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MAX-MIN INTUITIONISTIC LAPLACIAN FUZZY MATRIX OF AN INTUITIONISTIC FUZZY GRAPH

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Abstract

In this paper, we introduce the Max-Min intuitionistic laplacian fuzzy matrix $M_L(G)$ of an intuitionistic fuzzy graph. Also defined the extreme laplacian energy of $M_L(G)$. We also give the explicit expression for the coefficients of the characteristic polynomial of $M_L(G)$. These concepts are illustrated with real time examples.

Key Words: Intuitionistic fuzzy graph, Laplacian energy of Intuitionistic fuzzy graph.

1. Introduction

In this paper, we are concerned with simple graphs. Let G be a graph, possessing n vertices and m edges. Many kinds of matrices are associated with a graph. The spectrum of adjacency matrix is called the spectrum of the graph. The properties of the spectrum of a graph are related to the properties of the graph. The area of graph theory that deals with this is called spectral graph theory.

In 1973, the definition of fuzzy graph was introduced by Kafmann[8] from Zadeh's[14] fuzzy relations . In 1975, Rosenfeld [12] introduced another detailed definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. In 1999, the first definition of intuitionistic fuzzy graph was introduced by Atanassov [3]. In 1978, Gutmann [6] introduced the concept of energy of the graph as the sum of the absolute values of the eigen values of the adjacency matrix of the graph. In 2006, Gutmann [7] investigated the Laplacian energy of a graph as LE(G) = $\sum_{i=1}^{n}\left|\mu_i - \frac{2m}{n}\right|$ where $\mu_1, \mu_2, \ldots, \mu_n$ are the spectrum of laplacian matrix. In 2014 Sadegh Rahimi Sharbaf [13] introduced the Laplacian energy of a fuzzy graph and using this property, they defined the laplacian energy of fuzzy graph and the upper and lower bounds for the laplacian energy of the fuzzy graph.In [11], the adjacency matrix of an intuitionistic fuzzy graph was introduced and found the upper and lower bounds for the energy of an intuitionistic fuzzy graph are obtained. We introduced the laplacian energy of an Intuitionistic fuzzy graph and also obtained the lower and upper bounds for the laplacian energy of an intuitionistic fuzzy graph. In this paper, we organised as follows. In section 2, we give the required preliminaries, in section 3, we define the Max-Min intuitionistic laplacian fuzzy matrix $M_L(G)$ of an intuitionistic fuzzy graph and the extreme Laplacian energy of $M_L(G)$ is defined. We give the explicit expression for the coefficients of the characteristic polynomial of $M_L(G)$. In section 4, we illustrate these concepts with real time example. In section 5, we give the conclusion.

2. Preliminaries

2.1. Intuitionistic Fuzzy graph

Definition 1 :- [See 10]An intuitionistic fuzzy graph is defined as G=(V,E,μ,γ) where V is the set of vertices and E is the set of edges ,μ is a fuzzy membership function defined on V×V and γ is a fuzzy non-membership function we define $\mu(v_i, v_j)$ by μ_{ij} and $\gamma(v_i, v_j)$ by γ_{ij} such that

(i) $0 \le \mu_{ij} + \gamma_{ij} \le 1$ (ii) $0 \le \mu_{ij}, \gamma_{ij}, \pi_{ij} \le 1$ where $\pi_{ij} = 1 - \mu_{ij} - \gamma_{ij}$ .Hence $(V \times V, \mu, \gamma)$ is an Intuitionistic fuzzy graph.

Example 2:

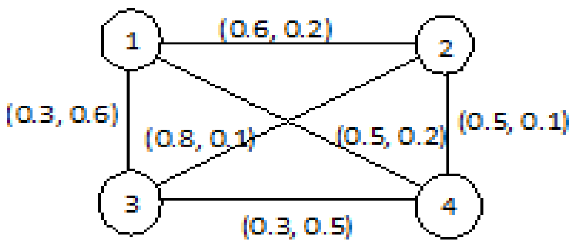

Figure 1: $\tilde{G}$, An intuitionistic fuzzy graph

Definition 3:-[See11] An intuitionistic fuzzy adjacency matrix of an intuitionistic fuzzy graph is defined as the adjacency matrix of the corresponding intuitionistic fuzzy graph. That is for an intuitionistic fuzzy graph G=(V,E,μ,γ), an intuitionistic fuzzy adjacency matrix is defined by A(IG)=[a_{ij}] where $a_{ij} = (\mu_{ij}, \gamma_{ij})$.Note that μ_{ij} represents the strength of the relationship between v_i and v_j and γ_{ij} the strength of non relationship between v_i and v_j.

Example 4:-For an intuitionistic fuzzy graph in fig 1. the adjacency matrix is

$$A\ (IG) = \begin{bmatrix} 0 & (0.6,0.2) & (0.5,0.2) & (0.3,0.6) \\ (0.6,0.2) & 0 & (0.5,0.1) & (0.8,0.1) \\ (0.5,0.2) & (0.5,0.1) & 0 & (0.3,0.5) \\ (0.3,0.6) & (0.8,0.1) & (0.3,0.5) & 0 \end{bmatrix}$$

Definition 5:- The adjacency matrix of an intuitionistic fuzzy graph can be written as two matrices one containing the entries as membership values and other containing non-membership values i.e. A (I (G)) =$\left[\left(\mu_{ij}\right),\left(\gamma_{ij}\right)\right]$ where

$$A\ (\mu_{ij}) = \begin{bmatrix} 0 & 0.6 & 0.5 & 0.3 \\ 0.6 & 0 & 0.5 & 0.8 \\ 0.5 & 0.5 & 0 & 0.3 \\ 0.3 & 0.8 & 0.3 & 0 \end{bmatrix} \text{ and } A\ (\gamma_{ij}) = \begin{bmatrix} 0 & 0.2 & 0.2 & 0.6 \\ 0.2 & 0 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0 & 0..5 \\ 0.6 & 0.1 & 0.5 & 0 \end{bmatrix}$$

2.2. Laplacian Energy of an Intuitionistic Fuzzy Graph

Definition 6:- The eigenvalues of an intuitionistic fuzzy adjacency matrix A (IG) is defined as (X, Y) Where X is the set of eigenvalues of A (μ_{ij}) and Y is the set of eigenvalues of

A(γ_{ij}).

Definition 7:- Let A (IG) be a adjacency matrix and D (IG) = [d_{ij}] be a degree matrix of IG=(V,E,μ,γ). The matrix L (IG) =D (IG)-A (IG) is defined as intuitionistic fuzzy Laplacian matrix of IG.

Here Laplacian matrix of an intuitionistic fuzzy graph can be written as two matrices one containing the entries as membership values and other containing non-membership values

i.e. L (IG) = $\left[\left(L\left(\mu_{ij}\right)\right),\left(L\left(\gamma_{ij}\right)\right)\right]$ where $L(\mu_{ij}) = \left|\lambda_i - \frac{\sum_{1\le i\le j\le n} \mu(u_i,u_j)}{n}\right|$ and $L(\gamma_{ij}) = \left|\theta_i - \frac{\sum_{1\le i\le j\le n} \gamma(v_i,v_j)}{n}\right|$.

Example 8:- The Laplacian matrix of the membership values of an intuitionistic fuzzy graph G_1 is

$$L\ [\mu_{ij}] = \begin{bmatrix} 1.4 & -0.6 & -0.5 & -0.3 \\ -0.6 & 1.9 & -0.5 & -0.8 \\ -0.5 & -0.5 & 1.3 & -0.3 \\ -0.3 & -0.8 & -0.3 & 1.4 \end{bmatrix} \text{ and } L\ [\gamma_{ij}] = \begin{bmatrix} 1.0 & -0.2 & -0.2 & -0.6 \\ -0.2 & 0.4 & -0.1 & -0.1 \\ -0.2 & -0.1 & 0.8 & -0.5 \\ -0.1 & -0.1 & -0.5 & 1.2 \end{bmatrix}$$

Definition 9:- Let L[μ_{ij}] be a intuitionistic fuzzy Laplacian matrix of IG=(V,E,μ,γ).The laplacian polynomial of intuitionistic fuzzy graph is the characteristic polynomial of its Laplacian matrix $\phi(IG,\mu)$=$\det\left(\tilde{\mu}I_n - L(IG)\right)$.The roots of $\phi(IG,\mu)$ is the intuitionistic fuzzy Laplacian eigen values of IG.

Theorem 10:- Let G be an intuitionistic fuzzy graph (without loops) with $|V|$ = n and $|E|$ = m and (IG) = $((\mu_{ij}),(\gamma_{ij}))$ be an intuitionistic fuzzy adjacency matrix and L(IG)= $(L(\mu_{ij}),L(\gamma_{ij}))$ be an intuitionistic fuzzy laplacian matrix of G then

$$\text{(i)}\; LE\left(\mu_{ij}(G)\right) \le \sqrt{\left(2\sum_{1\le i\le j\le n}\mu_{ij}^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}(G)}(u_i)-\frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)n}$$

$$\text{(ii)}\; LE\left(\gamma_{ij}(G)\right) \le \sqrt{\left(2\sum_{1\le i\le j\le n}\gamma_{ij}^2+\sum_{i=1}^{n}\left(d_{\gamma_{ij}(G)}(u_i)-\frac{2\sum_{1\le i\le j\le n}\gamma_{ij}}{n}\right)^2\right)n}$$

Theorem 11:- Let G be an intuitionistic fuzzy graph with $|V|$ =n and L(IG)= $(L(\mu_{ij}),L(\gamma_{ij}))$ be an intuitionistic fuzzy Laplacian matrix of G then

$$\text{(i)}\; LE\left(\mu_{ij}(G)\right) \ge 2\sqrt{\left(\sum_{1\le i\le j\le n}\mu_{ij}^2+\frac{1}{2}\sum_{i=1}^{n}\left(d_{\mu_{ij}(G)}(u_i)-\frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)}$$

$$\text{(ii)}\; LE\left(\gamma_{ij}(G)\right) \ge 2\sqrt{\left(\sum_{1\le i\le j\le n}\gamma_{ij}^2+\frac{1}{2}\sum_{i=1}^{n}\left(d_{\gamma_{ij}(G)}(u_i)-\frac{2\sum_{1\le i\le j\le n}\gamma_{ij}}{n}\right)^2\right)}$$

3. Max-Min Intuitionistic Laplacian fuzzy matrix of an intuitionistic fuzzy graph

In this section we define the Max-Min Intuitionistic Laplacian fuzzy matrix M(G) of an intuitionistic fuzzy graph. And the extreme energy of M(G) is defined . And we give the explicit expression for the coefficients of the characteristic polynomial of M (G).

Definition12:- Let $G=(V,E,\mu,\gamma)$ be an intuitionistic fuzzy graph for all node i, define $\alpha_j=\max_i \mu_{ij}$ and $\sigma_j=\min_i \gamma_{ij}$.

Definition13:- Let $G=(V,E,\mu,\gamma)$ be an intuitionistic fuzzy graph .The Max-min fuzzy matrix of an intuitionistic fuzzy graph is defined $M(G)=((r_{ij}),(s_{ij}))$

Where $r_{ij} = \begin{cases} \max(\alpha_i, \alpha_j) & if\ \mu_{ij} \neq 0 \\ 0 & otherwise \end{cases}$

and $s_{ij} = \begin{cases} \min(\sigma_i, \sigma_j) & if\ \gamma_{ij} \neq 0 \\ 0 & otherwise \end{cases}$

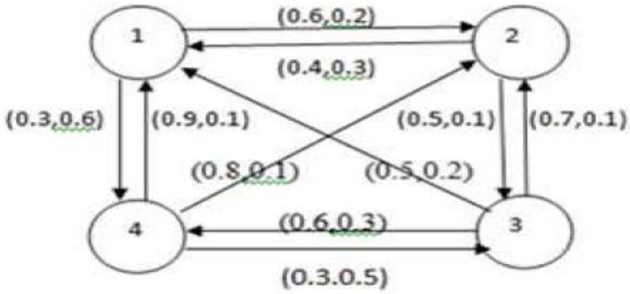

Figure 2: G1 , An Intuitionistic Fuzzy Graph

Example 14:- For the graph in Fig2, the max-min laplacian intuitionistic fuzzy matrix is

$$M(G) = \begin{bmatrix} 0 & (0.9,0.1) & 0 & (0.9,0.1) \\ (0.9,0.1) & 0 & (0.8,0.1) & 0 \\ (0.9,0.1) & (0.8,0.1) & 0 & (0.6,0.1) \\ (0.9,0.1) & (0.8,0.1) & (0.6,0.1) & 0 \end{bmatrix}$$

$$r_{ij} = \begin{bmatrix} 0 & 0.9. & 0 & 0.9 \\ 0.9 & 0 & 0.8 & 0 \\ 0.9 & 0.8 & 0 & 0.6 \\ 0.9 & 0.8 & 0.6 & 0 \end{bmatrix} \quad \text{and} \quad s_{ij} = \begin{bmatrix} 0 & 0.9. & 0 & 0.9 \\ 0.9 & 0 & 0.8 & 0 \\ 0.9 & 0.8 & 0 & 0.6 \\ 0.9 & 0.8 & 0.6 & 0 \end{bmatrix}.$$

Let $M(G) = ((r_{ij}),(s_{ij}))$ be the Max-Min Intuitionistic fuzzy matrix G .Let R=(r_{ij}) and S= (s_{ij}) .The characteristic polynomial of R and S is an equation of order n if G has n nodes. The characteristic polynomial of R of order-4 is $c_0\theta^4 - c_1\theta^3 + c_2\theta^2 - c_3\theta + c_4 = 0$

where $c_0 = 1, c_1 = tr(R), c_2 = \frac{1}{2}\left((trR)^2 - tr(R^2)\right), c_3 = \frac{1}{6}\left((trR)^3 - 3tr(R^2)(trR) + 2tr(R^3)\right)$ and

$c_4 = \det(R)$. Also we will give the explicit expression for the coefficients c_2 and c_3 .

Definition15:-Let M(G) be a max-min intuitionistic fuzzy matrix and $D(G) = (d_{ij})$ be a degree matrix of intuitionistic fuzzy graph. The Max-Min laplacian intuitionistic fuzzy matrix $M_L(G) = D(G) - M(G)$.

Here Max-Min Laplacian intuitionistic fuzzy matrix of an intuitionistic fuzzy graph can be written two matrices one containing the entries as membership values and other containing non membership Values i.e

$M_L(G) = \left[(R_L(\mu_{IJ})),(S_L(r_{ij}))\right]$

Example16:-The Max-Min Laplacian intuitionistic fuzzy matrix of the membership values of an intuitionistic fuzzy graph G_1 is

$$R_L\left(\mu_{ij}\right)=\begin{bmatrix} 2.7 & -0.9. & 0 & -0.9 \\ -0.9 & 3.0 & -0.8 & 0 \\ -0.9 & -0.8 & 2.6 & -0.6 \\ -0.9 & -0.8 & -0.6 & 2.9 \end{bmatrix} \text{ and } S_L\left(\gamma_{ij}\right)=\begin{bmatrix} 1.5 & -0.1. & 0 & -0.1 \\ -0.1 & 0.8 & -0.1 & 0 \\ -0.1 & -0.1 & 1.2 & -0.1 \\ -0.1 & -0.1 & -0.1 & 1.6 \end{bmatrix}$$

Definition17:-Let $R_L(\mu_{ij})$ be a max-min intuitionistic fuzzy matrix of I $IG=(V,E,\mu,\gamma)$.The Max-Min Laplacian polynomial of intuitionistic fuzzy graph is the characteristic polynomial of its laplacian matrix $\phi(IG,\mu)=\det(\mu I_n - M_L(IG))$.The roots of $\phi(IG,\mu)$ are the max-min fuzzy laplacian eigen values of IG.

Example 18:- for an intuitionistic fuzzy graph in fig 2,

Polynomial of $R(G_1)$ is $\lambda^4-11.2\lambda^3+44.37\lambda^2-70.735\lambda+34.1766$

Polynomial of $S(G_1)$ is $\theta^4-5.1\theta^3+9.52\theta^2-7.67\theta+2.234$

$spec\left(R_j\left(G_1\right)\right)=\{0.8358,\ 3.1230,\ 3.7449,\ 3.4963\}$

$spec\left(S_j\left(G_1\right)\right)=\{1.6661, 1.4724, 1.2071, 0.7544\}$

The characteristic polynomial of R_L of order 4 is $c_0\theta^4-c_1\theta^3+c_2\theta^2-c_3\theta+c_4=0$ where $c_0=1, c_1=tr(R_L)$, $c_2=\frac{1}{2}\left((trR_L)^2-tr\left(R_L{}^2\right)\right), c_3=\frac{1}{6}\left((trR_L)^3-3tr\left(R_L{}^2\right)(trR_L)+2tr\left(R_L{}^3\right)\right)$ and $c_4=\det(R_L)$.We gave the explicit expression for the coefficients c_2 and c_3.

Definition19:-Mutually adjacent: Let G be the given intuitionistic fuzzy graph. Two vertices v_i and v_j are said to be mutually adjacent if there is an edge from v_i to v_j and there is an edge from v_j to v_i .

Definition20:-Cyclic:-Let G be the given intuitionistic fuzzy graph. The vertices v_i, v_j and v_k are cycle if there is an edge from v_i to v_j, v_j to v_i and v_k to v_i.

Lemma21:- In the characteristic polynomial of R_L, $c_2=\sum_{1\le I\le J\le n}\left[r_{ii}r_{jj}-\left(r_{ij}\right)^2\right]$ if v_i and v_j are mutually adjacent.

Proof:-In general we have $c_2=\frac{1}{2}\left((trR_L)^2-tr\left(R^2\right)\right)$. Note that if v_i and v_j are mutually adjacent then $\gamma_{ij}=\gamma_{ji}$ otherwise any one γ_{ij} or γ_{ji} will be zero.

$$C_2 = \sum_{1\le i\le j\le n}\begin{vmatrix}\gamma_{ii} & \gamma_{ij}\\ \gamma_{ji} & \gamma_{jj}\end{vmatrix} = \sum_{1\le i\le j\le n}\left[\gamma_{ii}\gamma_{jj}-\gamma_{ij}\gamma_{ji}\right] = \sum_{1\le i\le j\le n}\left[\gamma_{ii}\gamma_{jj}-\left(\gamma_{ij}\right)^2\right]$$

Lemma22:- In the characteristic polynomial of R ,$C_3 = -\sum_{1\le i\le j\le n}\begin{vmatrix} r_{ii} & r_{ij} & r_{ik}\\ r_{ji} & r_{jj} & r_{jk}\\ r_{ki} & r_{kj} & r_{kk}\end{vmatrix}$ where the summation is taken over all i,j and k such that v_i, v_j and v_k are cyclic in G.

Proof:- In general we have $c_3 = \frac{1}{6}\left(\left(trR\right)^3 - 3tr\left(R^2\right)\left(trR\right) + 2tr\left(R^3\right)\right)$

$$= \sum_{1\le i\le j\le n}\begin{vmatrix} r_{ii} & r_{ij} & r_{ik}\\ r_{ji} & r_{jj} & r_{jk}\\ r_{ki} & r_{kj} & r_{kk}\end{vmatrix}$$

Theorem23 :- If $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ are eigen values of R then $\sum_{i=1}^{n}\lambda_i^2 = C_1^2 - 2C_2$.

Proof :-We know that $\left(\sum_{i=1}^{n}\lambda_i\right)^2 = \sum_{i=1}^{n}\lambda_i^2 + 2\sum_{1\le i\le j\le n}\lambda_i\lambda_j$

$$\left(\sum_{i=1}^{n}\lambda_i\right)^2 - 2\sum_{1\le i\le j\le n}\lambda_i\lambda_j = \sum_{i=1}^{n}\lambda_i^2 \longrightarrow (1)$$

By comparing the coefficients of λ^{n-2} in the characteristic polynomial $\prod_{i=1}^{n}\left(\lambda-\lambda_i\right) = \left|A-\lambda I\right|$

We get $\sum_{1\le i\le j\le n}\lambda_i\lambda_j = \sum_{1\le i\le j\le n}\left(\gamma_{ii}\gamma_{jj} - \left(\gamma_{ij}\right)^2\right) \longrightarrow (2)$

Equation (1) and (2) we get

$$\sum_{i=1}^{n}\lambda_i^2 = \left(\sum_{i=1}^{n}\lambda_i\right)^2 - 2\sum_{1\le i\le j\le n}\left(\gamma_{ii}\gamma_{jj} - \left(\gamma_{ij}\right)^2\right)$$

$$\sum_{i=1}^{n}\lambda_i^2 = C_1^2 - 2C_2$$

From the above discussion all the coefficients of the characteristic polynomial of the matrix R can be calculated .Now let us consider the matrix S whose characteristic polynomial is $d_0\theta^4 - d_1\theta^3 + d_2\theta^2 - d_3\theta + d_4 = 0$ here also $d_0 = 1$, $d_1 = tr(S)$, $d_2 = \sum_{1\le i\le j\le n}\left(s_{ii}s_{jj} - \left(s_{ij}\right)^2\right)$ if v_i, v_j are mutually adjacent , $d_3 = \sum_{1<i\le j\le n}\begin{vmatrix} s_{ii} & s_{ij} & s_{ik}\\ s_{ji} & s_{jj} & s_{jk}\\ s_{ki} & s_{kj} & s_{kk}\end{vmatrix}$ where the summation is

taken over all i,j,k such that v_i, v_j and v_k are cycle in G and $d_4 = \det(S)$ of $\theta_1, \theta_2, ..., \theta_n$ are the eigen values of S,then $\sum_{i=1}^{n} \theta_i^2 = d_1^2 - 2d_2$.

Definition-24:-Extreme laplacian energy of an intuitionistic fuzzy graph.-The extreme laplacian energy of an intuitionistic fuzzy graph $G = (V, E, \mu, \gamma)$ is defined by (LE_1, LE_2) where LE_1 is laplacian energy of max degree matrix of R and LE_2 is laplacian energy of the min degree matrix S.

Example 25:- The characteristic polynomial of R is $\lambda^4 - 11.2\lambda^3 + 44.37\lambda^2 - 70.735\lambda + 34.1766$.

Here $c_0 = 1, c_1 = tr(R), c_4 = \det(R)$=34.1766, $C_2 = \sum_{1 \le i \le j \le n} \left[r_{ii} r_{jj} - (r_{ij})^2 \right]$ if v_i and v_j are mutually adjacent. i.e.

$$C_2 = \{[(2.7)(3.0) - (0.9)^2] + [(3.0)(2.6) - (0.8)^2] + \{(2.6)(2.9) - (0.6)^2$$

$$+ [(2.7)(2.6) - 0] + [3(2.9) - 0][(2.7)(2.9) - (0.9)^2]\}$$

=7.29+7.16+7.18+7.02+8.7+7.02

= 44.37.

$$C_3 = \sum_{1 \le i \le j \le n} \begin{vmatrix} r_{ii} & r_{ij} & r_{ik} \\ r_{ji} & r_{jj} & r_{jk} \\ r_{ki} & r_{kj} & r_{kk} \end{vmatrix}$$

$$= \begin{vmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{vmatrix} + \begin{vmatrix} r_{11} & r_{14} & r_{13} \\ r_{41} & r_{44} & r_{43} \\ r_{31} & r_{34} & r_{33} \end{vmatrix} + \begin{vmatrix} r_{22} & r_{23} & r_{24} \\ r_{32} & r_{33} & r_{34} \\ r_{42} & r_{43} & r_{44} \end{vmatrix} + \begin{vmatrix} r_{44} & r_{42} & r_{41} \\ r_{24} & r_{22} & r_{21} \\ r_{14} & r_{12} & r_{11} \end{vmatrix}$$

= cyclic (123) + cyclic (143) +cyclic (234) and cyclic (421)

= 16.5780+19.30+18.063+16.794=70.735

Also $$\sum_{i=1}^{n} \lambda_i^2 = \left(\sum_{i=1}^{n} \lambda_i \right)^2 - 2 \sum_{1 \le i \le j \le n} \left(r_{ii} r_{jj} - (r_{ij})^2 \right)$$

$$= C_1^2 - 2C_2$$

= $(11.2)^2$-2(4437)=36.7

The characteristic polynomial of S is $\theta^4 - 5.1\theta^3 + 9.52\theta^2 - 7.67\theta + 2.234$.

Where $d_0 = 1, d_1 = tr(S) = 5.1, d_4 = \det(S)$=2.234

$$d_2 = \sum_{1 \le i \le j \le n} \left(s_{ii} s_{jj} - (s_{ij})^2 \right) = \{[1.5(0.8) - (0.1)^2] + [(0.8)(1.2) - (0.1)^2] + [(1.2)(1.6) - (0.1)^2]$$

$$+ \ [(1.6)(1.5)-(0.1)^2]+[(1.5)(1.2)-(0)^2]+[(0.8)(1.6)] \ \}$$

$$=1.19+0.95+1.91+2.39+1.8+1.28=9.52$$

$$d_3 = \sum_{1\le i\le j\le n} \begin{vmatrix} s_{ii} & s_{ij} & s_{ik} \\ s_{ji} & s_{jj} & s_{jk} \\ s_{ki} & s_{kj} & s_{kk} \end{vmatrix} = \begin{vmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{vmatrix} + \begin{vmatrix} s_{11} & s_{14} & s_{13} \\ s_{41} & s_{44} & s_{43} \\ s_{31} & s_{34} & s_{33} \end{vmatrix} + \begin{vmatrix} s_{22} & s_{23} & s_{24} \\ s_{32} & s_{33} & s_{34} \\ s_{42} & s_{43} & s_{44} \end{vmatrix} + \begin{vmatrix} s_{44} & s_{42} & s_{41} \\ s_{24} & s_{22} & s_{21} \\ s_{14} & s_{12} & s_{11} \end{vmatrix}$$

$$= 1.4120 + 2.8520 + 1.5110 + 1.8950 = 7.67$$

Also $\sum_{i=1}^{n}\theta_i^2 = \left(\sum_{i=1}^{n}\theta_i\right)^2 - 2\sum_{1\le i\le j\le n}\left(s_{ii}s_{jj} - \left(s_{ij}\right)^2\right)$

$$= d_1^2 - 2d_2$$

$$= (5.1)^2 - 2\ (9.52) = 6.97$$

Extreme laplacian energy of an Intuitionistic fuzzy graph is $(LE_1, LE_2) = (6.7284, 2.6)$.

4. Numerical examples

In this section we give the extreme Laplacian energy of an intuitionistic fuzzy graph through real time examples. We have taken the http://www.pantechsolutions.net/. This website is modelled as an intuitionistic fuzzy graph by considering the navigation of the customer. An intuitionistic fuzzy graph of this site for four different time periods is taken. For each of these periods the Max-Min intuitionistic laplacian fuzzy matrix is constructed and the extreme laplacian energy is calculated. These results are represented in the table.

Example 26:- In the website http://www.pantechsolutions.net/, we consider four links 1. Microcontroller-boards, 2. /log-in html, 3. /and 4. Project kits for the period July 16, 2014 to August 15,2014.

For an intuitionistic fuzzy graph in Figure 3, Extreme Laplacian energy = (1.45,1.8) but by the intuitionistic laplacian fuzzy adjacency matrix $A(G_2)$ of this graph , the laplacian energy = (0.5328,2.285).

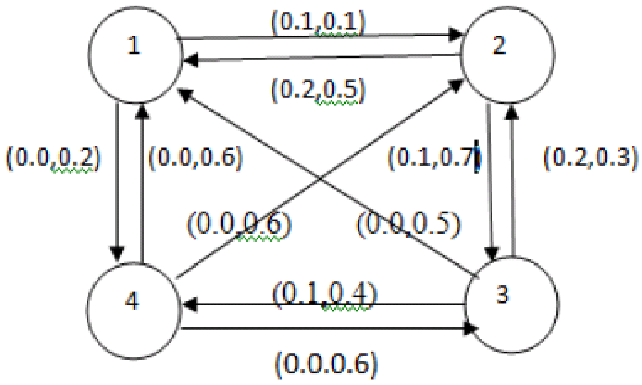

Figure 3: G_2, An Intuitionistic Fuzzy Graph

Example 27:- In the same website (mentioned above in example), we consider the same four links for the period August 16, 2014 to September 15, 2014.

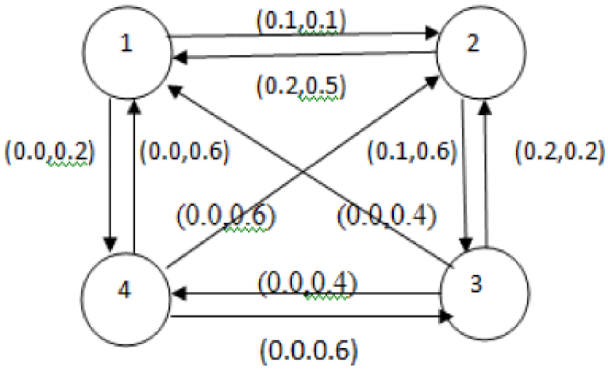

Figure 4: G_3, An Intuitionistic Fuzzy Graph

For an intuitionistic fuzzy graph in Figure 4, Extreme laplacian energy = (1.5,2.3) but by the intuitionistic laplacian fuzzy adjacency matrix A(G_2) of this graph , the laplacian energy = (0.6328,1.78).

Example-28:- In the same website (mentioned above in example), we consider the same four links for the period September 16, 2014 to October 15, 2014.

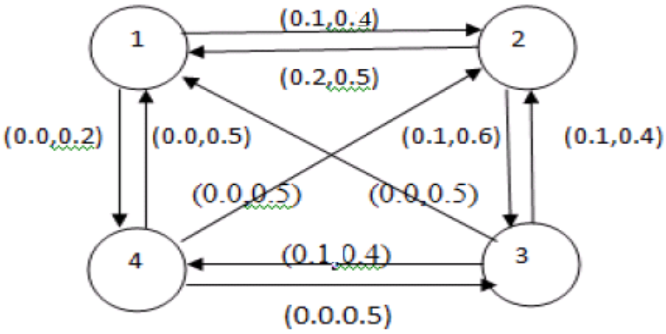

Figure 5: G_4, An Intuitionistic Fuzzy Graph

For an intuitionistic fuzzy graph in Figure 5, Extreme laplacian energy = (1.2,1.95) but by the intuitionistic laplacian fuzzy adjacency matrix A(G_4) of this graph , the laplacian energy = (0.4236,2.3).

Example 29:- In the same website (mentioned above in example), we consider the same four links for the period October 16, 2014 to November 15, 2014.

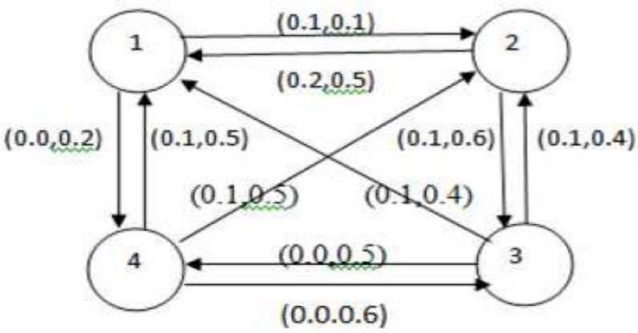

Figure 6: G_5 ,An Intuitionistic Fuzzy Graph

For an intuitionistic fuzzy graph in Figure 6,Extreme Laplacian energy = (1.1001,2.6) but by the intuitionistic laplacian fuzzy adjacency matrix $A(G_5)$ of this graph , the laplacian energy = (0.7236,2.15).

The following table represents the comparison of Laplacian energy of $A_L(G)$ and the Extreme laplacian energy of $M_L(G)$.

Period	Laplacian Energy of A_L (G)	Extreme Laplacian Energy of M_L(G)
May-Jun	(0.5328,2.285)	(1.45,1.8)
Jun-Jul	(0.6328,1.78)	(1.5,2.3)
Jul-Aug	(0.4236,2.3)	(1.2,1.95)
Aug-Sep	(0.7236,2.15)	(1.1001,2.6)

From the above table we can observe that the Laplacian energy of $A_L(G)$ for all the time periods is less than the Extreme laplacian energy of R in $M_L(G)$. Similarly the energy of Υ of $A_L(G)$ is greater than the Extreme laplacian energy S of $M_L(G)$.

5. Conclusion

In this paper we introduced the Max-Min intuitionistic laplacian fuzzy matrix $M_L(G)$ of an intuitionistic fuzzy graph G . The extreme laplacian energy of $M_L(G)$ is defined and also we give the explicit expression for the coefficients of the characteristic polynomial of $M_L(G)$. These concepts are illustrated through real time example. These results are compared with the Laplacian energy of an intuitionistic fuzzy graph.

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