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A STUDY ON FUNDAMENTALS OF Γ- SOFT SET RELATIONS

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Abstract

Soft set theory proposed by Molodtsov, it has been regarded as an effective Mathematical tool to deal with uncertainties. In this paper we studied the Γ- soft set relations, Γ– soft sub set relation, inverse Γ- soft set relation, different types of relations on Γ- soft set, Γ- soft set functions and some fundamental results are carried out.

Keywords: Γ- soft set relation**,** Γ– soft sub set relation, inverse Γ- soft set relation, Γ- soft set function

1. Introduction

The soft theory was first initiated by Molodtsov [1] as one of the important Mathematical tool to solve the problems with uncertainties. The modern set theory formulated byGeorge Contor is formulated the whole Mathematics. Soft set theory does not require the specialization of a parameter, instead it accommodates approximate descriptions of an object as its starting point. Molodtsov successfully applied the soft set theory into several directions, such as smoothness of functions, Riemann integration, theory of probability and on. Atharkharal and B. Ahmad [2] studied the notation of mapping on classes of fuzzy soft sets and properties of soft images and fuzzy sot inverse images of fuzzy soft sets. K.V. Babitha and J.J. Sunil [3] gave the theoretical aspects of soft sets by extending the notations of equivalence relations, composition of relations, and partition of functions of soft sets. NadalTahat et al. [7] introduced the concept of ordering on soft set relation and vague soft set. P.K. dass and R. Borgohain [5] applied fuzzy soft set in multi-observer, multi-criteria and decision making problems. Irfan Deli et al. [6] studied about relations on neutroscopic soft sets which allows to compose two neutroscopic soft sets and finally a decision making method on neutroscopic soft sets is presented. Pinaki Majumdar and S.K. Samanta[4] defined a generalized fuzzy sot sets in decision making problem and medical diagnosis problems. This paper is an attempt to study the theoretical aspects of Γ- soft relations and Γ- soft functions in which we defined different types of relations and functions on it with examples and also prove some theorems.

2. Preliminaries

2.1 Ternary semi ring:

A non-empty set S together with a binary operation called addition and ternary multiplication denoted by [.] is said to be a Ternary semi ring if S is an additive commutative semigroup satisfying the following conditions.

(i) [[a b c] d e] = [a [b c d] e] = [a b [c d e]]

(ii) [[a+b] c d] = [a c d] + [b c d]

(iii) [a [b + c] d] = [a b d] + [a c d]

(iv) [a b [c+ d]] = [a b c] + [a b d] , $\forall\, a, b, c, d \in S$.

2.2 Soft set:

Let U be an intial Universal set and E be set of parameters. Suppose that P(U) denotes the power set of U and A be a non- empty sub set of E. A pair (F,A) is called a soft set over U, where, $F : A \to P(U)$ is mapping.

2.3 Γ- Soft set:

Let U be the Universal set and P(U) be the power set of U. Let K and Γ be the sets of parameters attributes. The triode (F, A,Γ) is called a Γ- Soft set over the Universal set, U is (F, L,Γ) = $\{ F(a, \gamma) : a \in L, \gamma \in \Gamma\}$ where F is a mapping given by F : A X Γ → P(U) and L is the sub set of K.

2.4 Union of two Γ – soft sets:

Let (F, A,Γ) and (G, B,Γ) are two Γ- soft sets over a common Universal set, U then the union of these two Γ- soft sets is denoted by (F, A,Γ) ∪ (G, B,Γ) and is defined by

(F, A,Γ) ∪ (G, B,Γ) = (N, (A X Γ) ∪ (B X Γ)), where

N (e, γ) = {F (e, γ) if (e, γ) ∈ (A X Γ) - (B X Γ),

= { G (e, γ) if (e, γ) ∈ (B X Γ) - (A X Γ),

= { F (e, γ) ∪ G (e, γ) if (e, γ) ∈ (A X Γ)∩(B X Γ)

2.5 Intersection of two Γ – soft sets:

Let (F, A,Γ) and (G, B,Γ) are two Γ- soft sets over a common Universal set, U then the union of these two Γ- soft sets is denoted by (F, A,Γ) ∩ (G, B,Γ) and is defined by

(F, A,Γ) ∩ (G, B,Γ) = (N, (A X Γ)∩ (B X Γ)), where N (e, γ) = F (e, γ) or G (e, γ) for every (e, γ) ∈ (A X Γ)∩ (B X Γ).

3. Γ– soft set Relations

3.1 Γ– soft set Relation: Let (F, A,Γ) and (G, B,Γ) are two Γ- soft sets over a common Universal set, U. The relation between (F, A,Γ) to (G, B,Γ) is called Γ- soft set relation, (R, N,Γ) if R is a Γ- soft subset of (F, A,Γ) X (G, B,Γ), where (C,Γ) $\subseteq$(A,Γ) X (B,Γ) and R ((l, γ) , (m, γ)) = H ((l, γ), (m, γ))

Where, (H, (F, A) X (G, B)) = (F, A,Γ) X (G, B,Γ) .

3.2 Γ– Softsub set Relation:

A Γ- soft relation, R on (F, A,Γ) is a soft sub set of (F, A,Γ) X (F, B,Γ).

i.e., if (F, A,Γ) = { F(a, γ), F(b, γ),F(c, γ),…. } then F(a, γ) R F(b, γ) ↔ F(a, γ) X F(b, γ) ∈ R .

3.3 Domain :

The domain of a Γ- soft relation , R is the Γ- soft set (D, A_1,Γ) $\subseteq$(F, A,Γ), where $A_1 \subseteq$A and

A_1 = {(a, γ)∈ A X Γ…. ; H((a, γ), (b, γ)) ∈ R, for some (b, γ) ∈ B X Γ} and D(a_1, γ) = F(a_1, γ) ∀ (a_1, γ) ∈ A_1 X Γ.

3.4 Range:

The range of a Γ- soft relation , R is the Γ- soft set (E, B_1,Γ) $\subseteq$(G, B,Γ), where $B_1 \subseteq$B and B_1 = {(b, γ) ∈ B X Γ…. ; H((a, γ), (b, γ)) ∈ R, for some (a, γ) ∈ A X Γ } and E(b_1, γ) = G(b_1, γ) ∀ (b_1, γ) ∈ B X Γ.

3.5 Inverse:

The inverse of Γ- soft relation , R is denoted by R^{-1} is a Γ- soft relation from (G, B,Γ) to (F, A,Γ) defined by R^{-1} = { G(b, γ) X F (a, γ) : F (a, γ) R G (b, γ) }.

1. Examples:

Let U denotes set of five (5) Television sets given by U = {T_1, T_2, T_3, T_4, T_5 } and Γ is the parameter set which denotes the brand of television given by Γ = {brand – 1 , brand – 2 }. Let A and B are the parameter sets given by A = {a_1, a_2, a_3, a_4 } in which a_1 denotes' smart', a_2 denotes 'LCD', a_3 denotes 'LED' and a_4 denotes 'Plasma' and B = {b_1, b_2, b_3,b_4 } in which b_1 denotes 'curved' , b_2 denotes ' wega', b_3 denotes 'normal' and b_4 denotes 'trinitron'.

Let the Γ- soft set (F,A, Γ) is given by (F,A, Γ) = { F(a_1, γ_1) = {T_1, T_2}, F(a_1, γ_2) = {T_1, T_3T_4}, F(a_2, γ_1) = {T_4, T_5}, F(a_2, γ_2) = {T_2, T_4}, F(a_3, γ_1) = {T_1, T_5}, F(a_3, γ_2) = {T_1, T_4}} describes the televisions with different types of different brands.

Let $(G,A,\Gamma) = \{G(b_1,\gamma_1) = \{T_1, T_2, T_5\}, G(b_1,\gamma_2) = \{T_3, T_4, T_5\}, G(b_2,\gamma_1) = \{T_1, T_2\} G(b_2,\gamma_2) = \{T_1, T_2, T_5\}\}$ describes the televisions with different styles of different brands.

If we define the relation R from (F,A,Γ) to (FG,B,Γ) as follows

$F(a,\gamma)\ R\ G(b,\gamma) \leftrightarrow F(a,\gamma) \subseteq G(b,\gamma)$ then $R = \{F(a_1,\gamma_1) \times G(b_1,\gamma_1), F(a_2,\gamma_1) \times G(b_1,\gamma_2), F(a_2,\gamma_1) \times G(b_1,\gamma_2), F(a_3,\gamma_1) \times G(b_2,\gamma_2)\}$

(i) **Domain of R** $= (D,A_1,\Gamma) = \{(a_1,\gamma_1), (a_2,\gamma_1), (a_3,\gamma_1)\} \subseteq A \times \Gamma, \rightarrow (D,A_1,\Gamma) = (F,A,\Gamma)$.

(ii) **Range of R**$= (E,B_1,\Gamma) = \{(b_1,\gamma_1), (b_1,\gamma_2), (b_2,\gamma_2)\} \subseteq B \times \Gamma$ and $E(b,\gamma) = G(b,\gamma)$

Inverse Relation, $R^{-1} = \{G(b_1,\gamma_1) \times F(a_1,\gamma_1), G(b_1,\gamma_2) \times F(a_2,\gamma_1), G(b_2,\gamma_2) \times F(a_3,\gamma_3)\}$

Let R and Q be two Γ- soft set relations on a soft set (F,A,Γ), then

(a) $R \subseteq Q$ if $\forall (a,\gamma), (b,\gamma) \in A \times \Gamma, F(a,\gamma) \times F(b,\gamma) \in R \rightarrow F(a,\gamma) \times F(b,\gamma) \in Q$.

(b) The **complement** of R denoted as R^c and is defined by $R^c = \{F(a,\gamma) \times F(b,\gamma) : F(a,\gamma) \times F(b,\gamma)\}$

(c) The **union** of R and Q denoted by $R \cup Q$ and is defined by $R \cup Q = \{F(a,\gamma) \times F(b,\gamma) : F(a,\gamma) \times F(b,\gamma) \in R$ or $Q\}$

(d) The **intersection** of R and Q denoted by $R \cap Q$ and is defined by $R \cap Q = \{F(a,\gamma) \times F(b,\gamma) : F(a,\gamma) \times F(b,\gamma) \in R$ and $Q\}$

2. Example:

Let (F, A,Γ) is a Γ -soft set over a universal set, $U = \{u_1, u_2, u_3, u_4\}$, $A = \{a_1, a_2\}$, $\Gamma = \{\gamma_1, \gamma_2\}$. The Γ -soft set (F, A,Γ) is defined as $(F, A,\Gamma) = \{F(a_1,\gamma_1) = \{u_1, u_2, u_3\}; F(a_1,\gamma_2) = \{u_3, u_4\}; F(a_2,\gamma_1) = \{u_1, u_3, u_4\}; F(a_2,\gamma_2) = \{u_1, u_2, u_4\}\}$.

Let R be Γ –soft relation on (F, A,Γ) defined by $R = \{F(a_1,\gamma_2) \times F(a_2,\gamma_1); F(a_2,\gamma_1) \times F(a_2,\gamma_1), F(a_2,\gamma_2) \times F(a_2,\gamma_1)\}$, and Q be Γ–soft relation on (F, A,Γ)defined by$Q = \{F(a_1,\gamma_2) \times F(a_1,\gamma_1); F(a_2,\gamma_2) \times F(a_2,\gamma_1)\}$then

Complement of R, $R^c = \{F(a_1,\gamma_1) \times F(a_1,\gamma_1); F(a_1,\gamma_2) \times F(a_1,\gamma_2), F(a_2,\gamma_2) \times F(a_2,\gamma_2)\}$.

The union of R and Q is given by $R \cup Q = \{F(a_1,\gamma_2) \times F(a_2,\gamma_1); F(a_2,\gamma_1) \times F(a_2,\gamma_1), F(a_2,\gamma_2) \times F(a_2,\gamma_1), F(a_1,\gamma_2) \times F(a_1,\gamma_1); F(a_2,\gamma_2) \times F(a_2,\gamma_1)\}$

The intersection of R and Q is defined by $R \cap Q = \{F(a_2,\gamma_2) \times F(a_2,\gamma_1)\}$.

3.6 Types of Relations

Let R be a Γ- soft set relation on a soft set (F,A,Γ), then

(i) R is said to be **Reflexive** if F(a, γ) X F(a, γ) $\in$ R $\forall$ (a, γ) $\in$ A XΓ, where a $\in$ A and $\gamma \in \Gamma$.

(ii) R is said to be **Symmetric**if F(a, γ) X F(b, γ) $\in$ R $\rightarrow$ F(b, γ) X F(a, γ) $\in$ R $\forall$(a, γ), (b, γ) $\in$ A XΓ, where a, b $\in$ A and $\gamma \in \Gamma$.

(iii) R is said to be **Transitive** if F(a, γ) X F(b, γ) $\in$ R and F(b, γ) X F(c, γ) $\in$ R $\rightarrow$ F(a, γ) X F(c, γ) $\in$ R $\forall$ (a, γ), (b, γ), (a, γ)$\in$ A XΓ, where a, b, c$\in$ A and $\gamma \in \Gamma$.

(iv) R is said to be **Void** if no element of (F,A, Γ) is relate to any element of A XΓ.

Hence R = φ X φ, which is a sub set of A XΓ.

3. Example

Let (F, A,Γ) is a Γ -soft set over a universal set, U = { u_1, u_2, u_3}, A = { a_1, a_2}, Γ = { γ_1, γ_2}. The Γ -soft set (F, A,Γ) is defined as (F, A,Γ) = {F(a_1, γ_1), F(a_2, γ_2), F(a_3, γ_3)}.

Let R be a Γ –soft relation on (F, A,Γ) is defined byR = {F(a_1, γ_1) x F(a_2, γ_2)= F(a_3, γ_3), F(a_1, γ_1) x F(a_1, γ_1)= F(a_4, γ_2), F(a_2, γ_2) x F(a_1, γ_2)= F(a_5, γ_1)}, then R is called a Void Γ –soft relation.

4. Γ– soft set Function:

Γ– soft set Function

Let (F, A,Γ) and (G, B,Γ) be two non-empty Γ-soft sets over a universal set, U then the Γ-soft relation, f from (F, A, Γ) to (G, B, Γ) written as (F, A,Γ) $\rightarrow$ (G, B,Γ) is called a Γ-soft function if every element in the domain of f has a unique element in the range of f.

i.e., f (F(a,γ)) = G(b, γ).

5. Example

Let A = { a_1, a_2, a_3}, B = A = { b_1, b_2, b_3} and Γ = { γ_1, γ_2}. Consider the Γ-soft function f from (F, A, Γ) to (G, B, Γ) is given by

f = {F(a_1, γ_1) x G(b_2, γ_1), F(a_2, γ_1) x G(b_1, γ_1), F(a_2, γ_2) x G(b_2, γ_1), F(a_3, γ_2) x G(b_3, γ_2) } is a Γ-soft function.

If f = {F(a_1, γ_1) x G(b_2, γ_1), F(a_1, γ_1) x G(b_1, γ_1), F(a_2, γ_1) x G(b_2, γ_2), F(a_3, γ_2) x G(b_3, γ_2) } then f is not a Γ-soft function.

Let fbe a Γ –soft set function from (F, A, Γ) to (G, B, Γ). The function f is said to be **Injective**provided that for all (a_1, γ), (a_2, γ) in (F, A, Γ), whenever F ((a_1, γ)) = F ((a_2, γ)), then (a_1, γ) = (a_2, γ).

Equivalently, if (a_1, γ) ≠ (a_2, γ) → F ((a_1, γ)) ≠ F ((a_2, γ)).

6. Example

Let A = { a_1, a_2}, B = A = { b_1, b_2} and Γ = { γ_1, γ_2}. Consider the Γ-soft function f from (F, A, Γ) to (G, B, Γ) is given by f = {F(a_1, γ_1) x G(b_1, γ_1), F(a_2, γ_2) x G(b_2, γ_2), F(a_2, γ_1) x G(b_2, γ_2), F(a_1, γ_1) x G(b_2, γ_1) }. The function f is Injective.

A Γ –soft set function, f :(F, A, Γ) → (G, B, Γ) is said to be **Surjective** if for every G(b, γ) in (G, B, Γ) there is an F(a, γ) in (F, A, Γ) such that f (F(a, γ)) = G(b, γ).

A Γ –soft set function, f :(F, A, Γ) → (G, B, Γ) is said to be **Bijective** if it is both Injective and Surjective.

4.1 Theorem.

Let f : (F, A, Γ) → (G, B, Γ), be a Γ –soft set function and (F_1, A_1, Γ), (F_2, A_2, Γ) be the Γ –soft sets of (F, A, Γ), then

(i) (F, A_1, Γ)⊆(F, A_2, Γ) → f (F, A_1, Γ)⊆f (F, A_2, Γ)

(ii)f[(F, A_1, Γ)∪(F, A_2, Γ)] → f (F, A_1, Γ)∪f (F, A_2, Γ)

(iii)f[(F, A_1, Γ)∩(F, A_2, Γ)] → f (F, A_1, Γ)∩ f (F, A_2, Γ) may be equal if f is injective.

Proof: we have, , f : (F, A, Γ) → (G, B, Γ) is defined by f (F(a, γ)) = G(b, γ) for some F (a, γ) ∈(F, A, Γ) , G (b, γ) ∈ (G, B, Γ).

(i) suppose that f (F_1, A_1, Γ)⊆f (F_2, A_2, Γ), where (F_1, A_1, Γ), (F_2, A_2, Γ) be two Γ –soft sets of (F, A, Γ).

Let G (b, γ) ∈(F_1, A_1, Γ) → G (b, γ) = f (F(a, γ)) by definition of f for some F (a, γ) ∈(F, A_1, Γ).

= f (F(a, γ)) by definition of f for some

F (a, γ) ∈(F, A_2, Γ). (since(F, A_1, Γ)⊆ (F, A_2, Γ)

∴ G (b, γ) ∈ f (F_2, A_2, Γ).

∴f (F, A_1, Γ)⊆f (F, A_2, Γ).

(ii) let G (b, γ) ∈ f[(F, A_1, Γ)∪(F, A_2, Γ)]

→ G (b, γ) = f (F(a, γ)) by definition of f, for some F (a, γ) ∈(F, A_1, Γ) or F (a, γ) ∈(F, A_2, Γ).

→ G (b, γ) ∈(F, A_1, Γ) or G (b, γ) ∈(F, A_2, Γ).

∴ G (b, γ) ∈ f (F, A_1, Γ)∪f (F, A_2, Γ)

Hence f[(F, A_1, Γ)∪(F, A_2, Γ)] → f (F, A_1, Γ)∪f (F, A_2, Γ).

We know that $(F, A_1, \Gamma) \subseteq (F, A_1, \Gamma) \cup (F, A_2, \Gamma)$ and $(F, A_2, \Gamma) \subseteq (F, A_1, \Gamma) \cup (F, A_2, \Gamma)$.

$\rightarrow f(F, A_1, \Gamma) \subseteq f[(F, A_1, \Gamma) \cup (F, A_2, \Gamma)]$

$f(F, A_2, \Gamma) \subseteq f[(F, A_1, \Gamma) \cup (F, A_2, \Gamma)]$

$\therefore f[(F, A_1, \Gamma) \cup (F, A_2, \Gamma)] \subseteq f(F, A_1, \Gamma) \cup f(F, A_2, \Gamma)$.

From the above we conclude that $f[(F, A_1, \Gamma) \cup (F, A_2, \Gamma)] \rightarrow f(F, A_1, \Gamma) \cup f(F, A_2, \Gamma)$.

(iii) let $G(b, \gamma) \in f[(F, A_1, \Gamma) \cap (F, A_2, \Gamma)] \rightarrow G(b, \gamma) = f(F(a, \gamma))$, for some $F(a, \gamma) \in (F, A_1, \Gamma)$ and $F(a, \gamma) \in (F, A_2, \Gamma)$.

$\rightarrow G(b, \gamma) \in (F, A_1, \Gamma)$ and $G(b, \gamma) \in (F, A_2, \Gamma)$.

$\rightarrow G(b, \gamma) \in f(F, A_1, \Gamma) \cap f(F, A_2, \Gamma)$

$\therefore f[(F, A_1, \Gamma) \cap (F, A_2, \Gamma)] \subseteq f(F, A_1, \Gamma) \cap f(F, A_2, \Gamma)$.

Suppose that $G(b, \gamma) \in (F, A_1, \Gamma)$ and $G(b, \gamma) \in (F, A_2, \Gamma)$

$\rightarrow G(b, \gamma) \in f(F, A_1, \Gamma) \cap f(F, A_2, \Gamma)$

$\rightarrow G(b, \gamma) = f(F(a_1, \gamma))$, for some $(F(a_1, \gamma) \in (F, A_1, \Gamma)$ and

$G(b, \gamma) = f(F(a_2, \gamma))$, for some $(F(a_2, \gamma) \in (F, A_2, \Gamma)$

$\therefore f(F(a_1, \gamma)) = f(F(a_2, \gamma)) = G(b, \gamma)$

$\rightarrow (F(a_1, \gamma)) = (F(a_2, \gamma))$ when f is injective

$\rightarrow G(b, \gamma) \in f(F, A_1, \Gamma) \cap f(F, A_2, \Gamma)$

$\therefore f[(F, A_1, \Gamma) \cap (F, A_2, \Gamma)] \rightarrow f(F, A_1, \Gamma) \cap f(F, A_2, \Gamma)$.

If $f : (F, A, \Gamma) \rightarrow (G, B, \Gamma)$ be an Injective function, then the inverse of f is defined by f^{-1} which is called the inverse of f.

4.2 Theorem

If $f : (F, A, \Gamma) \rightarrow (G, B, \Gamma)$ is Bijective then $f^{-1} : (G, B, \Gamma) \rightarrow (F, A, \Gamma)$ is also Bijective, where (F, A, Γ), (G, B, Γ) be two Γ –soft sets defined over a common universal set.

Proof: let $G(b_1, \gamma) \neq G(b_2, \gamma)$, for some $(b_1, \gamma), (b_2, \gamma) \in (G, B, \Gamma)$ and

$f^{-1}(G(b_1, \gamma)) = F(a_1, \gamma)$ and $f^{-1}(G(b_2, \gamma)) = F(a_2, \gamma)$, for some $F(a_1, \gamma), F(a_2, \gamma)$ are in (F, A, Γ).

$\therefore f(F(a_1, \gamma)) = G(b_1, \gamma)$

$f(F(a_2, \gamma)) = G(b_2, \gamma)$.

$\rightarrow$ f (F(a_1, γ)) ≠ f (F(a_2, γ)), (since G(b_1, γ) ≠ G(b_2, γ))

$\rightarrow$ F(a_1, γ) ≠ F(a_2, γ), (since f is one-one)

∴ f^{-1} (G(b_1, γ)) ≠ f^{-1} (G(b_2, γ)).

Hence G(b_1, γ)) ≠ G(b_2, γ) $\rightarrow$ f^{-1} (G(b_1, γ)) ≠ f^{-1} (G(b_2, γ))

∴ f^{-1} is one-one.

Let F(a_1, γ)∈(F, A_1, Γ)

Since f is one-one then there exists a unique element G(b, γ) in (G, B, Γ) such that f (F(a, γ)) = G(b., γ) $\rightarrow$ f^{-1} (G(b, γ)) ≠ F(a, γ), for some F(a, γ) ∈(F, A, Γ).

∴for every element F(a, γ) in (F, A, Γ) there exists an element G(b., γ) in (G, B, Γ) such that

F(a, γ) = f^{-1} (G(b, γ)).

∴$f^{-1}$is onto $\rightarrow$ f^{-1} is both one-one and onto.

Hence $f^{-1}$is Bijective.

4.3 Theorem

Let f :(F, A, Γ) → (G, B, Γ), g : (G, B, Γ) → (H, C, Γ) be two bijective Γ –soft functions. Then gof :(F, A, Γ) → (H, C, Γ) is also Bijective and $(gof)^{-1}$= $f^{-1}og^{-1}$.

Proof: let F(a_1, γ) , F(a_2, γ) ∈(F, A, Γ), for some a_1, a_2∈ A, γ ∈Γ.

Suppose that F(a_1, γ) ≠ F(a_2, γ)

Since f is one-one then f (F(a_1, γ)) ≠ f (F(a_2, γ)).

Again, since g is one-one then g (f (F(a_1, γ))) ≠ g (f (F(a_2, γ)))

$\rightarrow$ g of (F(a_1, γ)) ≠ gof (F(a_2, γ))

∴ F(a_1, γ) ≠ F(a_2, γ) $\rightarrow$ gof (F(a_1, γ)) ≠ gof (F(a_2, γ))

Hence gof is one-one.

Let H(c, γ) ∈(H, C, Γ), since g is onto then there exists an element G(b, γ) in (G, B, Γ) such that g (G(b, γ)) = H(c, γ).

Again f is onto then for every G(b, γ) in (G, B, Γ) there exists an element F(a, γ) in (F, A, Γ) such that g (f (F(a_1, γ))) = H(c, γ), for every element H(c, γ) ∈(H, C, Γ).

$\rightarrow$ g of (F(a, γ)) = H(c, γ).

∴ for every element H(c, γ) in (H, C, Γ) there exists an element F(a, γ) in (F, A, Γ) such that g of (F(a, γ)) = H(c, γ).

∴g of is onto → g of is one-one and onto.

Hence gof is Bijective.

Since f, g and gof are Bijective then they are invertible.

Also we know that for any relations R and S we have $(SoR)^{-1} = R^{-1}$ o s^{-1}.

Since f and g are functions then we can verify $(gof)^{-1} = f^{-1}$ o g^{-1}.

4.4 Theorem

Let R beΓ –soft set relation from (F, A, Γ) to (G, B, Γ)and S beΓ –soft set relation from (G, B, Γ) to (H, C, Γ), then $(SoR)^{-1}= R^{-1}$ o S^{-1}

Proof: we have $(SoR)^{-1}$ is a Γ –soft set relation.

Let H(c, γ) ∈(H, C, Γ), F(a, γ) ∈(F, A, Γ)

Since R :(F, A, Γ) → (G, B, Γ), and S : (G, B, Γ) → (H, C, Γ) be Γ –soft set relations

∴SoR :(F, A, Γ) → (H, C, Γ) be a relation i.e., F(a, γ) (SoR) H(c, γ)

→ $(SoR)^{-1}$:(H, C, Γ) → (F, A, Γ) is also a relation, i.e., H(c, γ) $(SoR)^{-1}$F(a, γ).

From the above definitions this relations holds.

This is equivalent to G(b, γ) R^{-1} F(a, γ) and H(c, γ) S^{-1}G(b, γ).

Then H(c, γ) $(SoR)^{-1}$F(a, γ).

Hence $(SoR)^{-1} = R^{-1}$ o S^{-1}.

5. Conclusions

In this paper we studied the fundamental concepts on Γ- soft relations, Γ- soft functions, complement on Γ- soft relation and composition of mappingson Γ- soft functions with examples and also we proved some fundamental theoremson Γ- soft relations and functions. Further our work may support to study the Γ- soft graphs and its applications.

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