionistic fuzzy graph. That is for an intuitionistic fuzzy graph $G = (V, E, \mu, \gamma)$, an intuitionistic fuzzy adjacency matrix is defined by $A(IG) = [a_{ij}]$ where $a_{ij} = (\mu_{ij}, \gamma_{ij})$.Note that μ_{ij} represents the strength of the relationship between v_i and v_j and γ_{ij} the strength of non-relationship between v_i and v_j .

Definition 3.2.2 The adjacency matrix of an intuitionistic fuzzy graph can be written as two matrices one containing the entries as membership values and other containing membership values i.e., $A(I(G)) = [(\mu_{ij}),(\gamma_{ij})]$.

Definition 3.2.3 The eigenvalues of an intuitionistic fuzzy adjacency matrix $A(IG)$ is defined as (X, Y) where X is the set of eigenvalues of $A(\mu_{ij})$ and Y is the set of eigenvalues of $A(\gamma_{ij})$.

Definition 3.2.4 The Energy of an intuitionistic fuzzy graph $G=(V,E,\mu,\gamma)$ is defined as $\left(\sum_{\lambda_i\in X}|\lambda_i|,\sum_{\delta_i\in Y}|\delta|\right)$ where $\sum_{\lambda_i\in X}|\lambda_i|$ is defined as an energy of the non-membership matrix denoted by $E(\mu_{ij}(G))$ and $\sum_{\delta_i\in Y}|\delta_i|$ is defined as an energy of the non membership matrix denoted by $E(\gamma_{ij}(G))$.

3.3 Laplacian Energy of an Intuitionistic Fuzzy Graph [10]

Definition 3.3.1 Let $A(IG)$ be a adjacency matrix and $D(IG)=\lfloor d_{ij}\rfloor$ be a degree matrix of $IG=(V,E,\mu,\gamma)$.The matrix $L(IG)=D(IG)-A(IG)$ is defined as intuitionistic fuzzy Laplacian matrix of *IG*.

Definition 3.3.2 Let $L[\mu_{ij}]$ be a intuitionistic fuzzy Laplacian matrix of $IG=(V,E,\mu,\gamma)$.The laplacian polynomial of intuitionistic fuzzy graph is the characteristic polynomial of its Laplacian matrix $\phi(IG,\mu)$=det$(\tilde{\mu}I_n-L(IG))$.The roots of $\phi(IG,\mu)$ is the intuitionistic fuzzy Laplacian eigenvalues of *IG*.

Definition 3.3.3 Let $\tilde{G}=(V,E,\sigma,\mu)$ be an intuitionistic fuzzy graph with $|V|=n$ Vertices and $\lambda_1\geq\lambda_2\geq..\geq\lambda_n$ is the membership Laplacian eigenvalues of IG. The Laplacian energy intuitionistic fuzzy graph is defined as,

$$LE(\mu_{ij}(G))=\left|\lambda_i-\frac{2\sum_{1\leq i\leq j\leq n}\mu(u_i,u_j)}{n}\right|$$

The Laplacian energy of intuitionistic fuzzy graph $G=(V,E,\mu,\gamma)$is defined as

$$\left[LE(\mu_{ij}(G)),LE(\gamma_{ij}(G))\right]$$

3.4 Dominating Energy of an Intuitionistic Fuzzy Graph [7]

Definition 3.4.1 An arc (v_i,v_j) of an intuitionistic fuzzy graph G is called a Strong arc if $\mu_2(v_i,v_j)\leq\mu_1(v_i)\wedge\mu_1(v_j)$ and $\gamma_2(v_i,v_j)\leq\gamma_1(v_i)\wedge\gamma_1(v_j)$.

Definition 3.4.2 Let $G=(V,E)$ be an intuitionistic fuzzy graph. Let $u,v\in V$, we say that u dominates v in *G* if there exist a Strong arc between them. A subset $D\subseteq V$ is said to be dominating set in *G* if for every $v\in V-D$, there exist u in D such that *u* dominates *v*.

Definition 3.4.3A dominating set D of an intuitionistic fuzzy graph is said to be minimal dominating set if no proper subset of D is a dominating set. Minimum cardinality among all minimal dominating set is called the intuitionistic fuzzy domination number.

Definition 3.4.4 An intuitionistic fuzzy graph is defined as $G=(V,E,\mu,\gamma)$ where V is the set of vertices and E is the set of edges such that $\mu:V\times V\rightarrow[0,1]$, $\gamma:V\times V\rightarrow[0,1]$ denote the degree of membership and non-membership of $E\subseteq V\times V$ respectively and $0\le\mu(v_i,v_j)+\gamma(v_i,v_j)\le1$. Now define an intuitionistic fuzzy set $(\mu_1,\gamma_1):V\rightarrow[0,1]$ by $\mu_1(v_i)=\max_{v_j}(\mu(v_i,v_j)),\gamma_1(v_i)=\max_{v_j}(\gamma(v_i,v_j)),v_i\in V$.

Definition 3.4.5 An arc (v_i,v_j) of dominating intuitionistic fuzzy graph $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ is called a Strong arc if $\mu_2(v_i,v_j)\le\mu_1(v_i)\wedge\mu_1(v_j)and\gamma_2(v_i,v_j)\le\gamma_1(v_i)\wedge\gamma_1(v_j)$.

Definition 3.4.6 Let$G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a dominating intuitionistic fuzzy graph. Let $u,v\in V$, we say that u dominates$v$ in G if there exist a Strong arc from u to v. A subset $D\subseteq V$ is said to be dominating set in G if for every $v\in V-D$, there exist u in D such that u dominates v.

Definition 3.4.7 Let$G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a dominating intuitionistic fuzzy graph. A dominating intuitionistic fuzzy adjacency matrix $D(G)=(d_{ij})$ where

$$d_{ij}=\begin{cases}(\mu_{ij},\gamma_{ij}), & \text{if } (v_i,v_j)\in E\\ (1,1), & \text{if } i=j\ \&\ v_i\in D\\ 0, & \text{otherwise}\end{cases}$$

This dominating intuitionistic fuzzy adjacency matrix $D(G)$ can be written as $D(G)=(\mu_D(G),\gamma_D(G))$ where

$$\mu_D(G)=\begin{cases}\mu_{ij}, \text{if } (v_i,v_j)\in E\\ 1 \quad \text{if } i=j\ \&\ v_i\in D \text{ and}\\ 0, \quad \text{otherwise}\end{cases}$$

$$\gamma_D(G)=\begin{cases}\gamma_{ij}, \text{if } (v_i,v_j)\in E\\ 1 \quad \text{if } i=j\ \&\ v_i\in D\\ 0, \quad \text{otherwise}\end{cases}$$

3.5 Max-Min Intuitionistic Fuzzy Matrix of an Intuitionistic Fuzzy Graph [4]

Definition 3.5.1 If $(E(\mu_{ij}(G)))>(E(\gamma_{ij}(G)))$ i.e., If the number of visitors are maximum in an intuitionistic fuzzy network then the spreading rate of virus will be maximum. If $(E(\mu_{ij}(G)))>(E(\gamma_{ij}(G)))$ then the infection rate of an intuitionistic fuzzy network is defined as $\beta=\max_{i,j}\mu_{ij}$ and the curing rate of an intuitionistic fuzzy network is defined as $\delta=\max_{i,j}\gamma_{ij}$ .The ratio $\tau=\frac{\beta}{\delta}(\delta\ne0)$ is the effective spreading rate and $\tau_c=\frac{1}{\lambda_{\max}A(\mu_{ij})}$ where $\lambda_{\max}A(\mu_{ij})$ is the largest eigen value of the adjacency matrix $\lambda_{\max}A(\mu_{ij})$of an intuitionistic fuzzy network.

Definition 3.5.2 If $(E(\mu_{ij}(G))) < (E(\gamma_{ij}(G)))$ i.e., If the number of visitors are minimum in an intuitionistic fuzzy network then the spreading rate of virus will be minimum. If $(E(\mu_{ij}(G))) < (E(\gamma_{ij}(G)))$ then the infection rate of an intuitionistic fuzzy network is defined as $\beta = \min_{i,j} \mu_{ij}$ and the curingrate of an intuitionistic fuzzy network is defined as $\delta = \min_{i,j} \gamma_{ij}$ .The ratio $\tau = \frac{\beta}{\delta}(\delta \neq 0)$ is the effective spreading rate and $\tau_c = \frac{1}{\lambda_{\min} A(\mu_{ij})}$ where $\lambda_{\min} A(\mu_{ij})$ is the largest eigen value of the adjacency matrix $\lambda_{\min} A(\mu_{ij})$of an intuitionistic fuzzynetwork.

Definition 3.5.3 Let $G = (V, E, \mu, \gamma)$ be an intuitionistic fuzzy graph. $\forall$ node i, define $\alpha_j = \max_i \mu_{ij}$ and $\sigma_j = \underset{i}{mix} \gamma_{ij}$.

Definition 3.5.4 Let $G = (V, E, \mu, \gamma)$ be an intuitionistic fuzzy graph. The Max-Min intuitionistic fuzzy matrix of an intuitionistic fuzzy graph is defined as $M(G)((rij),(s_{ij}))$, where

$$r_{ij} = \begin{cases} \max(\alpha_i, \alpha_j), \text{if } \mu_{ij} \neq 0 \\ 0, \qquad \text{otherwise} \end{cases}$$

and

$$s_{ij} = \begin{cases} \min(\sigma_i, \sigma_j), \text{ if } \gamma_{ij} \neq 0 \\ 0, \qquad \text{otherwise} \end{cases}$$

4. Conclusion

In this paper we are willing to summarize types of energy fuzzy graph, structure of energy fuzzy graph and also types of intuitionistic energy fuzzy graph along with definition related to respective topics. We hope this paper will help the researcher to look the field of energy fuzzy graph at glance.

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