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2PRIMAL Γ-SEMIGROUPS

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Abstract: In this paper we introduce the notion of 2 primal Γ- semigroup. We obtain some chacterzationof 2 primal Γ- semigroup. We show that S be an SN Γ- semigroup. If S satisfies (S I) then S is a 2 primal Γ- semigroup.

Keywords: 2 primal Γ- semigroup, SN Γ- semigroup, intersection of factor property (IFP).

1. Introduction: In 1964, Nobusawa.N [1] introduced the notion of a Γ- ring, more general thana ring. In 1966, Barnes. E.W [2] weakend slightly the conditions in the definition of Γ- ring in the sense of Nobusawa. Many fundamentals results in ring theory have been extended to Γ- rings by different authors obtaining various authors generalization analogous to corresponding parts in ring theory. In 1981, Sen.M.K [3]and later in 1986, Sen and Saha [4,5,6] introduced the concept of the Γ- semigroup as a generalization of semigroup and ternary semigroup. Many classical notions of semigroup have extended to Γ- semigroups[7,8,9,10,11].

In this paper we introduced and study 2 primal Γ- semigroups extended and generalizing the results obtained for semigroups.

Definition 2.1:A Γ- semigroup*S* is said to be a 2 primal Γ- semigroup if and only if P (S) = N (S), where P (S) denotes the intersection of all prime ideals of the Γ – semigroup *S*.i.e. the prime radical of *S* and N (S) denotes the set of all nilpotent elements of *S*.

Definition 2.2: An ideal of a Γ- semigroup is said to be nil ideal if each of its elements is nilpotent.

Definition 2.3: A Γ - semigroup*S* is said to be *NI* if $N_r(S)$ = N (S), whereN r(S) denotes the nilradical (i.e. the sum all nil ideals) of the Γ- semigroup*S*.

Definition 2.4: A Γ - semigroup*S* is said to be *NCI* if N(S) contains a nonzero ideal of *S* whenever N (S) is nonzero.

Definition 2.5: A Γ - semigroup*S* is said to be SNΓ- semigroup. If N (S) = N_{Γ}(S), whereN $_{\Gamma}$(S) is the set of all strongly nilpotent elements of *S*.

Definition 2.6: A Γ - semigroup*S* is said to be reduced if it has no non zero nilpotent elements.

Proposition 2.7: Let *S* be any Γ- semigroup then P (S)⊆N (S).

Proof: Let *x*∈P (S). If possible, suppose for some γ ∈ Γ for all positive integer *n*, $(x\Gamma)^{n-1}x \neq 0$. Let H = $\{(x\Gamma)^{n-1}x / n$ is any positive integer$\}$. Then H is an *m*-system not containing zero. Then there exists a prime ideal P of S such that P ∩H = ∅. Then $x \notin P$. So $x \notin$ P (S) a contradiction. Hence $(x\Gamma)^{n-1}x = 0$ for all γ ∈ Γ and some positive integer *n*. Thus*x*∈N(S).SoP (S)⊆N (S).

Proposition 2.8: Every reduced Γ- semigroup S is 2 primal.

Proof: By Proposition 2.7, for any Γ- semigroup*S*, P (S)⊆N (S).Again since *S* is reduced, N (S) = {0}⊆P (S). Hence the result.

Definition 2.9: Let*A* be a nonempty subset of a Γ- semigroup*S*. The right annihilator *A* with respect to $\phi \subseteq \Gamma$ in *S*denoted by$r(A,\phi)$, is defined by$r(A,\phi) = \{s \in S / A\phi s = \{0\}\}$. In particular, if $\phi = \Gamma$ we denote $r(A,\phi)$ by $ann_R(A)$. Again if A = {*a*}. Then we denote $ann_R(A)$ by $ann_R(a)$.

Analogously, we can defined left annihilator $l(\phi, A)$ and for $\phi = \Gamma$ it is denoted by $ann_L(A)$.

Proposition 2.10: The right annihilator $r(A, \phi)$ of A with respect to ϕ in a Γ- semigruop S is a right ideal of S.

Definition 2.11: A Γ - semigroup S is said to be satisfy (SI) if for each *x*∈ S, $ann_R(a)$ is an ideal of S.

Lemma 2.12: For any Γ - semigroup of S is the following statements are equivalent.

(i) S satisfy (SI)

(ii) For any *x*,*y*∈ S, *x*Γ*y* = 0 implies *x*ΓSΓ*y* = 0.

Proof: (i) ⇒ (ii) Let *x*, *y*∈ S be such that *x*Γ*y* = 0. Then$y \in ann_R(x)$.Since $ann_R(x)$ is an ideal of S,SΓ$y \subseteq ann_R(x)$. Hence *x*ΓSΓ*y* = 0.

(ii)⇒ (i) By proposition 2.10, $ann_R(x)$ is a right ideal of S. Let $y \in ann_R(x)$ then*x* Γ *y* = 0 hence by (ii) *x*ΓSΓ*y* = 0. Which implies that SΓ$ann_R(x) \subseteq ann_R(x)$.Thus $ann_R(x)$ is a left ideal of S. Therefore $ann_R(x)$ is an ideal of S. Hence S satisfies (SI).

Proposition 2.13: Let S be an SNΓ- semigroup. If S satisfies (SI) then S is a 2 primal Γ-semigroup.

Proof: By proposition 2.7,P (S)⊆N (S). Let$x$∈N (S). Since S is an SNΓ- semigroup, then there exist a positive integer$n$ such that $(x\Gamma)^{n-1}x = 0$. If possible, let $x \notin$ P (S).Then $x \notin$ P for some prime ideal P of S.i.ex∈ S\P.Since P is prime,S\P is an m-system. Then there exist $s_1 \in$ S, $\alpha_1, \beta_1 \in \Gamma$ such that $x\alpha_1 s_1\beta_1 x \in$ S\P. Again since $x\alpha_1 s_1\beta_1 x$,x∈ S\P applying m-system property $x\alpha_1 s_1\beta_1 x\ \alpha_2 s_2\beta_2 x \in$ S\P, for some $\alpha_2, \beta_2 \in \Gamma$ and $s_2 \in$ S. Applying m-system property after finite step we have $x\alpha_1 s_1\beta_1 x\ \alpha_2 s_2\beta_2 x \ldots. x\alpha_{n-1} s_{n-1}\beta_{n-1} x \in$ S\P, for some $s_i \in$ S, $\alpha_i, \beta_i \in \Gamma$, where i = 1,2,......,(n-1). Since S satisfies (SI) and $(x\Gamma)^{n-1}x = 0$. By above lemma 2.12, $x\alpha_1 s_1\beta_1 x\ \alpha_2 s_2\beta_2 x \ldots. x\alpha_{n-1} s_{n-1}\beta_{n-1} x = 0 \in$ P a contradiction.Hence$x$∈P (S). This completes the proof.

Definition 2.14: An element $a \in S$ is called regular in a Γ- semigroup$S$ if $a \in a\Gamma S\Gamma a$.

Theorem 2.15: Let S be a regular Γ- semigroup, then the following statements are equivalent.

(i) S is 2-primal.

(ii) S is NI.

(iii) S is NCI.

(iv) S is reduced.

Proof: (i) ⇒ (ii)Since, S is 2- Primal, N (S)⊆P (S).So P (S) is a nil ideal of S. HenceP (S)⊆N_r(S)⊆N (S). Therefore N_r(S) = N (S). So S is NI.

(ii) ⇒ (iii) By (ii) N_r(S) = N (S) since N_r(S) is nonzero, S is NCI.

(iii) ⇒ (iv) Let S be a NCI Γ - semigroup. Suppose S is not reduced i.e. N (S)≠ 0. Then there exists a nonzero ideal, say I such thatI ⊆N (S). Let $x(\neq 0) \in$ I. Since S is regular, there exist $\alpha, \beta \in \Gamma$ and y ∈ S such that$x = x\alpha y\beta x$..... (1) then$x$ = $(x\alpha y)\beta(x\alpha y)\beta x$ = ...(2). Since,x∈ I, $x\alpha y$∈ I ⊆N (S). So there exists a positive integer n such that $((x\alpha y)\beta)^n(x\alpha y) = 0$. From (2), we have $x\alpha y = ((x\alpha y)\beta)^n(x\alpha y) = 0$ and so by(1),x = 0 a contradiction, therefore S is reduced.

(iv)⇒ (i) Follows from proposition 2.8.

Definition 2.16: A Γ - semigroup S is said to be a right symmetric if for x,y,z∈ S,$x\Gamma y\Gamma z = 0$ implies $x\Gamma z\Gamma y = 0$.

An ideal A of a Γ- semigroup S is said to be a right symmetric if$x\Gamma y\Gamma z$⊆ A implies $x\Gamma z\Gamma y$⊆ A for x,y,z∈ S.

Definition 2.17: A Γ - semigroup S is said to be a left symmetric if for x,y,z∈ S,$x\Gamma y\Gamma z = 0$ implies $y\Gamma x\Gamma z = 0$.

An ideal A of a Γ- semigroup S is said to be a left symmetric if $x\Gamma y\Gamma z\subseteq A$ implies $y\Gamma x\Gamma z\subseteq A$ for $x,y,z\in S$.

Lemma 2.18: Any Γ- semigroupwith out strongly nilpotent element is right symmetric and also left symmetric.

Proof: Let S be a Γ- semigroup having no strongly nilpotent elements. Let $x, y, z \in S$ be such that $x\Gamma y\Gamma z = 0$ then $z\Gamma(x\Gamma y\Gamma z)\Gamma x\Gamma y = 0$. i.e. $(z\Gamma x\Gamma y)\Gamma(z\Gamma x\Gamma y) = 0$. Since Shas no strongly nilpotent elements, $z\Gamma x\Gamma y = 0$.Hence $x\Gamma y\Gamma x\Gamma(z\Gamma x\Gamma y)\Gamma x\Gamma z = 0$ and so $x\Gamma y\Gamma x\Gamma z = 0$.Thus $y\Gamma x\Gamma z\Gamma y\Gamma(x\Gamma y\Gamma x\Gamma z)\Gamma y\Gamma x = 0$. Therefore $y\Gamma x\Gamma z\Gamma y\Gamma x = 0$ and so $x\Gamma z\Gamma(y\Gamma x\Gamma z\Gamma y\Gamma x)\Gamma z\Gamma y = 0$. i.e. $((x\Gamma z\Gamma y)\Gamma)^2(x\Gamma z\Gamma y) = 0$ and consequently $x\Gamma z\Gamma y = 0$.

Now by above $(y\Gamma x\Gamma z)\Gamma(y\Gamma x\Gamma z)\Gamma(y\Gamma x\Gamma z) = 0$. i.e. $((y\Gamma x\Gamma z)\Gamma)^2(y\Gamma x\Gamma z) = 0$. Since S is without strongly nilpotent $y\Gamma x\Gamma z = 0$. Hence S is left symmetric.

Definition 2.19:A one sided ideal A of a Γ- semigroup$S$ is said to have the intersection factors property or simply IFP. If for any $x, y \in S$, $x\Gamma y \subseteq A$ implies $x\Gamma S\Gamma y \subseteq A$.

Definition 2.20: For a prime ideal P of a Γ- semigroup S, we define.

$N(P) = \{a\in S : a\Gamma S\Gamma b\subseteq P(S)$ for some $b \in S\backslash P\}$.

$N_P = \{a\in S: a\Gamma b\subseteq P(S)$ for some $b \in S\backslash P\}$.

$\overline{N}_P = \{a\in S: (a\Gamma)^{n-1}a\subseteq N_P$ for some positive integer $n\}$

Proposition 2.21: Let S be a Γ –semigroup. Then for any prime ideal P of S, $N(P) \subseteq P$ and $N_P\subseteq \overline{N}_P$. Moreover, if S contains unity then $N(P) \subseteq N_P$.

Proof: Let $a\in N(P)$. Then $a\Gamma S\Gamma b\subseteq P(S)$for some $b \in S\backslash P$.Since $P(S)\subseteq P$ for any prime ideal P, $a\in P$. Hence $N(P) \subseteq P$. From definition, it is obvious, that $N_P\subseteq \overline{N}_P$. Let S be a Γ- semigroup with unity. Let $a\in N(P)$.Then $a\Gamma S\Gamma b\subseteq P(S)$ for some$b\in S\backslash P$.Since S is a Γ - semigroup with unity $a\Gamma S\Gamma b\subseteq P(S)$ implies that $a\Gamma b\subseteq P(S)$ where $b\in S\backslash P$.So $a\in N_P$. Hence $N(P) \subseteq N_P$.

Proposition 2.22: Let S be a Γ- semigroup and P be a prime ideal of S such that N_P is an ideal of S. Then N_P is a completely semiprime ideal of S if and only if $N_P = \overline{N}_P$.

Proof: Suppose N_P is acompletely semiprime ideal of S. Clearly,$N_P\subseteq \overline{N}_P$. Let $x\in \overline{N}_P$. Then $(x\Gamma)^{n-1}x\subseteq N_P$ for some positive integer n. As N_P is completelysemi prime ideal of S,$(x\Gamma)^{n-1}x\subseteq N_P\Rightarrow x\in N_P$. Therefore $N_P = \overline{N}_P$. The converse part is obvious.

Theorem 2.23: Let S be an SNΓ- semigroup. Then the following statements are equivalent.

(i) S is a 2 Primal Γ - semigroup.

(ii) P (S)is a completely semiprime ideal of S.

(iii) P (S) is a left and right symmetric ideal of S.

(iv) $a\Gamma b \subseteq$P (S) implies $b\Gamma S\Gamma a \subseteq$P(S)for some $a,b \in$ S.

Proof: (i) ⇒ (ii) Let$a\Gamma a \subseteq$P (S), where $a \in$ S. Since S is anSNΓ- semigroup and a 2 primal Γ- semigroup.$a\Gamma a \subseteq N_\Gamma$(S). Then there exists a positive integer n such that $((a\Gamma a)\Gamma)^{n-1}(a\Gamma a) = 0$. This implies that $(a\Gamma)^{2n-1}a = 0$. So $a \in N_\Gamma$(S) = N (S) = P (S). Therefore P (S) is a completely semiprime ideal of S.

(ii) ⇒ (iii) Let$x\Gamma y\Gamma z \subseteq$P(S) where $x,y,z \in$ S. Now $(z\Gamma x\Gamma y)\Gamma(z\Gamma x\Gamma y) = z\Gamma(x\Gamma y\Gamma z)\Gamma x\Gamma y \subseteq$P(S). Since P (S) is completely semiprime, $z\Gamma x\Gamma y \subseteq$P(S). Now $(x\Gamma y\Gamma x\Gamma z)\Gamma(x\Gamma y\Gamma x\Gamma z) = x\Gamma y\Gamma x\Gamma(z\Gamma x\Gamma y)\Gamma x\Gamma z \subseteq$P (S) as P (S) is an ideal of S. This implies that $(x\Gamma y\Gamma x\Gamma z) \subseteq$P (S). Again by similar argument, we have $(y\Gamma x\Gamma z\Gamma y\Gamma x)\Gamma(y\Gamma x\Gamma z\Gamma y\Gamma x) = y\Gamma x\Gamma z\Gamma y\Gamma(x\Gamma y\Gamma x\Gamma z)\Gamma y\Gamma x \subseteq$P(S)⇒ $(y\Gamma x\Gamma z\Gamma y\Gamma x) \subseteq$P (S)implies$(x\Gamma z\Gamma y\Gamma x\Gamma z\Gamma y)\Gamma(x\Gamma z\Gamma y\Gamma x\Gamma z\Gamma y) = x\Gamma z\Gamma(y\Gamma x\Gamma z\Gamma y\Gamma x)\Gamma z\Gamma y\Gamma x\Gamma z\Gamma y \subseteq$P (S)imples$x\Gamma z\Gamma y \subseteq$P (S) as P (S) is completely semiprime. HenceP (S) is a right symmetric ideal of S. Also $(y\Gamma x\Gamma z)\Gamma(y\Gamma x\Gamma z) = y\Gamma(x\Gamma z\Gamma y)\Gamma x\Gamma z \subseteq$P (S) implies $y\Gamma x\Gamma z \subseteq$P (S). Hence P (S)is a left symmetric ideal of S. Therefore,P (S)is a left and right symmetric ideal of S.

(iii) ⇒ (iv) Let $a\Gamma b \subseteq$ P (S), where $a,b \in S$. Suppose $s \in S$, then $S\Gamma a\Gamma b \subseteq$ P (S). As P (S) is right symmetric. $S\Gamma b\Gamma a \subseteq$ P (S). Also since P (S) is left symmetric, $b\Gamma S\Gamma a \subseteq$ P (S). Therefore $b\Gamma S\Gamma a \subseteq$ P (S).

(iv) ⇒ (i) We know thatP (S)⊆N (S). Let $a \in$N (S). Since S is an SNΓ- semigroup,$a \in N_\Gamma$(S). Then there exists a positive integer n such that $(a\Gamma)^{n-1}a = 0$. If possible, let $a \notin$P (S). Then$a \notin$ P for some prime ideal P of S.Then $a \in$ S\P.Proceeding as in the proof of proposition 2.13, we get $a\alpha_1 s_1\beta_1 a\ \alpha_2 s_2\beta_2 a\ ....\ a\alpha_{n-1}s_{n-1}\beta_{n-1}a \in$ S\P,where$s_i \in$ S; $\alpha_i, \beta_i \in \Gamma$; $i = 1, 2, ...(n-1)$. Now by (iv), as $(a\Gamma)^{n-1}a \subseteq$P (S) implies $a\Gamma S\Gamma a\Gamma S\Gamma a.....a\Gamma S\Gamma a \subseteq$P (S). i.e. $a\alpha_1 s_1\beta_1 a\ \alpha_2 s_2\beta_2 a\ ....\ a\alpha_{n-1}s_{n-1}\beta_{n-1}a \in$Pa contradiction. Thus $a \in$P (S).Therefore P (S) = N (S). Hence S is a 2 primal Γ-semigroup.

Theorem 2.24: Let S be an SNΓ- semigroup. The following statements are equivalent.

(i) S is a 2 Primal Γ - semigroup.

(ii) P (S) has the IFP.

(iii) N(P) has the IFP for each prime ideal P of S.

Moreover, if S contains unity, then (i) is equivalent to(iv)N(P) = $\overline{N}_P$ = $N_P$for each prime ideal P of S.

Proof: (i) ⇒ (ii) Let S be a 2 Primal Γ- semigroup. Let $a\Gamma b$⊆P (S)and s∈ S then S$\Gamma a\Gamma b$⊆P (S). Now by theorem 2.23(iii),P (S) is a left symmetric ideal of S. So $a\Gamma S\Gamma b$⊆P (S). Thus $a\Gamma S\Gamma b$⊆P (S) i.e.P (S) has the IFP.

(ii) ⇒ (iii) Let$a\Gamma b$⊆N(P) where P is a prime ideal of S.So $a\Gamma b\Gamma S\Gamma y$⊆P (S) for some y∈S\P. Since P (S) has the IFP, $a\Gamma S\Gamma b\Gamma S\Gamma y$⊆P (S). Therefore $a\Gamma S\Gamma b$⊆N(P). Thus N(P) has the IFP for each prime ideal P of S.

(iii) ⇒ (i) AlwaysP (S)⊆N (S). Let a∈N (S). Since S is an SN Γ- semigroup, a∈N_Γ(S) . So there exists a positive integer n such that $(a\Gamma)^{n-1}a = 0$. If possible, let $a \notin$ P (S), then $a \notin$ P for some prime ideal P of S. Then a∈ S\P.Proceeding as in the proof of proposition 2.13, we get $a\alpha_1 s_1\beta_1 a\ \alpha_2 s_2\beta_2 a \ldots . a\alpha_{n-1}s_{n-1}\beta_{n-1}a$ ∈ S\P, $s_i \in$ S, $\alpha_i, \beta_i \in \Gamma$, where $i = 1, 2, \ldots\ldots (n-1)$. Also since N(P) has the IFP and $(a\Gamma)^{n-1}a = 0 \subseteq$ N(P) implies that $a\alpha_1 s_1\beta_1 a\ \alpha_2 s_2\beta_2 a \ldots .$ $a\alpha_{n-1}s_{n-1}\beta_{n-1}a$ ∈N(P), i.e. $a\alpha_1 s_1\beta_1 a\ \alpha_2 s_2\beta_2 a \ldots . a\alpha_{n-1}s_{n-1}\beta_{n-1}a \in$ P[N(P) ⊆ P], a contradiction.Thus a∈P (S). Therefore P (S) = N (S). Hence S is a 2 Primal Γ- semigroup.

(i) ⇒ (iv) Let P be a prime ideal of S. Then by proposition 2.21,N(P) ⊆ N_P. Let $x \in N_P$. Then $(x\Gamma)^{n-1}x \subseteq N_P$, for some positive integer n. So there exists y∈S\P such that $(x\Gamma)^{n-1}x\Gamma b$⊆P (S). By theorem 2.23(iii), P (S) is a left and right symmetric ideal of S. Then we have $(x\Gamma y)\Gamma(x\Gamma y)\Gamma(x\Gamma y)\Gamma\ldots\Gamma(x\Gamma y)$ (n-times)⊆P (S) .

Again by theorem 2.23 (ii)P (S) is a completely semi prime ideal of S, then we have$x\Gamma y$⊆P (S). Now by (ii) P (S) has the IFP. Therefore $x\Gamma S\Gamma y$⊆P (S) which implies that x∈N(P). So $\overline{N}_P \subseteq N_P$ .Hence N(P) = N_P for each prime ideal P of S. By proposition 2.21,N(P)⊆N_P⊆ $\overline{N}_P$ and by above N(P) = $\overline{N}_P$ for each prime ideal P of S. Therefore N(P) = N_P = $\overline{N}_P$

(iv) ⇒ (i) Let N(P) = N_P = $\overline{N}_P$ for each prime ideal P of S. To prove the result it is sufficient to prove that N (S)⊆P (S). Let x∈N (S). Since S is an SNΓ - semigroup, x∈N_Γ(S). This implies that $(x\Gamma)^{n-1}x = 0$ for some positive integer n implies $(x\Gamma)^{n-1}x \subseteq N_P \Rightarrow x \in \overline{N}_P \Rightarrow x \in$ N(P) ⇒x∈ P. (By proposition 2.21) for each prime ideal P. Hence$x$∈P (S). Thus S is a 2 Primal Γ-semigroup.

Theorem 2.25: Let S be an SNΓ- semigroup with unity. Then the following statements are equivalent.

(i) S is a 2 Primal Γ- semigroup.

(ii) N(P) is a completely semiprime ideal of S for each prime ideal P of S.

(iii) N(P) is a left and right symmetric ideal of S for each prime ideal P of S.

(iv) $a\Gamma b \subseteq$ N(P) implies $b\Gamma S\Gamma a \subseteq$ N(P) for $a,b \in$ S and for each prime ideal P of S.

Proof: (i) ⇒ (ii) Let S be 2 Primal Γ- semigroup. Then by theorem 2.24 (iv).N(P) = N_P = $\overline{N}_P$ for each prime ideal P of S. Let $a \in$ S be such that $a\Gamma a \subseteq$ N(P). Then $a\Gamma a \subseteq \overline{N}_P$. Hence there exists a positive integer n such that $((a\Gamma a)\Gamma)^{n-1}(a\Gamma a) \subseteq N_P$. Therefore, $(a\Gamma)^{2n-1}a \subseteq N_P$. So $a \in \overline{N}_P$ = N(P). Hence N(P) is a completely semiprime ideal of S for each prime ideal P of S.

(ii) ⇒ (iii) Let $x\Gamma y\Gamma z \subseteq$N(P), where $x,y,z \in$ S. Now $z\Gamma x\Gamma y\Gamma z\Gamma x\Gamma y = z\Gamma(x\Gamma y\Gamma z)\Gamma x\Gamma y \subseteq$ N(P). Since N(P) is completely semiprime, $z\Gamma x\Gamma y \subseteq$ N(P). We have $x\Gamma y\Gamma x\Gamma z\Gamma x\Gamma y\Gamma x\Gamma z = x\Gamma y\Gamma x\Gamma(z\Gamma x\Gamma y)\Gamma x\Gamma z \subseteq$ N(P) $\Rightarrow x\Gamma y\Gamma x\Gamma z \subseteq$ N(P) implies $y\Gamma x\Gamma z\Gamma y\Gamma x\Gamma y\Gamma x\Gamma z\Gamma y\Gamma x = y\Gamma x\Gamma z\Gamma y\Gamma(x\Gamma y\Gamma x\Gamma z)\Gamma y\Gamma x \subseteq$ N(P)$\Rightarrow y\Gamma x\Gamma z\Gamma y\Gamma x \subseteq$ N(P) $\Rightarrow x\Gamma z\Gamma y\Gamma x\Gamma z\Gamma y\Gamma x\Gamma z\Gamma y\Gamma x\Gamma z\Gamma y = x\Gamma z\Gamma(y\Gamma x\Gamma z\Gamma y\Gamma x)\Gamma z\Gamma y\Gamma x\Gamma z\Gamma y \subseteq$ N(P) implies $x\Gamma z\Gamma y \subseteq$ N(P). Thus N(P) is a right symmetric ideal of S. Also $y\Gamma x\Gamma z\Gamma y\Gamma x\Gamma z = y\Gamma(x\Gamma z\Gamma y)\Gamma x\Gamma z \subseteq$ N(P) implies $y\Gamma x\Gamma z \subseteq$ N(P). N(P) is a left symmetric ideal of S. Therefore N(P) is a left and right symmetric ideal of S. (iii) ⇒ (iv) Let $a\Gamma b \subseteq$N(P), where $a,b \in$ S. Since N(P) is an ideal of S, for each $s \in$ S, $s\Gamma a\Gamma b \subseteq$ N(P). As N(P) is right symmetric $s\Gamma b\Gamma a \subseteq$ N(P). Also since N(P) is left symmetric $b\Gamma S\Gamma a \subseteq$ N(P). Therefore $b\Gamma s\Gamma a \subseteq$N(P).

(iv) ⇒ (i) We know P (S)⊆N (S). Let $a \in$N (S). Since S is an SNΓ-semigroup, $a \in N_\Gamma(S)$. Then there exists a positive integer n such that $(a\Gamma)^{n-1}a = 0$ if possible, let $a \notin$P (S). Then $a \notin$ P for some prime ideal P of S. Then $a \in$ S\P.Proceeding as in the proof of proposition 2.13, we get $a\alpha_1 s_1 \beta_1 a\ \alpha_2 s_2 \beta_2 a \ldots . a\alpha_{n-1} s_{n-1} \beta_{n-1} a \in$ S\P, for some $s_1, s_2, \ldots\ldots, s_{n-1} \in S$ and $\alpha_1, \alpha_2, \ldots\ldots, \alpha_{n-1}; \beta_1, \beta_2, \ldots., \beta_{n-1} \in \Gamma$. Now by (iv) $(a\,\Gamma)^{n-1}a(= 0) \subseteq$ N(P) implies $a\alpha_1 s_1 \beta_1 a\ \alpha_2 s_2 \beta_2 a \ldots . a\alpha_{n-1} s_{n-1} \beta_{n-1} a \in$N(P).i.e. $a\alpha_1 s_1 \beta_1 a\ \alpha_2 s_2 \beta_2 a \ldots . a\alpha_{n-1} s_{n-1} \beta_{n-1} a \in$P[N(P) ⊆ P], a contradiction. Thus $a \in$P (S).Therefore P (S) =N (S). Hence S is a 2 primal Γ - semigroup.

Note 2.26: Spec(S) and mSpec(S) denote the set of all prime and minimal prime ideals of S respectively.

Proposition 2.27: Let S be a Γ- semigroup then P (S) = $\bigcap_{P \in Spec\,(S)} N(P)$= $\bigcap_{Q \in Spec\,(S)} N(Q)$.

Proof: Let $x \in$ P (S) then $x\Gamma S \subseteq$P (S). Since P is proper S\P is nonempty. Let $b \in$ S\P. As P (S) is an ideal, $x\Gamma S\Gamma b \subseteq$P (S) for any prime ideal P of S. This implies that $x \in$N(P) for every prime ideal P of S. i.e. $x \in \bigcap_{P \in Spec\,(S)} N(P)$. So P (S) $\subseteq \bigcap_{P \in Spec\,(S)} N(P)$. Again N(P) ⊆ P for any prime ideal P of S. Hence $\bigcap_{P \in Spec\,(S)} N(P)) \subseteq \bigcap_{P \in Spec\,(S)} P$ = P (S). So P (S) = $\bigcap_{P \in Spec\,(S)} N(P)$. Now P (S)=$\bigcap_{Q \in Spec\,(S)} N(Q)$. This complete the proof.

Definition 2.28: For a prime ideal P of a Γ- semigroup S, we define. O(P) = {a∈ S : aΓSΓb = 0 for some b∈ S\P}.

O_P = {a∈ S: aΓb = 0 for some b∈ S\P}. $\overline{O}_P$ = {a∈ S: $(a\,\Gamma)^{n-1}a \subseteq O_P$ for some positive integer n}

Proposition 2.29: Let S be a Γ- semigroup. Then for any prime ideal P of S, O(P) ⊆ P and $O_P \subseteq \overline{O}_P$. Moreover, if S contains unity, then O(P) ⊆ O_P.

Proof: Let a∈O(P). Then aΓSΓb⊆P (S) for some b ∈ S\P.Since P (S) ⊆ P for any prime ideal P, a∈ P. Hence O(P) ⊆ P. From definition, it is obvious, that $O_P \subseteq \overline{O}_P$. Let S be a Γ- semigroup with unity. Let a∈O(P). Then aΓSΓb⊆P (S) for some b∈ S\P. Since S is a Γ - semigroup with unity aΓSΓb⊆ P (S) implies that aΓb⊆P (S) where b∈ S\P.So a∈ O_P. Hence O(P) ⊆ O_P

Proposition 2.30: Let S be a Γ- semigroup and P be a prime ideal of S such that O_P is an ideal of S. Then O_P is a completely semi prime ideal of S if and only if $O_P = \overline{O}_P$.

Proof: Suppose O_P is a completely semi prime ideal of S. Clearly, $O_P \subseteq \overline{O}_P$. Let $x \in \overline{O}_P$. Then $(x\,\Gamma)^{n-1}x \subseteq O_P$ for some positive integer n as O_P is completely semi prime ideal of S. $(x\Gamma)^{n-1}x \subseteq O_P \Rightarrow x \in O_P$. Therefore $O_P = \overline{O}_P$. The converse part is obvious.

Proposition 2.31: Let S be an SN Γ- semigroup then N (S) $\subseteq \bigcap_{P \in Spec\,(S)} \overline{O}_P \subseteq \bigcap_{Q \in mSpec\,(S)} \overline{O_Q}$

Proof: We first show that if P_1 and P_2 are two prime ideals of S such that $P_1 \subseteq P_2$, then $\overline{O}_{P_2} \subseteq \overline{O}_{P_1}$. Let $x \in \overline{O}_{P_2}$. Then $(x\,\Gamma)^{n-1}x \subseteq O_{P_2}$ for some positive integer n, which implies that $(x\,\Gamma)^{n-1}x\Gamma y = 0$ for some $y \in S \backslash P_2$. Since $P_1 \subseteq P_2$, $y \in S \backslash P_1$. So $(x\,\Gamma)^{n-1}x \subseteq O_{P_1}$ for some positive integer n. Thus $x \in \overline{O}_{P_1}$. Hence $\overline{O}_{P_2} \subseteq \overline{O}_{P_1}$. Let P be any prime ideal of S, then there exists a minimal prime ideal Q of S such that Q ⊆ P. Therefore $\bigcap_{P \in Spec\,(S)} \overline{O}_P \subseteq \bigcap_{Q \in mSpec\,(S)} \overline{O_Q}$. Let x∈N (S). Since S is an SN Γ- semigroup, $x \in N_\Gamma(S)$. So $(x\,\Gamma)^{n-1}x = 0$ for some positive integer n. Therefore $(x\,\Gamma)^{n-1}x \subseteq O_P$, for each prime ideal P of S. i.e. $x \in \overline{O}_P$ for each prime ideal P of S, which implies that

$x \in \bigcap_{P \in Spec\,(S)} \overline{O}_P$. Hence N (S) $\subseteq \bigcap_{P \in Spec\,(S)} \overline{O}_P \subseteq \bigcap_{Q \in mSpec\,(S)} \overline{O_Q}$.

Theorem 2.32: Let S be a Γ- semigroup with unity. Then the following statements are equivalent.

(i) S is a 2 Primal Γ- semigroup.

(ii) $\overline{O}_P \subseteq$ P for each prime ideal P of S.

(iii) N (S)= $\bigcap_{P\in Spec\,(S)} \overline{O_P}$ = P (S).

Proof: (i) ⇒ (ii) Let $x\in \overline{O_P}$. Then there exists a positive integer n such that $(x\,\Gamma)^{n-1}x\subseteq O_P$. So $(x\,\Gamma)^{n-1}x\Gamma y = 0$ for some $y\in S\backslash P$. i.e. $(x\,\Gamma)^{n-1}x\Gamma y\subseteq P(S)$, for some $y\in S\backslash P$ which implies that $(x\,\Gamma)^{n-1}x\subseteq N_P$ i.e. $x\in \overline{N_P}$. So $\overline{O_P} \subseteq \overline{N_P}$ for each prime ideal P of S. Also by theorem 2.24 (iv) $\overline{N_P}$ = N(P) for each prime ideal P of S. Again since N(P) ⊆ P for each prime ideal P of S. Thus $\overline{O_P} \subseteq$ P for each prime ideal P of S.

(ii) ⇒ (iii) Since $\overline{O_P} \subseteq$ P for each prime ideal P of S, $\bigcap_{P\in Spec\,(S)} \overline{O_P} \subseteq \bigcap_{P\in Spec\,(S)} P$ = P (S). Now by proposition 2.31, N (S) $\subseteq \bigcap_{P\in Spec\,(S)} \overline{O_P} \subseteq$P(S). Also P (S)⊆N (S). Therefore N (S) = $\bigcap_{P\in Spec\,(S)} \overline{O_P}$ = P (S).

(iii) ⇒ (i) Obvious.

Theorem 2.33: If $\overline{O_P}$ = P for each prime ideal P of an SN Γ- semigroup S with unity, then

(i) S is a 2 Primal Γ- semigroup.

(ii) $\overline{O_P}$ = N(P) for each prime ideal P of S.

(iii) Every prime ideal of S is minimal and completely prime.

Proof: (i) Since $\overline{O_P}$ = P, $\overline{O_P} \subseteq$ P. Hence by theorem 2.32 (ii), N (S) = P (S). i.e. S is 2 Primal. (ii) Since N(P) ⊆ P and $\overline{O_P}$ = P for each prime ideal P of S, N(P) $\subseteq \overline{O_P}$ for each prime ideal P of S. Since S is 2- Primal, by theorem 2.24 (iv), N(P) = $\overline{N_P}$ for each prime ideal P of S. Also $\overline{O_P} \subseteq \overline{N_P}$ for each prime ideal P of S. Thus $\overline{O_P} \subseteq$N(P) for each prime ideal P of S. Therefore $\overline{O_P}$ = N(P) for each prime ideal P of S.

(iii) Let P be a prime ideal of S. From (ii) and the given condition $\overline{O_P}$ = P we get N(P) = P for each prime ideal P of S. If Q is a minimal prime ideal of S containing P, then N(P) ⊆ N(Q) ⊆ Q ⊆ P = N(P). Thus P = Q. i.e. P is a minimal prime ideal of S. Let $a\Gamma b\subseteq$ P = N(P) and $a\notin$ P, Since $a\Gamma b\subseteq$ N(P) there exists $y\in S\backslash P$. Such that $(a\Gamma b)\Gamma S\Gamma y\subseteq$ P (S) i.e. $a\Gamma(b\Gamma S\Gamma y) \subseteq$ P (S) since P (S) has IFP (by theorem 2.24) . Thus $a\Gamma S\Gamma(b\Gamma S\Gamma y) \subseteq$P (S) ⊆ P. Now as $a\notin$ P, $b\Gamma S\Gamma y\subseteq$ P. Again since $y\notin$ P, b∈ P. So either $a\in$ P or $b\in$ P. Hence P is completely prime ideal of S.

Proposition 2.34: If S is a 2 Primal SN Γ - semigroup O_P = P for some prime ideal P, then P is a completely prime ideal of S, in particular O_P has the IFP.

Proof: Let $a\Gamma b \subseteq P = O_P$. If possible, let $a \notin P$. So there exists $y \in S \backslash P$ such that $(a\Gamma b)\Gamma y = 0$. Since S is a 2 Primal Γ - semigroup by theorem 2.24, P (S) has the IFP. Therefore $(a\Gamma b)\Gamma y (= 0) \subseteq$ P (S) implies that $(a\Gamma S\Gamma b)\Gamma S\Gamma y \subseteq$ P (S) ⊆ P. Since P is prime and $a \notin P$, $b\Gamma S\Gamma y \subseteq P$. Again since $y \notin P$, $b \in P$ as P is prime ideal of S. Therefore, either $a \in P$ or $b \in P$. Hence P is a completely prime ideal of S. Let $a\Gamma b \subseteq O_P$. Since $O_P = P$ and P is a completely prime ideal of S, either $a \in P$ or $b \in P$. Again since P is prime ideal of S, $a\Gamma S\Gamma b \subseteq P$. i.e. $a\Gamma S\Gamma b \subseteq O_P$ (since $O_P = P$). Hence O_P has the IFP.

Proposition 2.35: Let S be an SN Γ - semigroup. If O(P) has the IFP for each minimal prime ideal P of S, then S is 2 Primal Γ- semigroup.

Proof: Suppose O(P) has the IFP for each minimal prime ideal P of S. To prove S is a 2 Primal Γ- semigroup it is sufficient to show that N (S)) ⊆ P (S). Let $x \in$ N (S). Since S is an SN Γ - semigroup. Then $(x\,\Gamma)^{n-1}x = 0$ for some positive integer n. If possible, let $x \notin$ P (S), then there exists a prime ideal P of S such that $x \notin$ P i.e. $x \in S \backslash P$. As P is a prime ideal of S, S\P is an m-system of S. Proceeding as in the proof of proposition 2.13, we get $s_1, s_2, \ldots\ldots, s_{n-1} \in S$ and $\alpha_1, \alpha_2, \ldots\ldots, \alpha_{n-1} \in \Gamma; \beta_1, \beta_2, \ldots., \beta_{n-1} \in \Gamma$ such that $x\alpha_1 s_1 \beta_1 x\, \alpha_2 s_2 \beta_2 x \ldots.\, x\alpha_{n-1} s_{n-1} \beta_{n-1} x \in S \backslash P$. Also since O(P) has the IFP and $(x\,\Gamma)^{n-1}x(= 0) \subseteq$ O(P) implies that $x\Gamma S\Gamma x \ldots.. x\Gamma S\Gamma x$ i.e. inparticular $x\alpha_1 s_1 \beta_1 x\, \alpha_2 s_2 \beta_2 x \ldots.\, x\alpha_{n-1} s_{n-1} \beta_{n-1} x \in S \backslash P$. Also since O(P) has the IFP, and $(x\,\Gamma)^{n-1}x(= 0) \subseteq$ O(P) implies that $x\Gamma S\Gamma x \ldots.. x\Gamma S\Gamma x$i.e in particular $x\alpha_1 s_1 \beta_1 x\, \alpha_2 s_2 \beta_2 x \ldots.$ $x\alpha_{n-1} s_{n-1} \beta_{n-1} x \in$O(P). As O(P) ⊆ P, $x\alpha_1 s_1 \beta_1 x\, \alpha_2 s_2 \beta_2 x \ldots.\, x\alpha_{n-1} s_{n-1} \beta_{n-1} x \in P$, a contradiction. So $x \in$P (S). SoP (S)⊆N (S). Hence P (S) = N (S). i.e. S is a 2 Primal Γ- semigroup.

We now prove that if O_P is a prime ideal for each minimal prime ideal P of S, then the converse of the proposition 2.35 is true.

Theorem 2.36: Let S be an SN Γ - semigroup with unity such that O_P is a prime ideal of S for each minimal prime ideal P of S. Then O(P) has the IFP, for each minimal prime ideal P of S if and only if S is a 2 Primal Γ- semigroup.

Proof: Let S be an SNΓ -semigroup. Suppose S is a 2 Primal Γ- semigroup and P be minimal prime ideal of S such that O_P is a prime ideal of S. So $O_P\,\Gamma S \subseteq O_P$ and hence $O_P \Gamma S \Gamma y = 0$ for some $y \in S \backslash P$. Thus $O_P \Gamma S \Gamma y \subseteq P$. As P is a prime ideal of S and $y \notin P$, $O_P \subseteq P$. Again since O_P is a prime ideal of S and P is a minimal prime ideal of S, $O_P = P$. So by proposition 2.34, O_P has the IFP, for each minimal prime ideal P of S. We now prove that O(P) = O_P. Now by proposition 2.29, O(P) ⊆ O_P. Again let $x \in O_P$. So $x\Gamma S \subseteq O_P$ has O_P is an ideal. Then there exists $y \in S \backslash P$ such that $x\Gamma S\Gamma y$

= 0. Hence $x \in O(P)$. Thus $O_P \subseteq O(P)$. So $O(P) = O_P$, for each minimal prime ideal P of S. Therefore, O(P) has the IFP for each minimal prime ideal P of S. The converse part follows from the Proposition 2.35.

Theorem 2.37: Let O_P be prime ideal of an SN Γ- semigroup S with unity for each minimal prime ideal P of S. Then the following statement are equivalent:

(i) S is a 2 Primal Γ- semigroup.

(ii) O_P has the IFP for each minimal prime ideal P of S.

(iii) O_P is a completely semiprime ideal for each minimal prime ideal P of S.

(iv) O_P is a left and right symmetric ideal for each minimal prime ideal P of S.

(v) $a\Gamma b \subseteq O_P \Rightarrow b\Gamma S\Gamma a \subseteq O_P$ for $a,b \in$ S and for each minimal prime ideal P of S.

Proof: (i) ⇒ (ii) Follows from proposition 2.36, because $O(P) = O_P = P$.

(ii) ⇒ (iii) Let $a\Gamma a \subseteq O_P$. Since by (ii). O_P has the IFP, $a\Gamma S\Gamma a \subseteq O_P$. As O_P is a prime ideal of S, $a \in O_P$. Hence O_P is a completely semi prime ideal for each minimal prime ideal P of S.

The proofs of (iii) ⇒ (iv) and (iv) ⇒ (v) are similar to the proofs (ii) ⇒ (iii) and (iii) ⇒ (iv) of theorem 2.23 respectively.

(v) ⇒ (i) Let $a\Gamma b \subseteq P$ (S). Then $a\Gamma b \subseteq P$ for each minimal prime ideal P of S. Since O_P is a prime ideal of S for each minimal prime ideal of S, $O_P = P$, for each minimal prime ideal of S. Thus $a\Gamma b \subseteq O_P$ for each minimal prime ideal P of S. So by (v) $b\Gamma S\Gamma a \subseteq O_P = P$ for each minimal prime ideals P of S. Therefore $b\Gamma S\Gamma a \subseteq P$ (S). Hence by (iv) ⇒ (i) of theorem 2.23, so S is a 2 primal Γ- semigroup.

Theorem 2.38: Let O_P be a prime ideal for each minimal prime ideal P of an SN Γ - semigroup S. Then the following statements are equivalent:

(i) S is a 2 Primal Γ- semigroup.

(ii) O(P) has the IFP for each minimal prime ideal P of S.

(iii) Every minimal prime ideal of S is a completely prime ideal of S.

Proof: (i) ⇒ (ii) Follows from the proposition 2.36.

(ii) ⇒ (iii) Let P be a minimal prime ideal of S. Then O_P is a prime ideal of S. Then by the proof of the proposition 2.36, we get $O(P) = O_P = P$. Hence by (ii) P has the IFP. Suppose $a\Gamma b \subseteq P$. Since P has the IFP, $a\Gamma S\Gamma b \subseteq P$. Again since

P is prime either, $a\in$ P or $b\in$ P. Thus P is completely prime ideal of S.

(iii) ⇒ (i) By proposition 2.27, P (S) is the intersection of all minimal prime ideals of S and by (iii) each minimal prime ideal of S is a completely prime ideal of SN Γ- semigroup S. Then P(S) is completely prime ideal i.e. P (S) is a completely semi prime ideal of the SN Γ- semigroup S. Therefore by Theorem 2.23, S is a 2 Primal Γ - semigroup.

Theorem 2.39: Let O_P be a prime ideal of an SN Γ- semigroup S for every minimal prime ideal P of S then S is a 2-Primal Γ - semigroup if and only if P = O(P) = $\overline{O_P}$ for each minimal prime ideal P of S.

Proof: Suppose S is a 2 primal Γ - semigroup. Since O_P is a prime ideal for each minimal prime ideal P of S, by the proof of the Proposition 2.36, P = O(P) = O_P. Therefore by proposition 2.34, O_P = O(P) = P is a completely prime ideal and hence a completely semi prime ideal of S. Hence by proposition 2.30, $O_P= \overline{O_P}$. i.e. P = O(P) = $\overline{O_P}$.

Conversely, suppose P = O(P) = $\overline{O_P}$ for each minimal prime ideal P of S. By proposition 2.31, N (S) $\subseteq \bigcap_{P\in Spec\,(S)} \overline{O_P} \subseteq \bigcap_{Q\in mSpec\,(S)} \overline{O_Q} = \bigcap_{P\in Spec\,(S)} Q$= P(S). Also P (S)⊆N (S). So P (S) = N (S) hence S is a 2 primal Γ-semigroup.

Proposition 2.40: If O_P has the IFP for each minimal prime ideal P of an SN Γ- semigroup S, then $O_P\subseteq$ P for each minimal prime ideal P of S if and only if S is a 2 primal Γ - semigroup.

Proof: Suppose $O_P\subseteq$ P for each minimal prime ideal P of the Γ- Semigroup S. Let $x\in$N (S). Since S is an SN Γ-semigroup S, $(x\,\Gamma)^{n-1}x = 0$ for some positive integer n. If possible, let $x \notin$ P (S). So there exists a prime ideal P of S such that $x \notin P$. Since P is a prime ideal of S, S\P is a m- system. Proceeding as in the proof of proposition 2.13, we get $x\alpha_1 s_1\beta_1 x\ \alpha_2 s_2\beta_2 x \ldots. x\alpha_{n-1}s_{n-1}\beta_{n-1}x \in$ S\P for some $s_1,s_2,\ldots,s_{n-1}\in$ S and $\alpha_1,\alpha_2,\ldots,\alpha_{n-1}$, $\beta_1,\beta_2,\ldots,\beta_{n-1}\in \Gamma$. Again since O_P has the IFP and $(x\,\Gamma)^{n-1}x(= 0) \subseteq O_P$, $x\alpha_1 s_1\beta_1 x\ \alpha_2 s_2\beta_2 x \ldots. x\alpha_{n-1}s_{n-1}\beta_{n-1}x \in O_P$. Since $O_P\subseteq$P, $x\alpha_1 s_1\beta_1 x\ \alpha_2 s_2\beta_2 x \ldots. x\alpha_{n-1}s_{n-1}\beta_{n-1}x \in$ P, a contradiction. So x ∈P (S). Hence N (S) ⊆P (S).Also P (S) ⊆N (S). Thus P (S) = N (S). i.e. S is a 2 primal Γ - semigroup.

Conversely suppose S is a 2 primal SNΓ- semigroup and P is a minimal ideal of S. Let $x\in O_P$. Then there exists $y\in$ S\P such that $x\Gamma y$ = 0. So $x\Gamma y\subseteq$P (S). Since S is a 2 primal Γ- semigroup, by theorem 2.24, P (S) has the IFP. So $x\Gamma S\Gamma y\subseteq$P (S) which implies that $x\Gamma S\Gamma y\subseteq$ P. Since P is a prime ideal of S and $y \notin P$, $x\in$ P. Hence $O_P\subseteq$ P.

Conclusion: This concept is used in chemistry, Physical chemistry, electronics etc.

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