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NORMAL VAGUE IDEALS OF A Γ-NEAR RING

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Abstract

In this paper, we introduce and studied various properties on the concept of normal left(resp. right) vague ideal of a Γ-Near ring.

Key Words: Vague set, Vague cut, Normal Vague ideal Γ-Near ring.

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1. Introduction

We introduce and study the concept of normal left(resp. right) vague ideal of a Γ-Near ring and we prove that, a non-constant maximal element in the set of all normal left(resp. right) vague ideals of a Γ-Near ring M takes only two vague values [0, 0] and [1, 1] and we show that homomorphic image and inverse homomorphic image of a normal left(resp. right) vague ideal of M is also a normal left(resp. right) vague ideal of M .

2. Preliminaries

Definition 2.1: A Zero-Symmetric Γ-Near ring is a triple (M, +, Γ), where

(1) (M, +) is a group

(2) Γ is a non-empty set of binary operators on M such that for each $\alpha \in \Gamma$,

(M, +, α) is a near ring.

(3) $x\alpha(y\beta z) = (x\alpha y)\beta z$, for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

(4) $x\alpha 0 = 0$ for every $x \in M, \alpha \in \Gamma$.

Definition 2.2 fuzzy subset μ of a Γ-near ring M is called a fuzzy left(resp. right) ideal of M if for all x, y, a, b $\in$ M ; $\alpha \in \Gamma$ (1) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

(2) $\mu(y + x - y) \geq \mu(x)$

(3) $\mu(a\alpha(x + b) - a\alpha b) \geq \mu(x)(\text{resp.}\ \mu(x\alpha a) \geq \mu(x)$

Definition 2.3: A vague set A in the universe of discourse U is a pair (t_A, f_A), where $t_A : U \to [0,1]$, $f_A : U \to [0,1]$ are mappings such that $t_A(u) + f_A(u) \leq 1, \forall u \in U$. The functions t_A and f_A are called true membership function and false membership function respectively.

Definition 2.4: The interval $[t_A(u), 1 - f_A(u)]$ is called the vague value of u in A and it is denoted by $V_A(u)$ i.e., $V_A(u) = [t_A(u), 1 - f_A(u)]$.

Definition 2.5: A vague set A is contained in the other vague set B, $A \subseteq B$ if and only if $V_A(u) \leq V_B(u)$ i.e., $t_A(u) \leq t_B(u)$ and $1 - f_A(u) \leq 1 - f_B(u), \forall u \in U$.

Definition 2.6: Two vague sets A and B are equal written as A = B, if and only if $A \subseteq B$ and $B \subseteq A$ i.e., $V_A(u) \leq V_B(u)$ and $V_B(u) \leq V_A(u), \forall u \in U$.

Definition 2.7: The union of two vague sets A and B with respective truth membership and false membership functions t_A, f_A, t_B, f_B is a vague set C, written as $C = A \cup B$, whose truth membership and false membership functions are related to those of A and B by $t_C = \max\{t_A, t_B\}$ and $1 - f_C = \max\{1 - f_A, 1 - f_B\} = 1 - \min\{f_A, f_B\}$.

Definition 2.8: The intersection of two vague sets A and B with respective truth member- ship and false membership functions t_A, f_A, t_B, f_B is a vague set C, written as $C = A \cap B$, whose truth membership and false membership functions are related to those of A and B by $t_C = \min\{t_A, t_B\}$ and $1 - f_C = \min\{1 - f_A, 1 - f_B\} = 1 - \max\{f_A, f_B\}$.

Definition 2.9: The union and intersection of a family $\{A_i / i \in \Delta\}$ of vague sets of a set U are defined by

$$V_{\bigcup_{i\in\Delta} A_i}(u) = \sup_{i\in\Delta} V_{A_i}(u), \forall u \in U$$

$$V_{\bigcap_{i\in\Delta} A_i}(u) = \inf_{i\in\Delta} V_{A_i}(u), \forall u \in U.$$

Definition 2.10: A vague set A of a set U with $t_A(u) = 0$ and $f_A(u) = 1$, $\forall u \in U$ is called zero vague set of U.

Definition 2.11: A vague set A of a set U with $t_A(u) = 1$ and $f_A(u) = 0$, $\forall u \in U$ is called unit vague set of U.

Definition 2.12: Let A be a vague set of a universe U with true membership function t_A and false membership function f_A. For $\alpha, \beta \in [0,1]$ with $\alpha \leq \beta$, the (α, β)- cut or vague cut of a vague set A is the crisp subset of U is given by

$A_{(\alpha,\beta)} = \{x \in U / V_A(x) \geq [\alpha, \beta]\}$

i.e., $A_{(\alpha,\beta)} = \{x \in U / t_A(x) \geq \alpha$ and $1 - f_A(x) \geq \beta\}$.

Definition 2.13: The α-cut, A_α of the vague set A is the (α, α)-cut of A and hence given by $A_\alpha = \{x \in U / t_A(x) \geq \alpha\}$.

3. Normal Vague Ideals of Γ-Near rings

In this section, we introduce and study the concept of normal vague ideal of a Γ-Near ring and we prove that for a given vague ideal we construct a normal vague ideal which contains the given vague ideal. Also we prove that a non-constant maximal element in the set of all normal left(resp. right) vague ideals of a Γ-Near ring takes only two vague values [0, 0] and [1, 1]. Further we prove that homomorphic image and inverse homomorphic image of a normal vague ideal of M is also a normal vague ideal of M. For a given left(resp. right) vague ideal A of a Γ-Near ring M, $V_A(0)$ is the largest vague value of A.

Now, we introduce the following.

Definition 3.1: A vague set $A = (t_A, f_A)$ of M is said to be normal, if

$V_A(0) = [1,1]$ i.e., $t_A(0) = 1$ and $1 - f_A(0) = 1$.

The following theorem,

Theorem 3.2: Let $A = (t_A, f_A)$ be a vague set of M such that

$t_A(p) + f_A(p) \le t_A(0) + f_A(0),\ \forall p \in M$. Define $A^+ = (t_{A^+}, f_{A^+})$,

where $t_{A^+}(p) = t_A(p) + 1 - t_A(0)$ and $f_{A^+}(p) = f_A(p) - f_A(0),\ \forall p \in M$. Then A^+ is a normal vague set.

Proof : First we show that A^+ is a vague set. Let $p \in M$.

Now, $t_{A^+}(p) + f_{A^+}(p) = t_A(p) + 1 - t_A(0) + f_A(p) - f_A(0) \le 1$.

Thus A^+ is a vague set. Also $t_{A^+}(0) = 1$ and $f_{A^+}(0) = 0$.

Hence A^+ is a normal vague set.

Now, we have the following theorem.

Theorem 3.3: Let $A = (t_A, f_A)$ be a left(resp. right) vague ideal of M. Then the vague set A^+ is a normal left(resp. right) vague ideal of M, containing A.

Proof. : Let $p, q \in M$; $\gamma 1 \in \Gamma$. Now,

1) $V_{A^+}(p - q) = V_A(p - q) + [1,1] - V_A(0)$

$$\ge \min\{V_A(p), V_A(q)\} + [1,1] - V_A(0)$$

$$= \min\{V_A(p) + [1,1] - V_A(0), V_A(q) + [1,1] - V_A(0)\}$$

$$= \min\{V_{A^+}(p), V_{A^+}(q)\}$$

2) $V_{A^+}(q + p - q) = V_A(q + p - q) + [1,1] - V_A(0)$

$$\ge V_A(p) + [1,1] - V_A(0)$$

$$= V_{A^+}(p)$$

3) $V_{A^+}(a\gamma 1(p + b) - a\gamma 1 b) = V_A(a\gamma 1(p + b) - a\gamma 1 b) + [1,1] - V_A(0)$

$$\ge V_A(p) + [1,1] - V_A(0)$$

$$= V_{A^+}(p)$$

Also $V_{A^+}(0) = V_A(0) + [1,1] - V_A(0) = [1,1]$.

Thus A^+ is a normal left(resp. right) vague ideal of M.

Clearly $A \subset A^+$.

Corollary 3.4: If A is a left(resp. right) vague ideal of M satisfying

$V_{A^+}(p) = [0,0]$, for some $p \in M$. Then $V_A(p) = [0,0]$.

Theorem 3.5: A left(resp. right) vague ideal $A = (t_A, f_A)$ of M is normal if and only if $A^+ = A$.

Proof. : Suppose that A is normal left(resp. right) vague ideal of M. Let $p \in M$.

Then $t_{A^+}(p) = t_A(p) + 1 - t_A(0) = t_A(p) + 1 - 1 = t_A(p)$.

$f_{A^+}(p) = f_A(p) - f_A(0) = f_A(p) - 0 = f_A(p)$.

Thus $A^+ = A$.

The converse is obvious.

Theorem 3.6: Let $A = (t_A, f_A)$, $B = (t_B, f_B)$ be two left(resp. right) vague ideals of M. Then

1. $(A^+)^+ = A$

2. $(A \cap B)^+ = A^+ \cap B^+$.

3. $(A \cup B)^+ = A^+ \cup B^+$.

4. $A \subseteq B \Rightarrow A^+ \subseteq B^+$.

Proof. : Let $p \in M$.

1. $t_{(A^+)^+}(p) = t_{A^+}(p) + 1 - t_{A^+}(0) = t_{A^+}(p)$

(since A is left(resp. right) vague ideal $\Rightarrow A^+$ is normal).

$f_{(A^+)^+}(p) = f_{A^+}(p) - f_{A^+}(0) = f_{A^+}(p)$

(since A is left(resp. right) vague ideal $\Rightarrow A^+$ is normal).

Thus $(A^+)^+ = A^+ = A$ (from theorem: 3.5).

2. Now, we prove that $(A \cap B)^+ = A^+ \cap B^+$.

$t_{(A\cap B)^+}(p) = t_{A\cap B}(p) + 1 - t_{A\cap B}(0)$

$= \min\{t_A(p), t_B(p)\} + 1 - \min\{t_A(0), t_B(0)\}$

$= \min\{t_A(p) + 1 - t_A(0), t_B(p) + 1 - t_B(0)\}$

$= \min\{t_{A^+}(p), t_{B^+}(p)\}$

$= t_{A^+ \cap B^+}(p)$

Similarly, we can prove that $f_{(A\cap B)^+}(p) = f_{A^+ \cap B^+}(p)$.

Hence $(A \cap B)^+ = A^+ \cap B^+$.

3. Now, we prove that $(A \cup B)^+ = A^+ \cup B^+$.

$t_{(A\cup B)^+}(p) = t_{A\cup B}(p) + 1 - t_{A\cup B}(0)$

$= \max\{t_A(p), t_B(p)\} + 1 - \max\{t_A(0), t_B(0)\}$

$= \max\{t_A(p) + 1 - t_A(0), t_B(p) + 1 - t_B(0)\}$

$= \max\{t_{A^+}(p), t_{B^+}(p)\}$

$= t_{A^+ \cup B^+}(p)$

Similarly, we can prove that $f_{(A\cup B)^+}(p) = f_{A^+ \cup B^+}(p)$.

Hence $(A \cup B)^+ = A^+ \cup B^+$.

4. $t_{A^+}(p) = t_A(p) + 1 - t_A(0) \le t_B(p) + 1 - t_B(0) = t_{B^+}(p)$

$f_{A^+}(p) = f_A(p) - f_A(0) \le f_B(p) - f_B(0) = f_{B^+}(p)$

Hence $A^+ \subseteq B^+$.

Theorem 3.7: Let $A = (t_A, f_A)$ be a left(resp. right) vague ideal of M. If there exists a left(resp. right) vague ideal B of M satisfying $B^+ \subseteq A$, then A is normal.

Proof. : Assume that there exists a left(resp. right) vague ideal B of M satisfying $B^+ \subseteq A$. So, $[1, 1] = V_{B^+}(0) \le V_A(0)$.

We get $V_A(0) = [1, 1]$.

Thus A is normal.

Immediately we have the corollary.

Corollary 3.8: Let A be a left(resp. right) vague ideal of M. If there exists a left(resp. right) vague ideal B of M satisfying $B^{+} \subseteq A$, then $A^{+} = A$.

Proof. : By theorem: 3.7, A is normal and hence $A^{+} = A$.

Let N(M) denotes the set of all normal left(resp. right) vague ideals of M. Then it can be observe that the set N(M) is a poset under set inclusion.

Theorem 3.9: Let $A \in N(M)$ be a non-constant maximal element of

$(N(M), \subseteq)$. Then

A takes only two vague values [0, 0] and [1, 1].

Proof. : Let $A = (t_A, f_A)$ be a normal left(resp. right) vague ideal of M.

Then $V_A(0) = [1,1]$.

Let $p \in M$.

Suppose that $V_A(p) = [1,1]$.

We have to show that $V_A(p) = [0,0]$.

Assume that there exists $p_0 \in M$ such that $[0,0] < V_A(p_0) < [1,1]$. Define a

Vague Set B= (t_B, f_B) on M by

$$V_B(p) = \frac{V_A(p) + V_A(p_0)}{2}$$

for every $p \in M$.i.e,

$$t_B(p) = \frac{t_A(p) + t_A(p_0)}{2} \text{ and } f_B(p) = \frac{f_A(p) + f_A(p_0)}{2}$$

1) $V_B(p-q) = \frac{V_A(p-q)+V_A(p_0)}{2} \geq \frac{\min\{V_A(p),V_A(q)\}+V_A(p_0)}{2}$

$$= min\left\{\frac{V_A(p) + V_A(p_0)}{2}, \frac{V_A(q) + V_A(p_0)}{2}\right\}$$

$$= \min\{V_B(p), V_B(q)\}$$

2)$V_B(q+p-q)=\frac{V_A(q+p-q)+V_A(p_0)}{2} \geq \frac{V_A(p)+V_A(p_0)}{2}=V_B(p)$

3)

$V_B(a\ \gamma_1(p+b)-\ a\ \ \gamma_1\ \ b)\ =\frac{V_A(a\ \gamma_1(p+b)-a\ \ \gamma_1\ \ b)+V_A(p_0)}{2} \geq \frac{V_A(p)+V_A(p_0)}{2}=V_B(p)$

Thus B is left (resp. right) vague ideal of M.

Now$V_{B^+}(p)=V_B(p)+[1,1]-V_B(0)$

$=\frac{V_A(p)+V_A(p_0)}{2}+[1,1]-\frac{V_A(0)+V_A(p_0)}{2}$

$=\frac{V_A(p)+[1,1]}{2}$,

That implies $V_{B^+}(0)=\frac{V_A(0)+[1,1]}{2}$=[1,1].

Thus B^+ is a normal left (resp. right) vague ideal of M.

Now, $V_{B^+}(0)=[1,1]>V_A(p_0)$.

So, B^+ is a non-constant normal left(resp. right) vague ideal of M and hence

$B \in N(M)$.

Further, we have $V_{B^+}(p0)>V_A(p_0)$, it gives contradiction for A is maximal. Hence $V_A(p)=[0,0]$.

Thus A takes only two vague values [0, 0] and [1, 1].

Definition 3.10: A normal left(resp. right) vague ideal A of M is said to be complete normal if there exists $p \in M$ such that $V_A(p)=[0,0]$.

Let $C(M)$ denotes the set of all complete normal left(resp. right) vague ideals of M.

Clearly $C(M) \subseteq N(M)$ and that $(C(M),\subseteq)$ is a poset.

Theorem 3.11: Any non-constant maximal element of $(N(M),\subseteq)$ is also a maximal element of $(C(M),\subseteq)$.

Proof : Let A be a non-constant maximal element of $(N(M),\subseteq)$. Then A takes only two vague values [0, 0] and [1, 1].

i.e., $V_A(0)=[1,1]$ and $V_A(p)=[0,0]$, for some $p \in M$. That implies $A \in C(M)$.

Suppose $B \in C(M)$ such that $A \subseteq B$. So, $B \in N(M)$.

Since A is maximal in N(M) and $B \in N(M)$ with $A \subseteq B$, that gives $A = B$.

Hence A is maximal element in C(M).

Theorem 3.12: Let M_1 be a Γ_1-Near ring and M_2 be a Γ_2-Near ring and let f be a homomorphism of M_1 onto M_2. If B is a normal left (resp. right) vague ideal of M_2, then the inverse image of B, $f^{-1}(B)$ is a normal left(resp. right) vague ideal of M_1.

Proof.: From theorem: 3.12, $f^{-1}(B)$ is a left(resp. right) vague ideal on M_1.

Since B is normal, we have $V(0^1)= [1,1]^B$, where 0^1 is the zero element in M_2. Now, $V_{f^{-1}(B)}(0)= V_B(f(0))=V_B(0^1)=[1,1]$.

Hence $f^{-1}(B)$ is a normal left ideal (resp.right) vague ideal on M_1.

Theorem 3.13: Let M_1 be a Γ_1-Near ring and M_2 be a Γ_2-Near ring and let f be a homomorphism of M_1 onto M_2. If A is a normal left (resp. right) vague ideal of M_1 with Sup. Property, then the homomorphic image of A, f(A) is a normal left(resp. right) vague ideal of M_2.

Proof. From theorem: 3.13, f(A) is a left(resp. right) vague ideal of M_2.Since A is normal, $V_A(0) = [1,1]$.

Since f is epimorphism, there exists $0^l \in M_2$ such that $f(0)=0^l$.

Now $V_{f(A)}(0^1) = \sup_{r \in f^{-1}(0')} V_A(r) = \sup_{r \in f^{-1}(0')} V_A(0) = V_A(0) = [1,1]$.

Hence f(A) is a normal left(resp. right) vague ideal of M2.

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References:

1. H.Khan, M.Ahmad and Ranjit Biswas, 2007, "On Vague Groups", International journal of Comput Ational Cognition, Vol.5, No.1, 27-30.
2. John N Mordeson and D.S.Malik, "Fuzzy Commutative Algebra", World Scientific Publishing Co. Pte. Ltd.
3. K.T.AtAnassov, 1986, "Intuitionistic Fuzzy Sets", Fuzzy Sets and Systems, 20, 87-96.

4. L.A.Zadeh, 1965, "Fuzzy sets", Information and Control 8, 338-353.

5. M.K.Rao, 1995, "Γ-semiring 1", Southeast Asian Bulletin of Maths, 19, 49-54.

6. Nobusawa. N, "On generalization of the ring theory ", Osaka J. Math. 1, 185-190.

7. N.Ramakrishna, "On a product Vague Groups", International journal of Computational Cognition(communicated).

8. N.Ramakrishna, 2008, "Vague Normal Groups", International journal of Computational Cognition, Vol. 6, No. 2, 10-12.

9. Rajesh Kumar, "Fuzzy Algebra", University Press, University of Delhi, Delhi-110007.

10. Ranjit Biswas, June(2006), "Vague Groups", International journal of Computational Cognition, Vol. 4, No. 2, 20-23.

11. T.Eswarlal, Sepember(2008) "Vague Ideals and Normal Vague Ideals insemirings", International journal of Computational Cognition, Vol. 6, No. 3, 60-65.

12. W.L. Gau and D.J. Buehrer, 1993, "Vague Sets", IEEE Transactions on systems, man and cybernetics, Vol. 23, No.2, 610-613.

13. Y.B.Jun and CH.Park, 2007, "Vague Ideal in Substraction Algebra", International Mathematical Forum, 59(2), 2919-2926.

14. Y.Bhargavi and T.Eswarlal, "Fuzzy Γ-semirings", accepted for publication in International Journal of Pure and Applied Mathematics. gives the necessity condition for a vague set to be normal vague set.

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