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# SOLVING TWO STAGE SOLID TRANSPORTATION PROBLEMS

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**Abstract**

Based on the plane-line method [7], a new solution procedure for solving a two stage solid transportation problem is proposed. A set of optimal solutions to the two stage solid transportation problem is provided by the proposed solution procedure such that the minimum total transportation costs of the solid transportation problem in one stage and two stage are the same. Numerical example is presented to illustrate the proposed solution procedure. The proposed solution procedure helps the decision makers in the transportation problems with three parameters by supporting them in the decision making process.

**Keywords**: Solid transportation problem, optimal solution, Plane-line method, two stage solid transportation problem.

## 1 Introduction

The solid transportation (ST) problem is introduced by Shell [10] as an extension of a classical transportation problem [3]. Its bounds are given on three parameters, namely, supply, demand and conveyance. In many industrial problems, a homogeneous product is shipped from an origin to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, ships and so on.

The ST problem was solved by Haley [2] using an extension of the plane transportation algorithm based on MODI method. In Haley's method, the ST problem requires $m+n+l-2$ non-negative basic feasible solution for applying MODI method. Patel and Tripathy [8] developed a computationally superior method for solving ST problems with mixed constraints. Pandian and Anuradha [5] and Anuradha [1] proposed the min zero-min cost method for finding an optimal solution of solid transportation problems directly in which minimum cost and minimum zero are considered, but MODI method is not used. A new method namely, plane-line method has been presented by Pandian and Kavitha [7] for solving

ST problems directly in which all parameters are considered at each step and minimum costs are not taken for final allotment. In some circumstances due to storage constraints, destinations are not able to receive the quantity in excess of their minimum demand. In such case, a single shipment is not possible to supply the full demand of the destinations. Therefore, items are shipped to destinations from the origins in two stages. Initially, the minimum demands of the destinations are shipped from the origins. After consuming a part of whole of this initial shipment, they are prepared to receive the excess quantity in the second stage. Such type of transportation problems are known as two stage transportation problems. The main objective of the two stage transportation problem is to determine a shipment plan from the origins to the destinations in two stages such that the total shipping costs in the two stages is minimum.

A polynomial bound algorithm for two stage time minimization problems was proposed by Sonia and Rita Malhotra [11]. Sonia et al.[12] presented a polynomial time algorithm for solving time minimization problems in two stages. A parametric approach was developed to solve a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers by Nagoor Gani and Abdul Razak [4]. Ritha and Merline Vinotha [9] proposed a method for finding a best compromise solution to a multiobjective two stage fuzzy transportation problem using geometric programming approach. Pandian and Natarajan [6] proposed a method for solving two stage transportation problem such that the total shipping costs in single stage and two stages are the same.

In this paper, we present a new solution procedure for solving two stage ST transportation problems which is based on plane-line method [7]. The proposed solution approach is easy to understand and apply and also, provides more than one optimal solutions to the ST problem. The solution procedure approach is demonstrated by a numerical example. The current study is an extension of the two stage classical transportation problem [6]. The proposed solution approach helps the decision makers for solving transportation models in two stages of their real life situations involving three parameters and to choose an appropriate solution according to their current situation.

**2. Two Stage Solid Transportation Problem**

Now, the mathematical model of a ST problem is given as follows:

(G) Minimize $w = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} t_{ijk}$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} t_{ijk} = a_i, \quad i \in I$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} t_{ijk} = b_j, \quad j \in I$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ijk} = e_k, \quad k \in K$$

$t_{ijk} \geq 0$, for all i, j and k and integers

where $I = \{1,2,...,m\}$, $J = \{1,2,...,n\}$, $K = \{1,2,...,l\}$, $c_{ijk}$ is the unit transportation cost from the origin $i$ to the destination $j$ by means of the conveyance $k$ and $t_{ijk}$ is the amount of quantity shipped from the origin $i$ to the destination $j$ by means of the kth conveyance. $a_i$ is the amount of available quantity at the origin $i$, $b_j$ is the amount of demand at the destination $j$ and $e_k$ is the amount of quantity shipped by conveyance $k$.

Now, due to storage issue in the destinations, the problem (G) can be modified as a two stage ST problem as given below:

(P) Minimize $z = z_1 + z_2$

Subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}x_{ijk} \leq z_1$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}y_{ijk} \leq z_2$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq a_i, \quad i \in I$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = k_j, \quad j \in J$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} x_{ij} = t_k, \quad k \in K$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} y_{ijk} = a_i - \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}, \quad i \in I$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} y_{ijk} = b_j - k_j, \quad j \in J$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} y_{ijk} = v_k - t_k, \quad k \in K$$

$x_{ijk}, y_{ijk} \geq 0, i \in I$, $j \in J$ and $k \in K$ and integers where $k_j$ is the minimum requirement (maximum storage capacity) of the jth destination, $t_k$ is the minimum (maximum amount of the ) quantity shipped by the conveyance k, $x_{ijk}$ is the amount of quantity shipped from the origin $i$ to the destination $j$ by means of the $k$th conveyance in the stage I and $y_{ijk}$ is the amount of quantity shipped from the origin $i$ to the destination $j$ by means of the $k$th conveyance in the stage II. $z_1$ is the total cost transportation in the stage I and $z_2$ is the total cost transportation in the stage II.

The following theorem is established a relation between the optimal solutions of the two ST problems (G) and (P).

**Theorem 1:** An optimal solution of the problem (P) is obtained from an optimal solution of the problem (G). Further, the optimal objective value of the problem (G) and the optimal objective value of the problem (P) are the same.

**Proof:** Let $\{t^{\circ}_{ijk}, i\in I,\ j\in J \text{ and } k\in K\}$ be an optimal solution of the problem (G) with objective value $w^{\circ}$.

Let $\{x^{\circ}_{ijk}, i\in I, j\in J \text{ and } k\in K\}$ and $\{y^{\circ}_{ijk}, i\in I, j\in J \text{ and } k\in K\}$ be two sets of positive real numbers which are generated from the set $\{t^{\circ}_{ijk}, i\in I, j\in J \text{ and } k\in K\}$ such that

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x^{\circ}_{ijk} \le a_i,\ i\in I$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x^{\circ}_{ijk} = k_j,\ j\in J$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} x^{\circ}_{ijk} = t_k,\ k\in K$$

$$y^{\circ}_{ijk} = t^{\circ}_{ijk} - x^{\circ}_{ijk},\quad i\in I,\ j\in J \text{ and } k\in K .$$

Let $z^{\circ}_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}x^{\circ}_{ijk}$ and $z^{\circ}_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}y^{\circ}_{ijk}$.

Then, $z^{\circ}_1 + z^{\circ}_2 = w^{\circ}$

Clearly, $\{z^{\circ}_1, z^{\circ}_2, x^{\circ}_{ijk} \text{ and } y^{\circ}_{ijk}, i\in I, j\in J \text{ and } k\in K\}$ is a feasible solution to the problem (P).

Suppose that $\{z^{\circ}_1, z^{\circ}_2, x^{\circ}_{ijk} \text{ and } y^{\circ}_{ijk}, i\in I, j\in J \text{ and } k\in K\}$ is not an optimal solution to the problem (P).

Then, there exists a feasible solution $\{z_1, z_2, x_{ijk} \text{ and } y_{ijk}, i\in I, j\in J \text{ and } k\in K\}$ to the problem (P) such that $z = z_1 + z_2 < w^{\circ}$.

Since $\{z_1, z_2, x_{ijk} \text{ and } y_{ijk}, i\in I, j\in J \text{ and } k\in K\}$ is a feasible solution to the problem (P), the set $\{t_{ijk} = x_{ijk} + y_{ijk}, i\in I, j\in J \text{ and } k\in K\}$ is a feasible solution to the problem (G).

Now, $w = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}t_{ijk} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}(x_{ijk} + y_{ijk})$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}y_{ijk} \le z_1 + z_2 < w^{\circ}$$

Which contradicts the hypothesis " $\{t^{\circ}_{ijk}, i \in I\ , j \in J$ and $k \in K\}$ is an optimal solution of the problem (G)".

Therefore, $\{z_1^{\circ},\ z_2^{\circ},\ x^{\circ}_{ijk}$ and $y^{\circ}_{ijk}, i \in I,\ j \in J$ and $k \in K\}$ is an optimal solution to the problem (P) and the optimal objective value of the problem (G) and the optimal objective value of the problem (P) are the same.

Hence, the theorem is proved.

### 2.1 A Solution Procedure for Solving Two Stage Solid Transportation Problems

We, now introduce a new solution procedure for finding a set of optimal solutions for two stage ST problems.

The proposed solution procedure proceeds as follows.

**Step 1.** Determine an optimal solution of the given ST problem without considering the destination's capacity constraints by the plane-line method [7].

**Step 2.** Find a set of possible solutions to the Stage I and Stage II problems of the given two stage ST problem using the optimal solution obtained in the Step.1. by splitting the items of the each allotment cell such that the destination's capacity constraints are satisfied.

**Step 3.** The combined solution(s) of the set of optimal solutions obtained from the Step 2. Yields various possible optimal solutions to the given two stage ST problem.

The following numerical example is presents to illustrate the proposed solution procedure for solving the ST problem in two stages.

**Example 2.1** In a paramedical company, a product is produced in three factories and it is send to three destinations by three different modes of conveyances. The unit shipping cost, supply, demand and conveyance capacity are given below:

| | | | | | | | | | | Capacity |
|---|---|---|---|---|---|---|---|---|---|---|
| Conveyance | C1 | | | C1 | | | C1 | | | 11 |
| | | C2 | | | C2 | | | C2 | | 14 |
| | | | C3 | | | C3 | | | C3 | 9 |
| | D1 | | | D2 | | | D3 | | | Supply |
| S1 | 4 | 7 | 8 | 3 | 9 | 7 | 6 | 7 | 2 | 11 |
| S2 | 4 | 2 | 6 | 1 | 3 | 8 | 8 | 4 | 5 | 13 |
| S3 | 8 | 1 | 3 | 4 | 7 | 3 | 5 | 6 | 4 | 10 |
| Demand | 7 | | | 15 | | | 12 | | | |

with minimum requirement ( maximum capacity ) of the destinations are $k_1 = 7$ ; $k_2 = 10$ and $k_3 = 6$ respectively.

Now, by the plane-line method [7], we obtain the optimal solution to the given solid transportation problem without considering the minimum demand constraints.

| | | | | | | | | | | Capacity |
|---|---|---|---|---|---|---|---|---|---|---|
| Conveyance | C1 | | | C1 | | | C1 | | | 11 |
| | | C2 | | | C2 | | | C2 | | 14 |
| | | | C3 | | | C3 | | | C3 | 9 |
| | D1 | | | D2 | | | D3 | | | Supply |
| S1 | | | | 2 | | | | | 9 | 11 |
| S2 | | | | 9 | 4 | | | | | 13 |
| S3 | | 7 | | | | | | 3 | | 10 |
| Demand | 7 | | | 15 | | | 12 | | | |

and the minimum total transportation cost is Rs.70.

Now, since maximum capacity of destinations are $b_1 = 7$; $b_2 = 10$ and $b_3 = 12$, we can arrange the optimal solution of the STP in the following ways.

| | | | | | | | | | | Capacity |
|---|---|---|---|---|---|---|---|---|---|---|
| Conveyance | C1 | | | C1 | | | C1 | | | 11 |
| | | C2 | | | C2 | | | C2 | | 14 |
| | | | C3 | | | C3 | | | C3 | 9 |
| | D1 | | | D2 | | | D3 | | | Supply |
| S1 | | | | 2 | | | | | 9 | 11 |
| S2 | | | | 9 | 4 | | | | | 13 |
| S3 | | 7 | | | | | | 3 | | 10 |
| Demand | 7(5) | | | 15(10) | | | 12(12) | | | |

Now, the given two stage transportation problem has the following 14 different optimal solutions with minimum total transportation cost is Rs. 70.

| S.No. | Stage-I Optimal solutions | Stage-II Optimal solutions |
|---|---|---|
| 1 | $x_{312} = 5$ ; $x_{121} = 1$; $x_{221} = 9$; $x_{222} = 0$; $x_{133} = 9$; $x_{332} = 3$ with total transportation cost is Rs.53 | $x_{312} = 2$ ; $x_{121} = 1$; $x_{221} = 0$; $x_{222} = 4$; $x_{133} = 0$; $x_{332} = 0$ with total transportation cost is Rs.17 |
| 2 | $x_{312} = 5$ ; $x_{121} = 2$; $x_{221} = 8$; $x_{222} = 0$; $x_{133} = 9$; $x_{332} = 3$ with total transportation cost is Rs.55 | $x_{312} = 2$ ; $x_{121} = 0$; $x_{221} = 1$; $x_{222} = 4$; $x_{133} = 0$; $x_{332} = 0$ with total transportation cost is Rs.15 |
| 3 | $x_{312} = 5$ ; $x_{121} = 0$; $x_{221} = 9$; $x_{222} = 1$; $x_{133} = 9$; $x_{332} = 3$ with total transportation cost is Rs.53 | $x_{312} = 2$ ; $x_{121} = 2$; $x_{221} = 0$; $x_{222} = 3$; $x_{133} = 0$; $x_{332} = 0$ with total transportation cost is Rs.17 |
| 4 | $x_{312} = 5$ ; $x_{121} = 1$; $x_{221} = 8$; $x_{222} = 1$; $x_{133} = 9$; $x_{332} = 3$ with total transportation cost is Rs.55 | $x_{312} = 2$ ; $x_{121} = 1$; $x_{221} = 1$; $x_{222} = 3$; $x_{133} = 0$; $x_{332} = 0$ with total transportation cost is Rs.15 |
| 5 | $x_{312} = 5$ ; $x_{121} = 2$; $x_{221} = 7$; $x_{222} = 1$; $x_{133} = 9$; $x_{332} = 3$ with total transportation cost is Rs.57 | $x_{312} = 2$ ; $x_{121} = 0$; $x_{221} = 2$; $x_{222} = 3$; $x_{133} = 0$; $x_{332} = 0$ with total transportation cost is Rs.13 |

| | | |
|---|---|---|
| 6 | $x_{312}=5$ ; $x_{121}=0$; $x_{221}=8$; $x_{222}=2$; $x_{133}=9$; $x_{332}=3$ with total transportation cost is Rs.55 | $x_{312}=2$ ; $x_{121}=2$; $x_{221}=1$; $x_{222}=2$; $x_{133}=0$; $x_{332}=0$ with total transportation cost is Rs.15 |
| 7 | $x_{312}=5$ ; $x_{121}=1$; $x_{221}=7$; $x_{222}=2$; $x_{133}=9$; $x_{332}=3$ with total transportation cost is Rs.57 | $x_{312}=2$ ; $x_{121}=1$; $x_{221}=2$; $x_{222}=2$; $x_{133}=0$; $x_{332}=0$ with total transportation cost is Rs.13 |
| 8 | $x_{312}=5$ ; $x_{121}=2$; $x_{221}=6$; $x_{222}=2$; $x_{133}=9$; $x_{332}=3$ with total transportation cost is Rs.59 | $x_{312}=2$ ; $x_{121}=0$; $x_{221}=3$; $x_{222}=2$; $x_{133}=0$; $x_{332}=0$ with total transportation cost is Rs.11 |
| 9 | $x_{312}=5$ ; $x_{121}=0$; $x_{221}=7$; $x_{222}=3$; $x_{133}=9$; $x_{332}=3$ with total transportation cost is Rs.57 | $x_{312}=2$ ; $x_{121}=2$; $x_{221}=2$; $x_{222}=1$; $x_{133}=0$; $x_{332}=0$ with total transportation cost is Rs.13 |
| 10 | $x_{312}=5$ ; $x_{121}=1$; $x_{221}=6$; $x_{222}=3$; $x_{133}=9$; $x_{332}=3$ with total transportation cost is Rs.59 | $x_{312}=2$ ; $x_{121}=1$; $x_{221}=3$; $x_{222}=1$; $x_{133}=0$; $x_{332}=0$ with total transportation cost is Rs.11 |
| 11 | $x_{312}=5$ ; $x_{121}=2$; $x_{221}=5$; $x_{222}=3$; $x_{133}=9$; $x_{332}=3$ with total transportation cost is Rs.61 | $x_{312}=2$ ; $x_{121}=0$; $x_{221}=4$; $x_{222}=1$; $x_{133}=0$; $x_{332}=0$ with total transportation cost is Rs.9 |
| 12 | $x_{312}=5$ ; $x_{121}=0$; $x_{221}=6$; $x_{222}=4$; $x_{133}=9$; $x_{332}=3$ with total transportation cost is Rs.59 | $x_{312}=2$ ; $x_{121}=2$; $x_{221}=3$; $x_{222}=0$; $x_{133}=0$; $x_{332}=0$ with total transportation cost is Rs.11 |
| 13 | $x_{312}=5$ ; $x_{121}=1$; $x_{221}=5$; $x_{222}=4$; $x_{133}=9$; $x_{332}=3$ with total transportation cost is Rs.61 | $x_{312}=2$ ; $x_{121}=1$; $x_{221}=4$; $x_{222}=0$; $x_{133}=0$; $x_{332}=0$ with total transportation cost is Rs.9 |
| 14 | $x_{312}=5$ ; $x_{121}=2$; $x_{221}=4$; $x_{222}=4$; $x_{133}=9$; $x_{332}=3$ with total transportation cost is Rs.63 | $x_{312}=2$ ; $x_{121}=0$; $x_{221}=5$; $x_{222}=0$; $x_{133}=0$; $x_{332}=0$ with total transportation cost is Rs.7 |

**Remark 2.1** These optimal solutions are very much helpful to decision makers when they are handling the ST problems in two stages at the time of taking decision on the shipping plan more comfortably.

## 3 Conclusions

In real life situations, the two-stage ST problem occurs due to minimum capacity of destinations. The main aim of the ST problem is to find the two schedules for shipping the minimum requirements to the destinations from the sources in stage-I and the left out quantity is supplied to the destinations in stage-II, from the sources which have surplus quantity left after the completion of stage-I such that the combined total transportation cost is minimum. The proposed solution procedure based on plane-line method [7] provides more than one optimal solutions for a two stage transportation problem. When the decision makers are handling various types of logistic problems involving three parameters with destination capacity parameters, the proposed solution procedure provides more optimal schedules and can be served an important tool for them to choose a suitable optimal solution according to their current situation.

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