ISSN: 0975-766X
CODEN: IJPTFI
Available Online through ***Research Article***
www.ijptonline.com

A CASE STUDY ON WAVELET ANALYSIS AND ITS APPLICATIONS

[1]V Elamaran, [2]Har Narayan Upadhyay, [1]G Rajkumar
[1]Assistant Professor, Dept., of ECE, SEEE, SASTRA University, Thanjavur, Tamil Nadu, India.
[2]Associate Dean, Dept., of ECE, SEEE, SASTRA University, Thanjavur, Tamil Nadu, India.
Email: elamaran@ece.sastra.edu

Received on 08-08-2015 *Accepted on 27-08-2015*

Abstract

Wavelet analysis has become a significant computational tool in signal processing and image processing applications. The wavelet analysis is renowned for its successful approach to the problem of analyzing a signal in both time domain and frequency domain. Analyzing non stationary signal had been a big challenge for various transform techniques. The transform techniques such as Fourier Transform (FT), short time Fourier Transform (STFT) are failed in analyzing non-stationary signals. But wavelet transform (WT) techniques can be able to analyze both stationary and non stationary signals in an effective manner. WT is capable to analyze one dimensional signal like audio signals and two dimensional signals like images. In this paper wavelet transform techniques and its applications are discussed in detail and the experimental results are shown using MATLAB simulation software tool.

Key Words:

Data decomposition, Frequency domain, Non-stationary signals, Time domain, Wavelet, Wavelet Analysis.

Introduction

The signals are need to be transformed to various domains in order to extract information. Most of the signals in practice are time domain signals. Often, the information that cannot be seen in time domain can be readily seen in frequency domain. Fourier transform is the most popular transform to obtain frequency information of the signal. But the Fourier transform is not suitable for analyzing non-stationary signals, due to its poor time resolution (Robi polikar).

It has been shown that wavelets can approximate time varying non-stationary signals with better time and frequency resolutions (Mulgrew et al, 2003). Wavelet can be visualized as a brief oscillation with amplitude that begins at zero, increases, and then decreases back to zero (Brani Vidakovic and Peter Mueller). A wavelet function has two main properties,

- The function should be oscillatory or should have wavy appearance
- Most of the energy is confined to a finite duration

Wavelet analysis is the breaking of a signal into shifted and scaled version of original wavelet called mother wavelet. Several families of wavelets are available for analysis. Some of the most useful wavelet families are daubechies, haar, biorthogonal, symlet, coiflets, symlets, morlet, mexian hat, meyer (Chun-Lin, Liu, 2010; Smith, 2003). Wavelets are capable to analyze localized area of a larger signal.

Continuous wavelet transform (CWT) measures the similarity between the signal and an analyzing function called wavelet. The CWT compares signal to the shifted and compressed or stretched versions of a wavelet. Wavelet decomposition is capable of analyzing a signal at different resolution levels. Discrete wavelet transform (DWT) analysis is more efficient and accurate. The DWT can be represented by octave-band tree structure (J.K.M. Vetterli, 1995). This scheme can be implemented by decomposing the original signal into approximation and detail sub bands by passing it through the series of low pass and high pass filters.

The low frequency content is called approximation and high frequency content is called detail. The decomposition process can be iterated by decomposing the successive approximations. The Wavelet packet decomposition (WPD) is a generalization of discrete wavelet transform (Mac A. Cody, 1994). Optimal sub band tree structure is used to represent WPD. The signal is passed through more filters than the DWT. In this scheme both approximation and detail sub band are decomposed in further iterations. Wavelet analysis can be applied to one dimensional data (eg: Audio signals, biological signals) as well as two dimensional data (eg: Images). Though wavelet processing has wide applications, this paper focuses on compression and de-noising.

Materials and Methods

Wavelet Decomposition

In Wavelet decomposition, a signal is decomposed by passing it through series of low pass and high pass filters. The low pass filter extracts the coarse signal information called approximation. On the other hand, the high pass filter extracts the sharp transitions and noisy components of the signal called detail (Manikandan and Madheswaran, 2007). After the filtering process the resultant signals are down sampled to maintain good resolution.

The decomposition process can be iterated by further decomposing the approximation and it is continued until the desired level is reached. A three level wavelet decomposition tree structure is shown in Fig 1. Necessary modifications have to be done in the deepest level of decomposition and then the original signal can be reconstructed from decomposed coefficients using bottom-up approach.

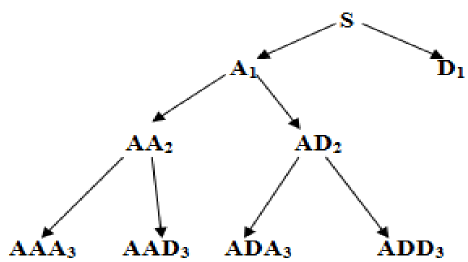


Figure 1: Wavelet decomposition tree structure

Decomposition of Non-Stationary Signals

The stationary concept of signal has paramount importance in signal analysis. The signals whose frequency content do not change in time are called stationary signals i.e., frequency content of stationary signals do not change in time. In this case all frequency components exist at all times. For example the signal,

$$x(t)=\sin(2\pi*25*t)+\sin(2\pi*50*t)+\sin(2\pi*100*t) \quad (1)$$

is a stationary signal since it has frequencies of 25, 50 and 100 HZ at all time instants. In contrary, the signal whose frequency content constantly varies in time is called non-stationary signals. The pictorial representation of non-stationary signal is shown in Fig 2(a) which is the concatenation of 20, 40 and 60 HZ frequencies. By analyzing a signal, Fourier transform can be able to give what frequency components exist in the signal. But it cannot give at what time instance the frequency occurs. Hence it is not suitable for analyzing non-stationary signal (Cristi, 2004). Wavelets are able to analyze the non-stationary signal in efficient manner where the Fourier transform fails. By choosing appropriate wavelet, single level wavelet decomposition can be performed on non-stationary signal in order to analyze it. The decomposition results in approximation and detail sub bands.

The approximation shown in Fig 2(b) contains all the frequency components since the highest frequency component (60HZ) in the non-stationary signal is much below the Nyquist limit for the sampling rate of the signal(2500/2=1250HZ). Whereas the detail shows the points of change of signal i.e., it has detected the points in time at which the frequency of the signal changes as shown in Fig 2(c).

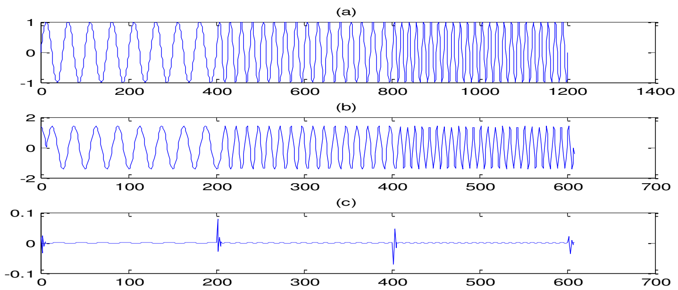


Figure 2: Wavelet decomposition of Non-stationary signal (a), approximation (b) and detail (c).

Detecting Signal Disontinuities

The signal may undergo abrupt changes such as a jump or a sharp change in the first or second derivative. Detecting the abrupt changes is one of the major problems in signal processing. Wavelet analysis helps in detecting signal discontinuities as the wavelets are capable to analyze localized area of the signal. This analysis is carried out to determine:

- At which time the change occurs
- Amplitude of the change
- Type of the change

A signal with discontinuities is shown in Fig 3(a). The discontinuities can be detected by performing single level wavelet decomposition. Short wavelets have to be chosen for decomposition since the short wavelets are more effective in finding signal rupture.

Fig.3 (b) and Fig.3(c) shows the first level of approximation and detail sub bands respectively. The detail sub band shows exactly where the breakdown occurs. So the discontinuities are detected near the samples 50 and 150.

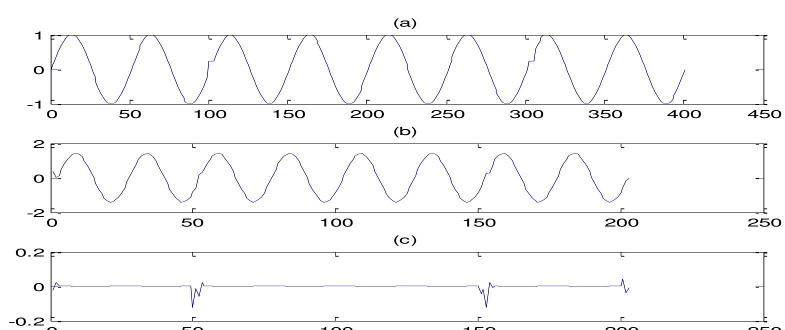


Figure 3: Detecting signal discontinuities (a) Original signal with discontinuities (b) first level approximation (c) first level detail.

Signal De-Noising

De-noising is one of the major applications of the wavelet analysis. The signals may corrupted by various factors during acquisition, processing, compression and transmission. Most of the signals experience the effect of noise (Abomhar et al., 2010; Li and Jiang, 2013). So de-noising the signal is the important process. De-noising one dimensional using wavelet analysis is as follows:

Unlike previous sections the signal de-noising needs more levels of decomposition. Wavelet analysis is performed on the noisy signal by applying wavelet decomposition. Here the level of decomposition is four. In wavelet analysis the approximate sub band contains the low frequency components called signal component and detail sub band contains the high frequency components made of noise.

The Fig. 4 shows the original signal which is corrupted by noise and approximate sub band of four decomposition levels. The noise decreases as the level of decomposition increases. At each level the noise is lower than the previous level. The noise is completely removed at the fourth level as shown in fourth level of approximation in Fig. 4.

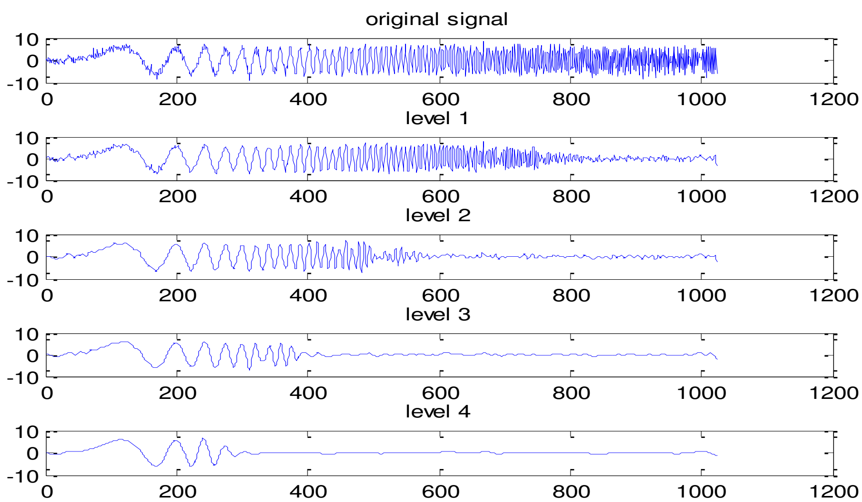


Figure 4: De-noising one dimensional signal.

Image Compression

Image compression is another important application of wavelet analysis. Digital image has to be compressed for easy storage and transmission (Russ and Russ, 2007). Image compression can be done with less computational complexity and without compromising the image quality by applying discrete wavelet transform.

The discrete wavelet transform first decomposes the given image into approximate and detail coefficients. Here the compression is done by thresholding. Thresholding is divided into two categories as follows:

- Hard threshold
- Soft threshold

Hard threshold seems to be natural. Soft threshold shrinks coefficients above the threshold in absolute value. The threshold method has to be chosen according to the application. Hard threshold is chosen for image compression.

The steps involved in image compression are as follows:

- Choose appropriate wavelet and decomposition level. Perform decomposition to the chosen level.
- The threshold is selected and applied to the detail coefficients of each level. Coefficients below the threshold are set to zero.
- Wavelet reconstruction is computed using the approximation coefficients and modified detail coefficients of the deepest level.

Peak signal to noise ratio (PSNR) is estimated as a quality measurement to determine the performance of the technique. The measure of peak signal to noise ratio is defined by the formula:

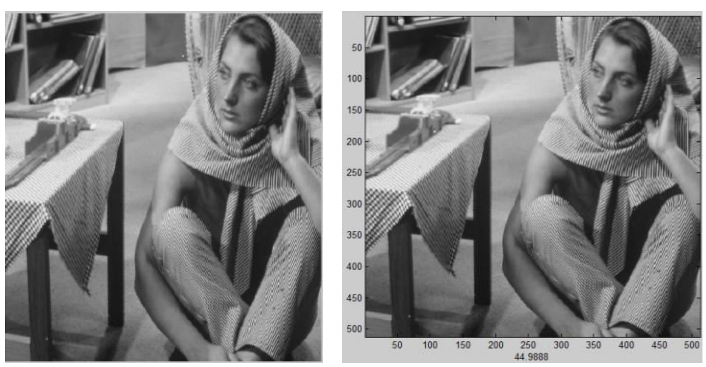
$$\text{PSNR in dB} = 10\log\left(\frac{255^2}{MSE}\right) \qquad (2)$$

where,

$$\text{MSE} = \frac{1}{mn}\sum_{xy}(f'(x,y) - f(x,y))^2 \qquad (3)$$

Let f(x, y) be the original image of size m x n and f'(x, y) be the compressed image of same size.

The image compression using discrete wavelet transform is applied to various images and the results with PSNR values as a performance measure are shown in Fig. 5 and Fig. 6.



(a) (b)

Figure 5: (a) Original image (b) compressed image with PSNR 44.9888

(a) (b)

Figure 6: (a) Original image (b) compressed image with PSNR 51.3908.

Wavelet Packet Decomposition

Wavelet packet decomposition is a generalization of wavelet decomposition method. Wavelet packet is a collection of wavelet functions. The wavelet packets provide more flexibility in terms of scale and position than a wavelet. In wavelet packet transform the decomposition process is iterated by decomposing both approximation and detail subbands. Hence, N level of decomposition yields 2N nodes as a result (Logashanmugam and Ramachandran, 2008).

Using suitable wavelet packet, signal is decomposed into approximate and detail sub-bands. The low frequency components which give the signal its identity are preserved in approximate coefficients.

The high frequency components together with harmonic interference and other noisy components are represented in detail coefficients. These subbands are further decomposed and the decomposition continues until the desired level is reached.

A three level wavelet packet decomposition tree structure is shown in Fig 7. Necessary modifications can be done in the deepest level and the signal can be reconstructed using bottom up approach as in wavelet decomposition method. Several applications are available for wavelet packet decomposition. Few more applications are described in the following subsequent sections.

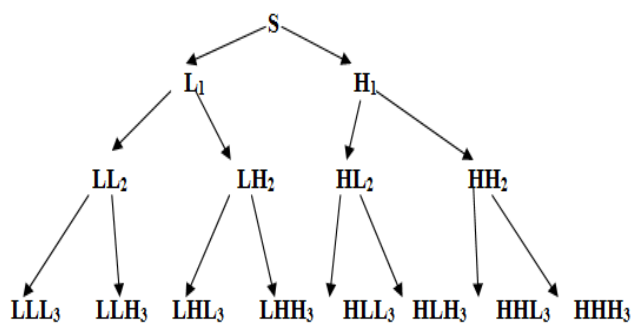


Figure 7: Wavelet packet decomposition tree structure.

Denoising One Dimensional Signal

One dimensional signal can be de-noised by wavelet decomposition but de-noising by wavelet packet decomposition is much faster and accurate. This is because, here both approximate and detail coefficients are further decomposed and analyzed. Suitable wavelet packet has to be chosen for decomposition and analysis. Here "db4" wavelet packet is used for de-noising the one dimensional signal.

Fig 8 shows the noisy signal and the de-noised signal. In this de-noising method, noisy signal is decomposed and various wavelet packet coefficients are obtained. Here the de-noising is done by thresholding. Thresholding is a process in which the coefficients above the threshold value are eliminated. This process is carried out in the deepest level of decomposition. Then the signal can be reconstructed and de-noised signal is obtained.

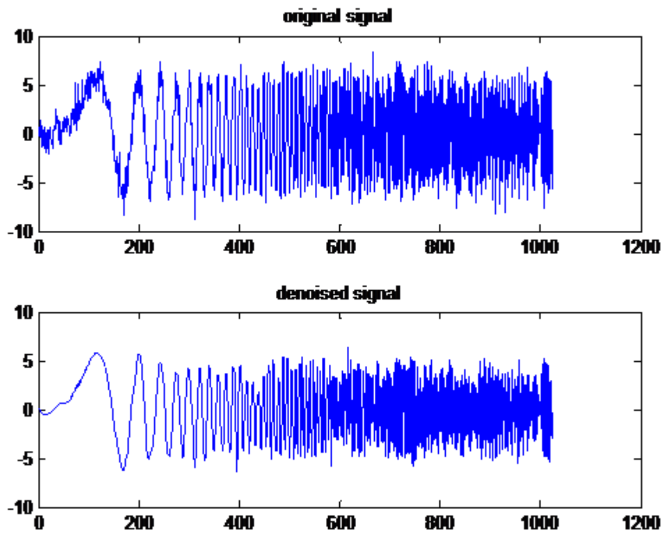


Figure 8: Signal de-noising using wavelet packet decomposition.

Denoising Two Dimensional Signals

Two dimensional signals can be well analyzed by wavelet packet decomposition. In most of the cases the wavelet packet decomposition is used to compress the two dimensional signals called images. The wavelet compression is much better than JPEG compression since it avoids small artifacts and well suited for detecting discontinuities. Various level of decomposition is performed on two dimensional signals according to the applications. First level image decomposition yields four nodes as a result. Each node can be further decomposed and each node yields four nodes. Hence second level of decomposition yields sixteen nodes as a result. The level of decomposition is chosen according to the application.

Single level decomposition is shown in Fig. 8. Fig 9 shows the two level decomposition with sixteen nodes. Third level decomposition with sixty four nodes is shown in Fig 10.

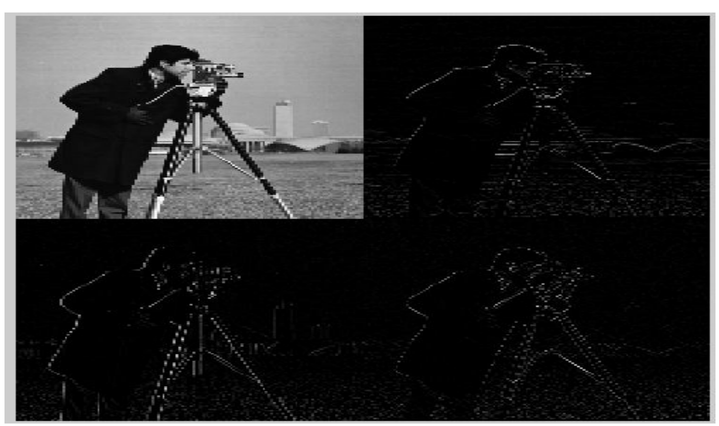


Figure 9: Single level decomposition.

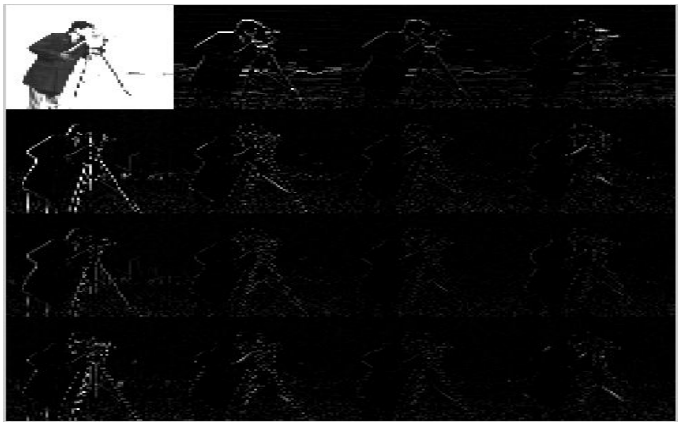


Figure 10: Two level decomposition.

Results

The one dimensional wavelet analysis is tested on an audio signal. De-noising which is one of the applications of wavelet decomposition is performed on a tone and the simulation results are shown in figure. The tone to be de-noised is shown in Fig. 11. Approximation sub bands of four level of decomposition are shown in Fig.12. At fourth level the noise is removed.

The one dimensional wavelet packet analysis is also tested on the audio signals. The harmonic distortion in an audio signal can be removed using wavelet packet decomposition. The results are shown in figures.

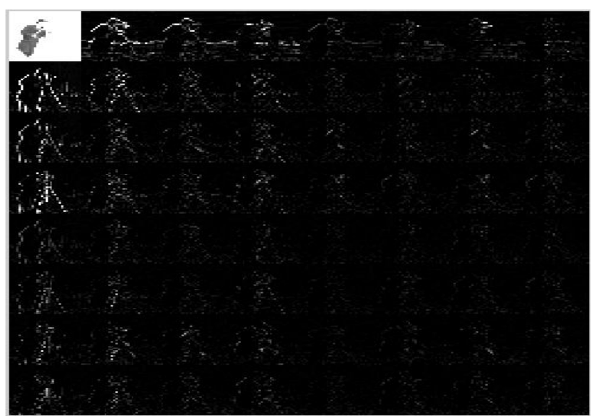


Figure 11: Three level decomposition.

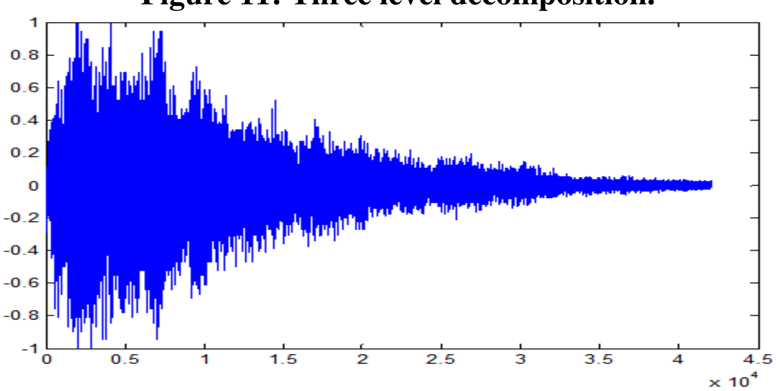


Figure 12: Tone to be de-noised.

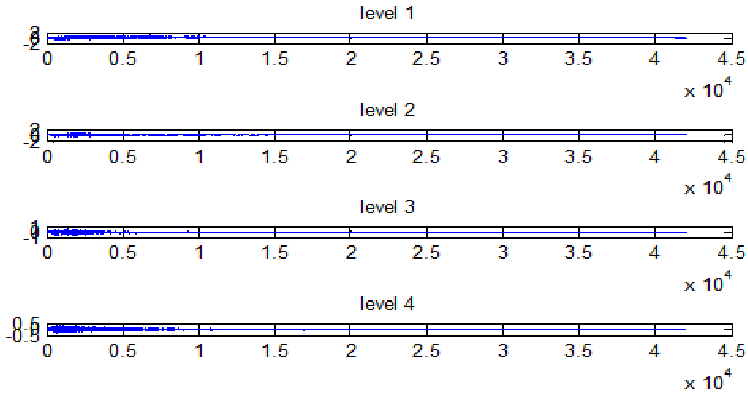


Figure 13: Approximation sub-bands

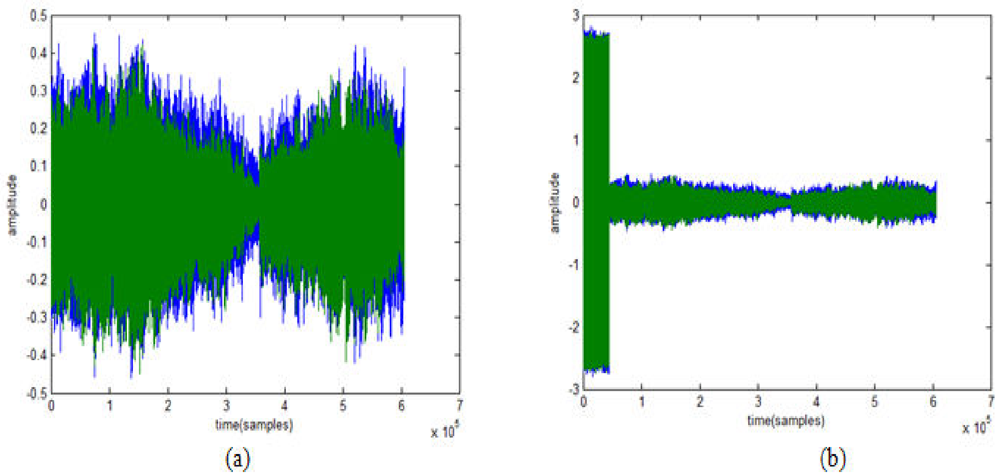


Figure 14: (a) original audio signal. (b) Contaminated audio signal.

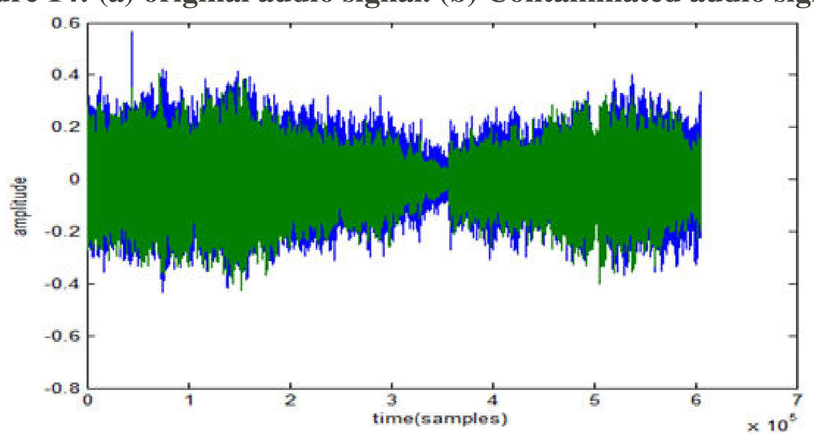


Figure 15: Reconstructed audio signal.

Discussions

Harmonics Removal

Wavelet packet decomposition is a discrete wavelet transform provides a powerful tool for signal analysis. Using suitable wavelet, signal is decomposed into approximate and detail subbands. The low frequency components which give the signal its identity are preserved in approximate coefficients. The high frequency components together with harmonic interference and other noisy components are represented in detail coefficients. These subbands are further decomposed and the decomposition continues till the cut-off frequency of approximate subband becomes smaller than the fundamental frequency of the harmonics. The harmonic distortion can be eliminated simply by thresholding the detail coefficients when the signal to noise ratio (SNR) is large enough. The degree of distortion increases with decrease in the level of SNR. The bottom level subbands have to be chosen for processing to remove the harmonics.

Mean of the wavelet coefficients in a sub-band can be referred as baseline of the coefficients [7]. If the signal is contaminated by harmonics, non-zero baseline may appear in detail subband. The harmonic interference is solely responsible for the occurrence of non-zero baseline, when a special constraint condition is satisfied. The harmonics can be removed by shifting the baseline back to zero. The shifting of baseline does not disturb the original signal components. The PSNR of the audio signal in Fig. 13 is 51.73 dB whereas SNR of the harmonics removed audio signal in Fig. 14 is 79.27dB. Various music signals and speech signals are tested and the PSNR is tabulated as shown in Table 1 and Table 2.

Table 1: PSNR results for music.

Test Signals	PSNR (dB)	
	Before removal of harmonics	After removal of harmonics
Music 1	52.27	73.99
Music 2	54.51	81.20
Music 3	54.85	83.18
Music 4	54.84	77.20

Table 2: PSNR results for speech

Test Signals	PSNR (dB)	
	Before removal of harmonics	After removal of harmonics
Speech 1	53.67	74.94
Speech 2	52.58	72.34
Speech 3	51.61	65.37
Speech 4	47.16	77.58

Conclusions

This study helps teaching and student community a lot to understand better with wavelet analysis and its applications. This work can be further extended to much more advanced topics such as time-frequency analysis, multi rate signal processing and spectral estimation too..

Acknowledgement

The author would like to thank Dr. Har Narayan Upadhyay, Professor and Dr.R.John Bosco Balaguru, Professor, Electronics and Communication Engineering Deoartment, SASTRA University, Thanjavur for constant encouragement and valuable guidance during our research work.

References:

1. Bernard Mulgrew, Peter Grant and John Thompson, Digital Signal Processing: Concepts and Applications. 2nd Edition, Palgrave Macmillan, U.K., 2003.
2. E. Logashanmugam and R. Ramachandran, An Improved Algorithm for Image Compression Using Wavelets and Contrast Based Quantization Technique, Information Technology Journal, Vol 7, pp.180-184, 2008.
3. Jae S.Lim and Alav V.Oppenheim, Advanced Topics in Digital Signal Processing, Prentice-Hall, Englewood Cliffs, NJ, 1988.
4. Li Tan and Jean Jiang, Digital Signal Processing: Fundamentals and Applications. Academic Press, 2013.
5. M. Abomhara, O.O. Khalifa, O. Zakaria, A.A. Zaidan, B.B. Zaidan and A. Rame, Video Compression Techniques: An Overview, Journal of Applied Sciences, Vol 10, pp.1834-1840, 2010.
6. S.Manikandan and M.Madheswaran, Implementation of Active Noise Filter for Real-time Noise Reduction Using the TMS320C5402 DSP Kit, Journal of Applied Sciences, Vol 7, pp.2750-2754, 2007.
7. Smith, Stevan W., Digital Signal Processing: A Practical Guide for Engineers and Scientisits, Newnes Publishers, 2003.
8. V.Elamaran, Kalagarla Abhiram and Narreddi Bhanu Prakash Reddy, FPGA Implementation of Audio Enhancement using Xilinx System Generator, Journal of Applied Sciences, Vol 14, pp.1972-1977, 2014.
9. V. Elamaran, K. Narasimhan, P.V.M. Vijayabhaskar and G. Shiva, 2013, Comparison of Wavelet Filters in Hybrid Domain Watermarking, Research Journal of Information Technology, Vol 5, pp.393-401, 2013.
10. Vellaiappan Elamaran and Angam Praveen, Comparison of DCT and Wavelets in Image Coding, Proceedings of the International Conference on Computer Communications and Informatics, Coimbatore, India, pp.1-4, 2012.

Corresponding Author:

V Elamaran*,

Email: elamaran@ece.sastra.edu