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UNDERSTANDING DIGITAL FILTERS-FROM THEORY TO PRACTICE USING MATLAB AND SIMULINK

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Abstract

In the field of signal and image processing, digital filters play a vital role especially in audio engineering. So the way of understanding to design a simple filter to the much complex filter is essential for engineers who work in the field of digital signal processing (DSP). This elegant study convey the fundamentals of digital filters working principle in terms of impulse response and pole-zero analysis. The filter characteristics can be completely derived from the impulse response of the system function. The design of filters is not a mathematical process but should be logically concluded from the location of poles and zeros and understanding the corresponding frequency response. This study aims to highlight the important connections between the filter characteristics and analysis of pole-zero pattern with the corresponding effects in the frequency spectrum using Matlab and Simulink.

Keywords: Digital filter, Impulse response, Matlab, Pole-zero plot, Simulink.

Introduction

Digital signal processing has many advantages compared with the analog signal processing like repeatable, physically stable, and programmable. It has more versatility like storing and saving the information much easier than analog signal processing. Any DSP system contains an anti-aliasing filter at the beginning to avoid aliasing [1] [2]. The sample-and-hold circuit and analog-to-digital converter are responsible to produce the sampled digital signal which is applied finally into the DSP processor. The filtered or processed data out from the DSP processor again fed into the digital-to-analog converter followed by a reconstruction filter to obtain the real time output data [3] [4]. In audio engineering field, the digital filters play a crucial role in obtaining the processed speech or audio. Thus the design of frequency selective filters such as low-

pass, high-pass, band-pass, and band-stop filters are major work for the DSP programmers. Thus the understanding of the basics of filter, impulse response, pole-zero analysis are essential part for the engineers. This study convey the tips, tricks with fundamental concepts behind discrete-time signal processing [4] [5]. The first and foremost concept to be known is the linear convolution technique for linear-shift invariant (LSI) systems. This method obtain the response of the system for any arbitrary input data samples. But the convolution method is the difficult one since it requires flipping, shifting, multiplication, and addition finally. The same response of a LSI system can be made by the techniques like Z-Transform, Discrete-Time Fourier Transform, and Discrete Fourier Transform also. Since the convolution in the time domain would be equal to the multiplication in the frequency domain, the DTFT and DFT methods are most popular [6]. The DTFT method is more suitable with paper-and-pencil work not with a digital hardware since the frequency response is infinite. To work with finite samples, the DTFT output is sampled which becomes the DFT technique. These DFT samples are handled with digital hardware like computers easily [7] [8]. Similarly the correlation techniques are most popular in applications like radar signal processing to determine the distance of the target from the base station. Almost all steps of the convolution are done except flipping the second sequence. So if we flip the second sequence and the convolution is performed with the first sequence, the resultant data would be the cross-correlation. This cross-correlation technique is the most powerful in digital communication receivers as a matched filter [9]. The primary objective of the study is, as stated before, to explain the design of basic filters from the location of poles and zeroes and study the impact of positioning of poles and zeroes on the frequency spectrum. Any DSP system or filter contains adders, multipliers and delay elements [10]. This study is organized as follows. Section 1 illustrates low pass and high pass filter design from pole-zero plot. Section 2 describes simple notch filter and multi notch filter using impulse response and also comb filter. Section 3 gives a cascade combination of notch and comb filter designed to act as a band pass filter.

Materials and Methods

1. Low-Pass and High-Pass Filters

1.1 Understanding Filters by Zero location

Basically, the gain is provided by the poles and the attenuation is occurred by the zeros. The pole-zero placement is the simplest technique to design filters for the simpler specifications [11]. The following examples convey the importance of the locations of the poles and zeros which are reflected in the frequency response of the system.

Example – 1: $H_1(z) = 1 + 0.5z^{-1}$

Here the zero is located at -0.5 and the pole is at the origin as always with causal finite impulse response (FIR) filter. This shows the stability of the FIR filters always. Fig. 1 shows the pole-zero plot for this system and the frequency response is obtained in Fig. 2.
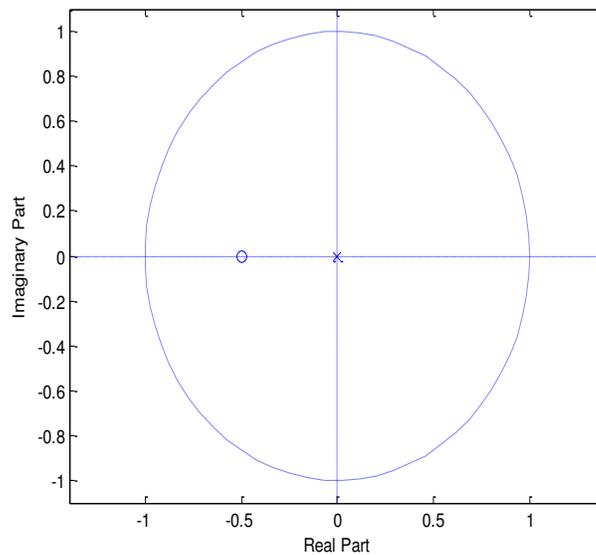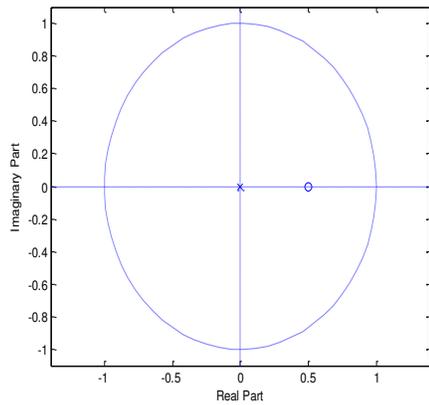

Fig. 1.Pole-zero plot of H$_1$(z).

The magnitude response depends directly on the distance between the zero and e^{jw}, which is a unit circle. It is observed that as ω aries from 0 to □, the distance from the location at -0.5 to the unit-circle (e^{jw}) decreases [12]. Since DC terms and low-frequency terms have the more distance to the zero location, the gain is more for low-frequencies and hence is a low-pass filter. This is appraent in the Fig. 2.
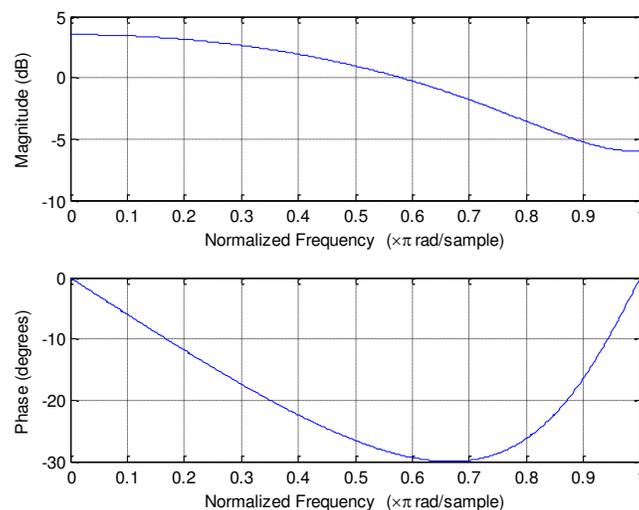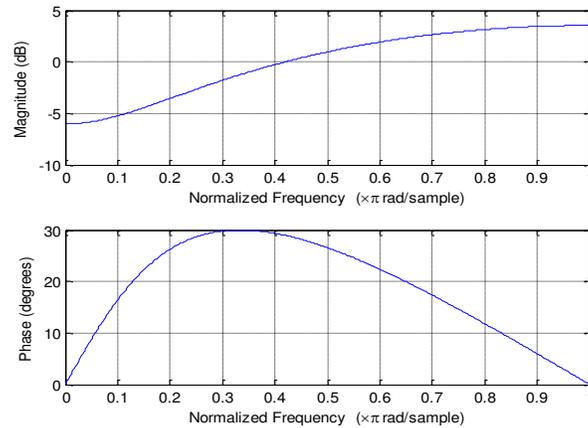

Fig. 2. Frequency response of H$_1$(z).

Example –2: $H_2(z) = 1 - 0.5z^{-1}$

Here the zero is located at + 0.5 and the distance for the high frequency points to the zero location are maximum. So this system function works as a high-pass filter. Fig. 3 shows the pole-zero plots for this system and the frequency response is obtained in Fig. 4.

Fig. 3. Pole-zero plot of H₂(z).

Fig. 4. Frequency response of H₂(z).

Thus, it is evident that, for a low-pass, there is a maximum gain at $\omega = 0$ and gain decreases with increasing ω. For a high-pass, there is a least gain at $\omega = 0$ and gain increases with increasing ω [13].

1.2 Understanding Filters by Pole location

The Infinite Impulse Response (IIR) filters have poles which determine the feedback filter's behavior like the stability and frequency response.

Example – 1: $H_3(z) = z/(z – 0.6)$

Here the pole is located at +0.6 and the distance from the frequency point on the unit semicircle to the location at +0.6 is minimum for the DC term and low-frequencies. Since the gain is inversely proportional to this distance, it is maximum for low frequencies and thus behaves as a low-pass filter [14]. The pole-zero plot and the frequency response results are obtained in Fig.5 and 6 respectively.

Example – 2: $H_4(z) = z/(z + 0.6)$

Here the pole is located at − 0.6 and the filter now behaves in a opposite manner. That is the distance between the high frequency points on the unit semicircle to the pole is minimum and thus provide more gain [15]. This high-pass filter's pole-zero plot and the frequency response are shown in Fig. 7 and 8 respectively.
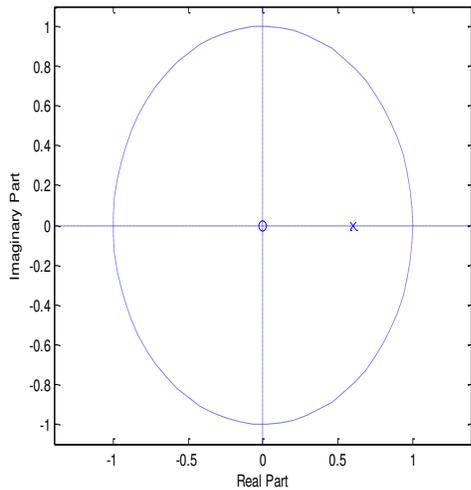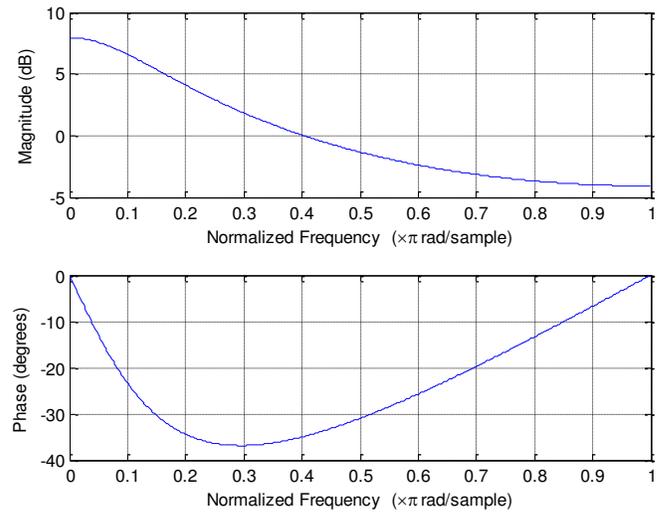
Fig. 5.Pole-zero plot of H₃(z).
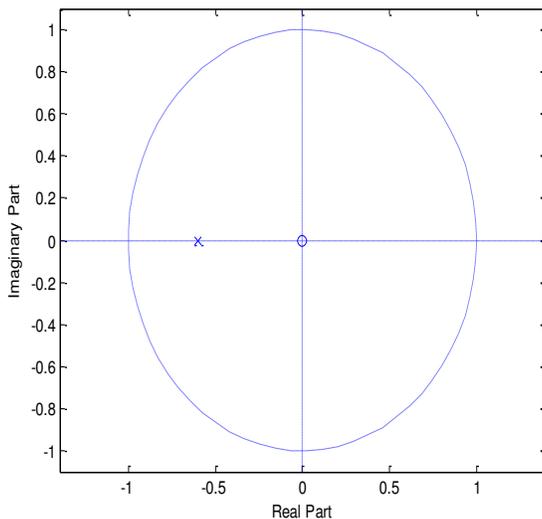

Fig. 6. Frequency response of H₃(z).
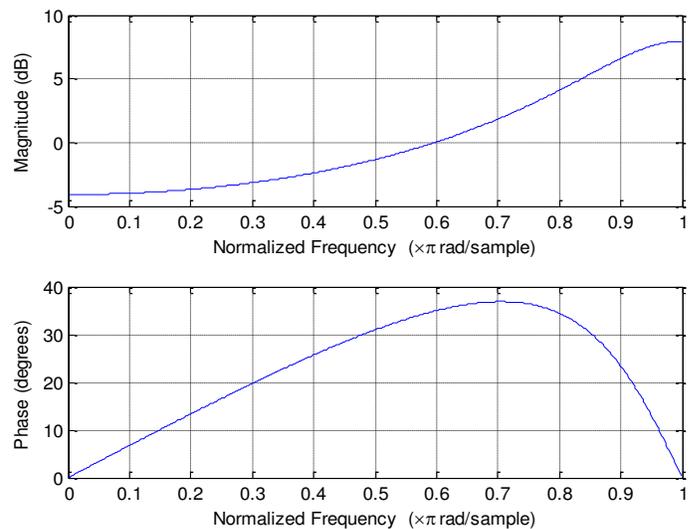

Fig. 7. Pole-zero plot of H₄(z).

Fig. 8. Frequency response of H₄(z).

2. Notch and Comb Filter

2.1 Simple Notch Filter

A notch filter passes most frequencies unaltered, but attenuates those in a specific range to very low levels [16]. For example, zeros are placed at $z = + j$ and $z = - j$ to reject the $F_s/4$ frequencies. So, the system function becomes as $1 + z^{-2}$, which can be modeled using Simulink. Fig. 9 shows the spectrum of the generated audio tone frequencies of 5512 Hz (Fs/8), 11025 Hz (Fs/4), 16538 Hz (3Fs/8), and 22050 Hz (Fs/2) with a sampling frequency of 44100 Hz. Fig. 10 shows the corresponding Simulink model and the notch filter output is shown in Fig. 11 which does not have the Fs/4 tone, i.e., 16538 Hz.
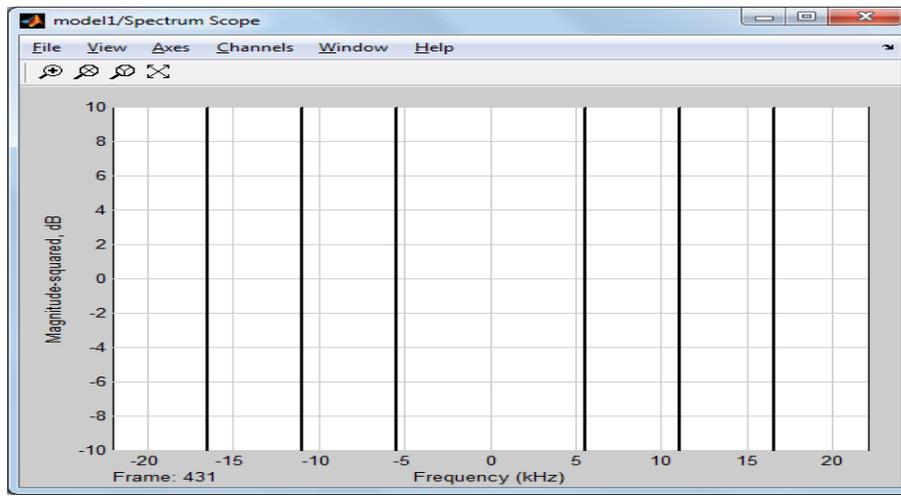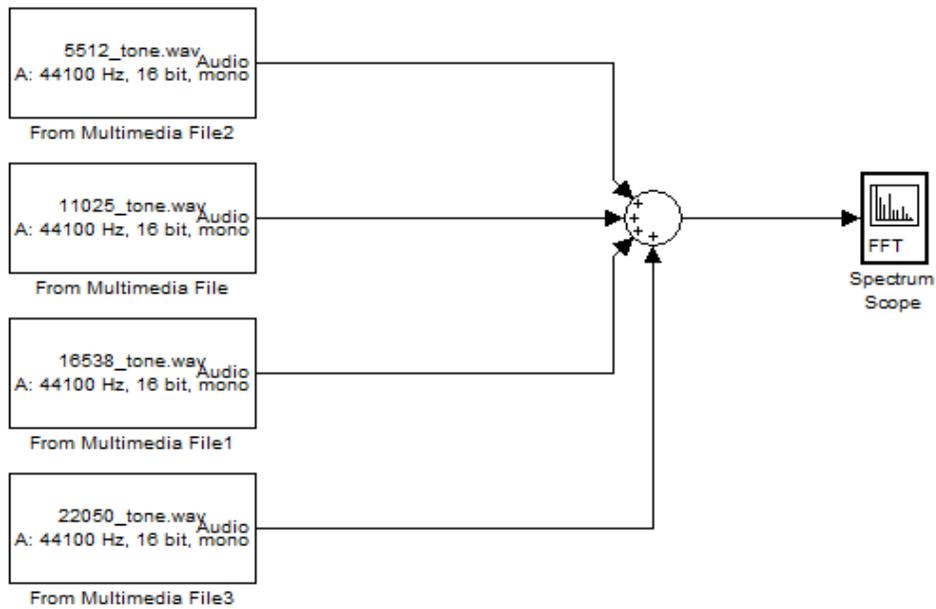
Fig. 9. The spectrum plot of the input signal.

Fig. 10. Simulink model of a notch filter to reject Fs/4 frequency.

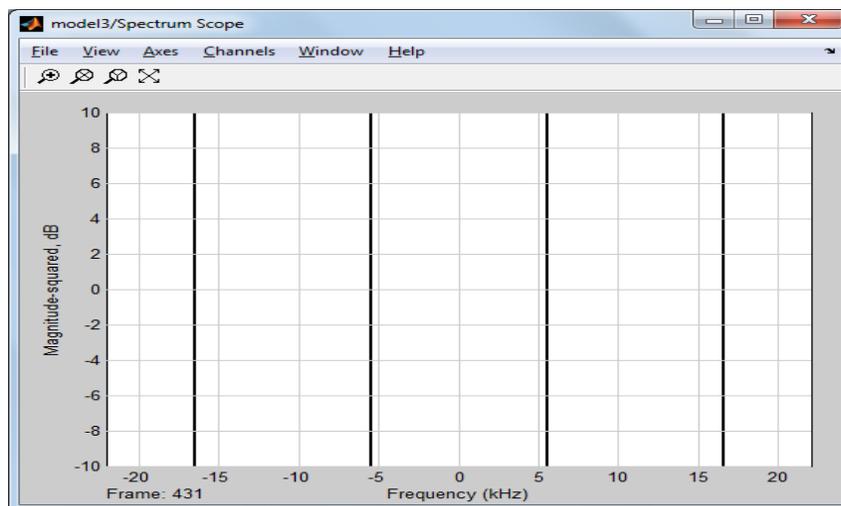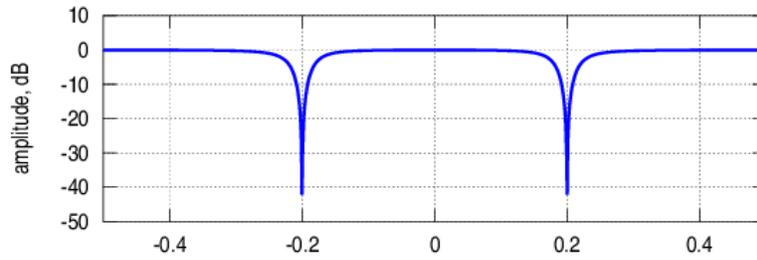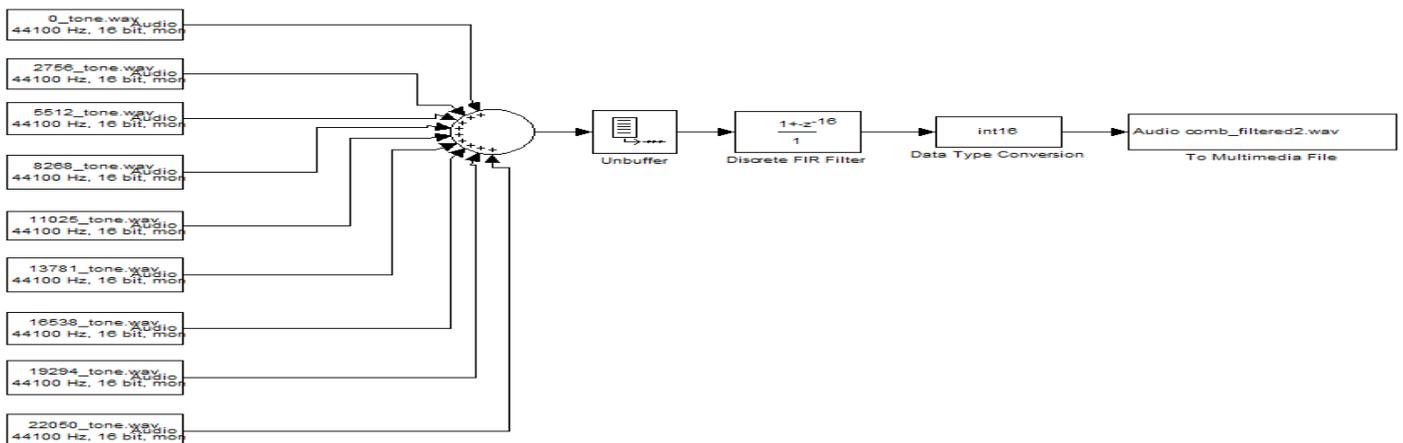**Fig. 11. Spectrum view of the absence of the frequency Fs/4.**

**Fig 12. The response of a multi-notch filter.**

2.2 Multi-notch filter

This is an extension of notch filter used to reject a number of specific frequencies. An advantage of multi notch filter is that it gives equal magnitude response to the allowed band of frequencies unlike comb filter [13]. A specific technique used in multi notch filter is placing of poles near the zeroes and not at the origin. This is a unique feature of multi notch filter. Below given response is an example of the expected frequency response of a multi notch filter.
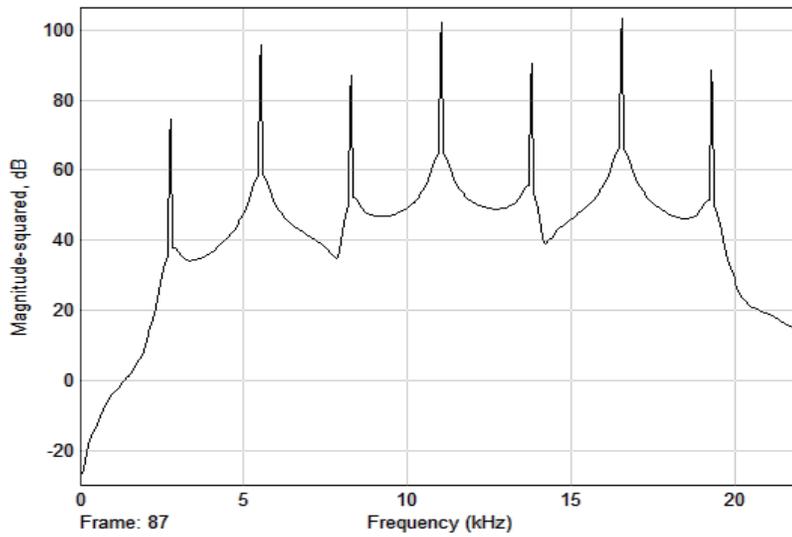
2.3 Comb filter

Comb filter differs from notch filter as it does not give flat response to the allowed frequencies [17]. There is a peak in response in between any two rejected frequencies. The frequency response of a comb filter consists of regularly spaced spikes, thus giving the appearance of a comb. Since the system function is $1 - z^{-16}$, this filter rejects $0F_s/16$, $1Fs/16$, $2Fs/16$, $3Fs/16$, till $8Fs/16$ frequencies and allows $1Fs/32$, $3Fs/32$, $5Fs/32$, till $15Fs/32$ frequencies. This filter is constructed as in Fig.13. The input spectrum view and the response of the filter are shown in Fig. 14 and

Fig. 15 respectively. The frequency spectrum of the output signal of a comb filter does not show the peaks at the rejected frequencies.

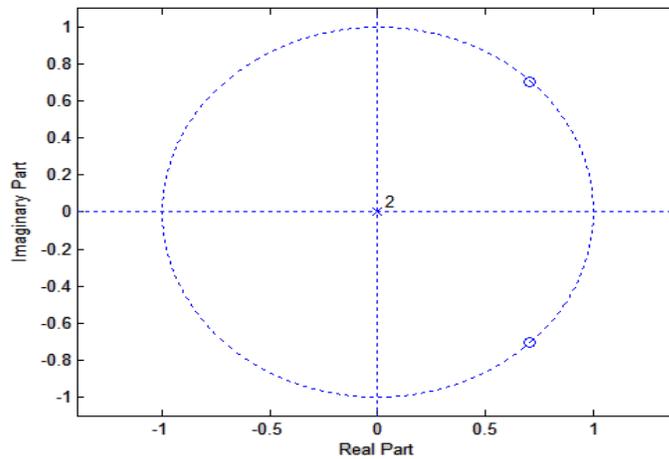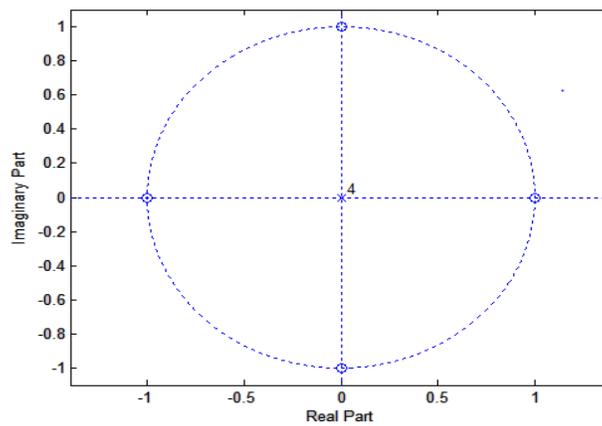
Fig. 13. Simulink model of a comb filter
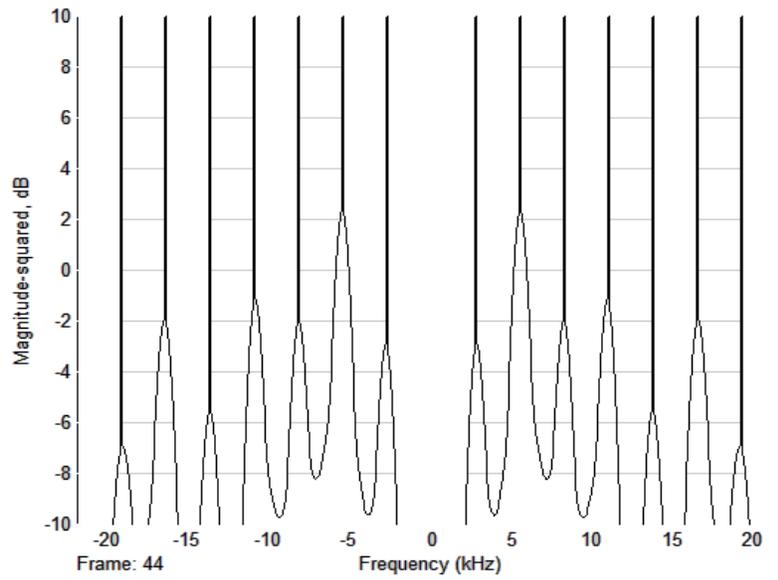
Fig. 14. The frequency spectrum view of the input to the comb filter.

Fig. 15. Frequency spectrum of output signal.

**Fig. 16. Pole zero plot of notch filter designed to reject Fs/8.**

Fig. 17. Pole zero plot of comb filter designed to reject dc, Fs/4, Fs/2.

3. Complex Filters Using Simple Filters

3.1 Band pass filter

A complex filter can be designed easily by connecting simple filters in cascade. The cascade combination of a notch and comb filter can be used to produce a band pass filter. The resulting filter functionality can be understood by studying the pole zero plot of the individual filters [1]. Again pole-zero placement technique helps us to design such a band-pass filter. Fig. 16 shows a notch filter which rejects the frequency, Fs/8. Fig. 17 shows a comb filter designed to reject the frequencies dc, Fs/4 and Fs/2. These two are cascaded to form a band-pass filter and the resulting system pole-zero plot is obtained in Fig. 18.
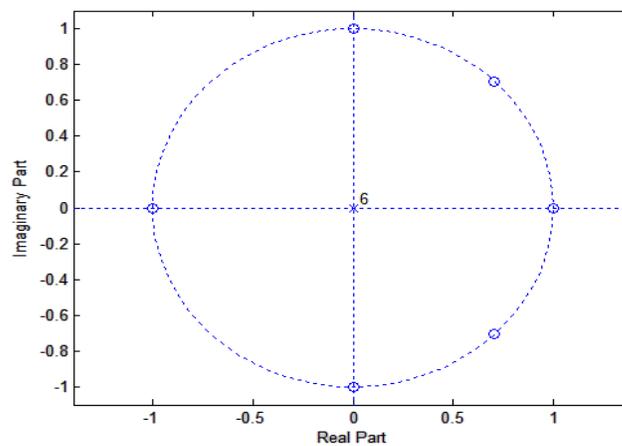
Fig. 18. Pole zero plot of the resulting band pass filter.

From the pole zero locations, it is evident that the frequencies in the band Fs/4 and Fs/2, –Fs/4 and –Fs/2 are allowed, thus forming the pass band. This can be visualized from the frequeency spectrum of the above designed filter shown below in Fig. 19.
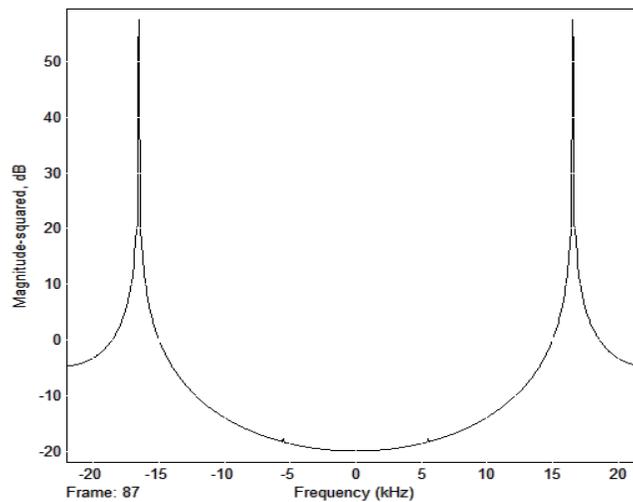
Fig. 19. Frequency response of a band-pass filter.

Results and Discussion

Fundamentals of digital filters working principle in-terms of pole-zero analysis and impulse response are discussed in this study. Various filters like low-pass, high-pass, Notch filter, Comb filter, and complex filters are considered in the discussion. Based on zero-location, low-pass and high pass filters are obtained. Fig.1 and Fig.2 shows the pole-zero plot and frequency response of low-pass filter, and Fig.3 and Fig.4 shows the pole-zero plot and frequency response of high-pass filter respectively. Similarly, high-pass and low pass filters are designed based on the pole-location. Fig.5 and Fig.6 depicts the frequency response and pole-zero plots of low-pass filter, and Fig.7 and Fig.8 shows the frequency response pole-zero plot and of high-pass filter respectively. In the case of notch filter it leaves most frequencies unchanged but certain frequency is attenuated to low level. Fig.9 exhibits the audio tone frequency spectrum and Fig.10 presents the corresponding Simulink model. Fig.11 demonstrates the output of notch filter. Multi-notch filter is the extension of a notch filter. The frequency response of multi-notch filter is exhibited in Fig.12. The next filter into consideration is the comb filter. This filter differs from notch filter because no flat response. This filter results in a peak between any two rejected frequency. A Simulink model of a comb filter is established as in Fig.13. The frequency spectrum view of input to the comb filter is shown in Fig.14. The output signal frequency spectrum is given in Fig.15. A band-pass signal is obtained by cascading a notch and comb filter. A notch filter which rejects the frequency, Fs/8 is shown in Fig.16. Similarly, Fig. 17 depicts a comb filter designed to reject the frequencies dc, Fs/4 and Fs/2 .By cascading the results of Fig.16 & 17 pole-zero plot for band-pass filter is obtained and is shown in Fig.18. Finally, the frequency response of a band-pass filter is shown in Fig.19.

Conclusions

To understand the digital signal processing fundamentals, this study conveys an elegant approach to reduce the gap between theory and practice using Matlab and Simulink. Many examples exemplify the concept behind the impact of the pole-zero locations of a system. The best method to learn and study signal processing fundamentals is by practice with demonstrations which helps reducing also the gap between the industry and academy. Matlab and Simulink tools are used to design filters with simple specifications here. This study helps student community for better understanding the importance of poles and zeros of a system. Other tools like, filter design analysis tool (FDA tool), filter visualization tool (fvtool), and signal processing tool (sptool) which are part of the Matlab software can also be used to design efficient filters. This study further can be extended to help the student community to understand FIR and IIR filter design techniques better.

References:

1. Terell, T.J., Lik-Kwan,S., "Digital Signal Processing – a student guide", Macmillan, 1996.

2. Chakka Sri Harsha Kaushik, Tadvarthi Gautam, V Elamaran, "A Tutorial Review on Discrete Fourier Transform with Data Compression Application", Proc. of the International Conference on Green Computing, Communication and Electrical Engineering (ICGCCEE 2014), pp.1-6, Mar. 2014.

3. Vellaiappan Elamaran and Angam Praveen, Comparison of DCT and Wavelets in Image Coding, Proceedings of the International Conference on Computer Communications and Informatics, Coimbatore, India, pp.1-4, 2012.

4. V.Elamaran, Kalagarla Abhiram and Narreddi Bhanu Prakash Reddy, FPGA Implementation of Audio Enhancement using Xilinx System Generator, Journal of Applied Sciences, Vol 14, pp.1972-1977, 2014.

5. Roberto Cristi, "Modern Digital Signal Processing", Brooks/Cole publishers, 2004.

6. John G Proakis ,Dimitris G Monalakis, "Digital Signal Processing, Principles , Algorithms and Applications", Fourth Edition , Pearson Education, New Delhi, 2006.

7. J.Lim, A.Oppenheim, "Advanced Topics in Signal Processing", Prentice Hall, 1988.

8. Oppenheim A.V ,Schaffer R.W, "Discrete-Time Signal Processing", Second Edition, Prentice Hall, New Delhi, 2002.

9. Krzysztof Sozanski, "A Linear-phase IIR Filter for Audio Signal Interpolator", Proc. of the International Conference on Signal Processing Algorithms, Architectures, Arrangements, and Applications, pp.65-69, 2013.

10. Li Tan, "Digital Signal Processing – Fundamentals and Applications," Academic Press, Elsevier publishers, 2008.

11. Emmanuel C.Ifeachor, Barrie W.Jervis, "Digital Signal Processing: A practical Approach", Second Edition, Prentice Hall of India, New Delhi, 2002.

12. Edumand Lai, "Practical Digital Signal Processing for engineers and technicians", Newnes, 2004.

13. B.Porat, "A Course in Digital Signal Processing", Wiley, 1997.

14. Kruthiventi S.S.Srinivas, Kishore prahallad, "An FIR Implementation of Zero Frequency Filtering of Speech Signals", IEEE Transactions on Audio Speech and Language Processing, Vol 20,no.9, pp.2613-2617, 2012.

15. Sanjit K.Mitra, "Digital Signal Processing – A Computer Based Approach", Third Edition, Mc-Graw Hill, 2007.

16. Bernard Mulgrew, Peter Grant, John Thompson, "Digital Signal Processing : Concepts and Applications", 2nd Edition, Palgrave Macmillan, 2003.

17. Smith, Stevan W., "Digital Signal Processing : a practical guide for engineers and scientists", Newnes, 2003.

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