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MULTI PERSPECTIVE METRICS FOR FINDING SCHEDULES TO FUZZY ALLOCATION PROBLEMS

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Abstract

The investigation of metrics in multiple perspectives is dealt in this paper for a fuzzy allocation problem (FAP). By portraying the problem in a graphical view, its corresponding metrics in terms of graph theory is discussed. The application can also be analyzed in management perspective through which the solutions in reality are discussed. These approaches can be served as an essential tool for the decision makers when they are dealing different varieties of logistics problems.

Keywords: Fuzzy Allocation problem, bipartite graph, ranking method, perfect matching, optimal schedule

1. Introduction

Resource allocation is used to assign the available resources in an economic way. When the resources to be allocated are scarce, a well-planned action is necessary for a decision-maker to attain the optimal utility. If the supplying sources and the receiving agents are limited, the best pattern of allocation to get the maximum return or the best plan with the least cost, whichever may be applicable to the problem, is to be found out. This class of problems is termed as allocation problems. Allocation problem (AP) has been used in a variety of application contexts such as personnel scheduling, manpower planning, resource allocation and so on. The ultimate goal of the allocation problem is to a allocate number of jobs to number of persons at a minimum cost or time.However in real life, the parameters of AP are imprecise data instead of fixed real data because time or cost for doing a job by a facility might vary due to different reasons.In fuzzy decision making, the ranking of fuzzy number plays a vital role. Ranking of fuzzy numbers was first proposed by Jain [2]. Dominance of fuzzy numbers can be explained by many ranking methods of these, Robust's ranking method [12] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in [0,1].Cottrell [11] discussed some performance measures that concerns vary according to the freight transport providers.Nagarajan and Solairaju[4] solved the assignment problem with fuzzy costs under Ranking techniques. Ruth and Uthra[6] discussed an assignment problem with fuzzy costs represented by linguistic numbers.Srinivasan and Geetharamani [8] proposed a new method for solving FAP by using Robust's ranking function. Jahir and Jayaraman [1] proposed an algorithm based on the ranking method for solving fuzzy AP.Thorani and Sankar [10] discussed fuzzy assignment problem with generalized fuzzy numbers. Senthil Kumar and Hussain [7] investigated an assignment problem in which cost coefficients are triangular intuitionistic fuzzy numbers. Kotwal and Dhope [3] studied an unbalanced AP using modified approach. Thiruppathiand Iranian [9] discussed an innovative method for finding optimal solution to AP.

The primary focus of this paper is to investigate the metrics of the fuzzy allocation problem in multiple perspectives. This paper is organized as follows: In Section 2 we discuss some basic concepts of fuzzy theory. Section 3 projects the basics of the FAP metrics. In section 4, a simple illustration is chosen to discuss the possibility of metrics using the graph theory models and management structures. Finally the paper is concluded in section 5.

2. Preliminaries

We need the following definitions of fuzzy set, fuzzy number and membership function which can be found in [13].

2.1Definition:

Let A be a classical set and $\mu_A(x)$ be a membership function from A to [0,1]. A fuzzy set $\tilde{A}$ with the membership function $\mu_A(x)$ is defined by

$\tilde{A} = \{ (x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1] \}$.

2.2 Definition: A Fuzzy set $\tilde{A}$ is called positive if its membership function is such that $\mu_{\tilde{A}}(x) = 0$ for all $x \le 0$.

2.3 Definition:

A Fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number of its member ship function has the following conditions:

(i) $\mu_{\tilde{A}}(x) : R \to [0,1]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $(-\infty, a] \cup [c, \infty)$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on [a,b] and strictly decreasing on [b,c].

(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in b$ where $a \le b \le c$.

2.4 Definition: A fuzzy number $\tilde{A}$ is denoted as a triangular fuzzy number by (a_1, a_2, a_3) and its membership function $\mu_{\tilde{A}}(x)$ is given as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & \text{if} \quad a_1 \le x \le a_2 \\ \dfrac{x-a_3}{a_2-a_3} & \text{if} \quad a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

2.5 Definition: The α -cut of afuzzy number $A(x)$ is defined as $A(\alpha)=\{x / \mu(x) \ge \alpha, \alpha \in [0,1]\}$

2.6Robust's ranking method: The Robust's ranking is defined as $R(\tilde{t})=\int_0^1 0.5(t_\alpha^L, t_\alpha^U)\,d\alpha$, where (t_α^L, t_α^U) is the α-level cut of the fuzzy number $\tilde{t}$.

Robust's ranking technique satisfies compensation, linearity and additive property which provides results that are consistent with human intuition.

3. Fuzzy Allocation Problem

Mathematically fuzzy allocation problem can be stated as

(P) Minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{t}_{ij} x_{ij}$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \; j = 1, 2, \ldots, n \qquad (1)$$

$$\sum_{j=1}^{n} x_{ij} = 1, \; i = 1, 2, \ldots, n \qquad (2)$$

$$x_{ij} = 0 \text{ (or) } 1, \text{ for all } i \text{ and } j \qquad (3)$$

Where $\tilde{t}_{ij}$ denotes the time used to complete the j^{th} task by i^{th} person.

3.1. Metric Allocation Problem

Measuring the finite solutions of the AP is performed by evaluating various metrics. Thus the metrics AP evolve with the considerations of graphs and managerial metrics. AP can also be modeled as complete bipartite graph whose edges are joined between machines and tasks which represent nodes of the graph. The processing time of jth task by ith machine is denoted as weight on the (i,j)th edge. Our aim is to obtain an optimal and next to optimal matching of complete bipartite graph. We need the following definitions which can be found in Narsingh Deo[5].

Definition 3.1.1: Any graph $G=(V,E)$ consists of two sets of objects namely vertex set V and edge set E where an edge $e_k = (v_i, v_j)$ is identified by an unordered pair of vertices v_i and v_j.

Definition 3.1.2: A graph G is called bipartite if its vertex set V can be decomposed into two disjoint subsets V_1 and V_2 such that every edge in G joins a vertex in V_1 with a vertex in V_2. Further it is said to be complete bipartite if every vertex in V_1 is adjacent to every other vertex in V_2.

Definition 3.1.3: A matching in a graph is a subset of edges in which no two are adjacent. If a matching includes all the vertices then it is said to be a perfect matching.

4. Illustration

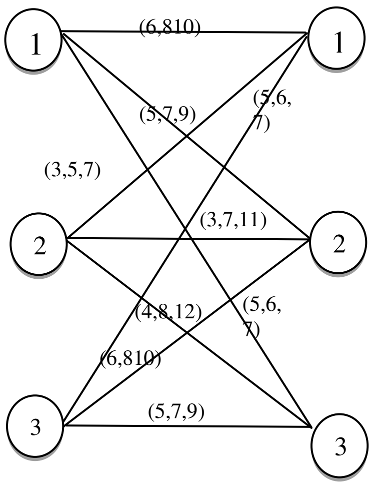
A company has three machines available for allocations to three tasks. Any machine can be allocated to any task and each task requires processing by one machine. The goal is to locate the optimal schedule and next to optimal schedule for the allocation problem. The time $\tilde{t}_{ij}$ required to set up each machine for the processing of each task is given in table I.

Table-I: Fuzzy Time (Hours) Matrix.

Machine↓	Tasks		
	T1	T2	T3
M1	(6,8,10)	(5,7,9)	(5,6,7)
M2	(3,5,7)	(3,7,11)	(4,8,12)
M3	(5,6,7)	(6,8,10)	(5,7,9)

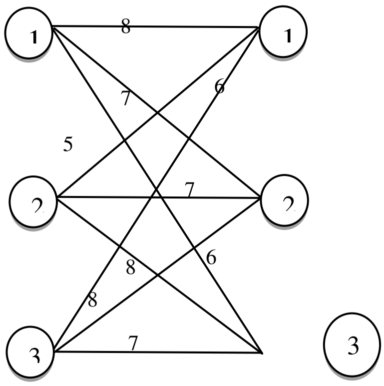
4.1. Metrics in Graphical FAP

Any FAP in Table I can be efficiently modeled as a complete bipartite graph G. The machines and tasks are represented as nodes. For each pair of nodes in graph G, there are metrics that define the fuzzy time between them, which are represented as arcs.



Graph: G

The above graph G is a complete fuzzy bipartite graph which can be converted to the following bipartite graph by using the ranking index.



Graph: $K_{3,3}$

The above complete bipartite graph $K_{3,3}$ has 6_{C_2} perfect matching's, of which the following are the optimal and next to optimal perfect matching's.

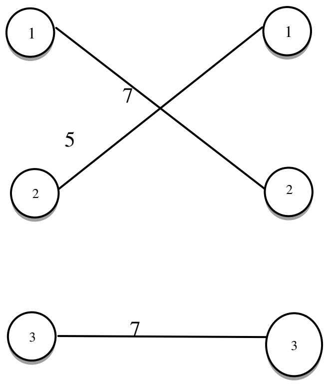
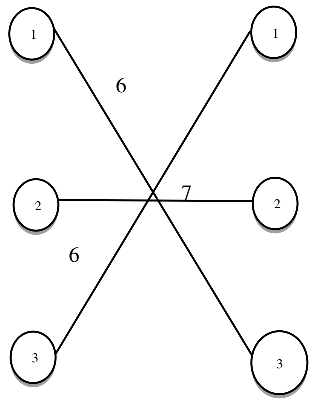
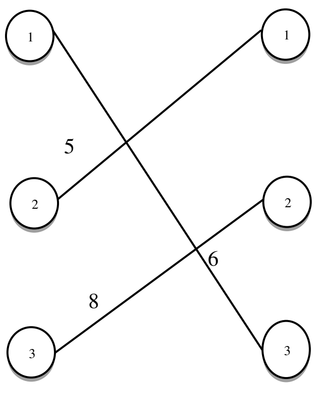
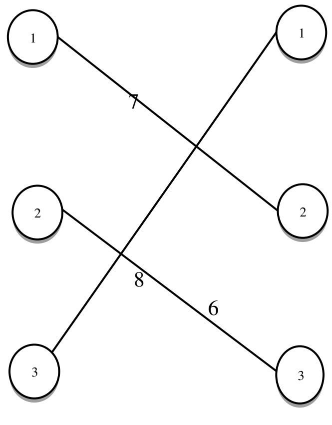
 Perfect matching M1 with weight 19	 Perfect matching M2 with weight 19
 Perfect matching M3 with weight 19	 Perfect matching M4 with weight 21

Table II: Perfect Matching

From the Table II, perfect matching's M1, M2 and M3 are the optimal perfect matching's having weight 19 and the matching M4 is the next to optimal perfect matching with weight 21.

4.2. Management Metrics

Metrics are an objective means of measuring performance and effectiveness. They are crucial in the strategic planning process for optimizing the application of resources to achieve the criterion. For the well-organized management of the FAP, the processing time is considered as metrics. The given FAP in Table Iis viewed as fuzzy processing time criterions between machines and tasks. Replace the fuzzy time $\tilde{t}_{ij}$ by their respective ranking indices which is given in Table III.

Table III: Time (Hours) Matrix

	Tasks		
Machine ↓	T1	T2	T3
M1	8	7	6
M2	5	7	8
M3	6	8	7

Now optimal and next to optimal schedules can be obtained by the following procedure. Find the minimum time in each of the row of the time matrix then determine all possible minimum allocations and their times. Obtain the minimum of the time of all possible minimum allocations which is the optimal schedule. The next to optimal schedule can be obtained by choosing other alternative allocations which yield schedule that are very closer to the optimal schedule. All possible minimum allocations and their corresponding objective values to the reduced allocation problem and the given fuzzy allocation problem is given in Table IV.

Table IV: Schedules and their objective values.

Sl. No	Schedule	Table III Objective value (Hours)	Table I Objective value(Hours)
1	$M1 \to T2,\ M2 \to T1, M3 \to T3$	19	(13,19,25)
2	$M1 \to T3,\ M2 \to T2, M3 \to T1$	19	(13,19,25)
3	$M1 \to T3,\ M2 \to T1, M3 \to T2$	19	(14,19,24)
4	$M1 \to T2,\ M2 \to T3, M3 \to T1$	21	(14,21,28)

From Table IV, the schedules $M1 \to T2, M2 \to T1, M3 \to T3$; $M1 \to T3,\ M2 \to T2, M3 \to T1$; $M1 \to T3,\ M2 \to T1, M3 \to T2$ are identified as the optimal schedule and $M1 \to T2,\ M2 \to T3, M3 \to T1$ is identified as the next to optimal schedule of the given problem.

5. Conclusion

This paper provides the set of schedules for fuzzy allocation problem by analyzing the metrics in different perspectives such as graphical view and management approach. The obtained schedules help the decision makers to assess the economical activities and make suitable managerial decision while they are dealing with various types of allocation problems.

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