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TOTAL EXCELLENT, JUST TOTAL EXCELLENT AND TOTAL VERY EXCELLENT ON TOTAL DOMINATION DOT-STABLE GRAPHS

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Abstract

A set D of vertices of a graph G is a total dominating set if every vertex of G is adjacent to some vertex in D. The minimum cardinality of a total dominating set of G is the total domination number denoted by γ_t (G). A graph G is said to be total domination dot stable, if identifying any pair of adjacent vertices the total domination number remains unchanged abbreviated as (TDDS). In this paper, we initially proved some basic theorems on total domination dot stable graph. Further we proved the resulting graph of TDDS is total excellent, just total excellent and total very excellent graphs or not.

Key words: total domination, total domination dot stable, total excellent, just total excellent, total very excellent.

1. Introduction

Let G = (V, E) be a simple graph with vertex set V of order n = |V| and edge set E of size m = |E|. A thorough study of the concept of domination in graphs we refer [1]. The open neighbourhood of v is, N (v) = {u / uv $\in$ E (G)} and the closed neighbourhood of v is N [v] = N (v) $\cup$ {v}. The private neighbourhood of v $\in$ D is, Pn [v, D] = N (v) – N (D – {v}). A vertex of degree one is called pendant vertex and its neighbour is called a support vertex. A set D $\subset$ V is a dominating set of G if every vertex in V – D is adjacent to some vertex in D. The minimum cardinality of dominating set is the domination number of G denoted by γ(G). A total dominating set is a set D of G if every vertex of G is adjacent to some vertex in D. The minimum cardinality of the total dominating set is the total domination number of G denoted by γ_t(G) is called γ_t – set. A vertex v is said to be good if there is a γ_t – set of G containing v. If there is no γ_t – set of G containing v, then v is said to be a bad vertex. A vertex v is said to be, down vertex if γ_t(G – u) < γ_t(G), level vertex if γ_t(G – u) = γ_t(G) and up vertex if γ_t(G – u) > γ_t(G). A vertex v is said to be selfish in the MDS D, if v is needed only to dominate itself. A graph G is said to be total excellent if given any vertex v then there is a γ_t – set of G containing v. A graph G is said to be just total excellent if each u $\in$ V, there is unique γ_t – set of G containing u. The total excellent graph G is said to be total very excellent if there is a γ_t – set D of G such that to each vertex u $\in$ V – D $\exists$ a vertex v $\in$ D such that D – {v} $\cup$ {u} is a γ_t – set of G. A γ_t – set D of G satisfying this property is called a total very excellent γ_t – set of G.

Graphs for which a given graph parameter changes (respectively, remains the same) upon a graph modification are called critical (respectively, stable). Both criticality and stability of the domination number of a graph have been studied for various graph modifications including adding an edge, removing an edge, and removing a vertex. Henning and Rad [2] investigated criticality of the total domination number when the graph G is modified by dotting (identifying) any two adjacent vertices decreased the total domination number. The graph formed from G by identifying u and v is denoted as G$\bullet$ uv. The G$\bullet$ uv is obtained from G by deleting u and v adding a new vertex, denoted by uv, that is adjacent to all of the vertices of G – u – v that were originally adjacent to either of u or v. Stephanie A. Rickett, T.W. Haynes [3] were introduced the concept of total domination dot stable graphs. A graph G is said to be total domination dot stable if identifying any pair of adjacent vertices the total domination number remains unchanged. ie., γ_t (G$\bullet$ uv) = γ_t (G), $\forall$ u, v $\in$ V (G), u adjacent to v.

2. Known Results

M.A. Henning and Nader Jafari Rad have proved in [2],

Preposition 2.1

A graph G is γ_t DC if and only if for any two adjacent vertices u and v, either u or v is a γ_t – critical vertex or there exists a γ_t – set S containing both u and v such that B ({u, v}) $\cap$ S $\neq$ ϕ.

M. Yamuna and A. Jeeva have proved in [4]

Theorem 2.2

G is TDDS iff $\forall$ u, v $\in$ D, B ({u, v}) $\cap$ D = ϕ , where D is a γ_t – set for G **.**

M. Yamuna and N. Sridharan were proved in [5],

Theorem 2.3

A graph G is JTE iff ,

a. γ_t (G) divides n.

b. d_t (G) = n / γ_t (G).

c. G has exactly n / γ_t (G) distinct γ_t – sets.

Remark [5]

If $G \neq \overline{K}_n$ is JTE then $| Pn (u, D) | \geq 2 \ \forall \ u \in D$ where D is any γ_t – set of G.

3. Main Results

In this section, we consider TDDS graph and proved some basic results and theorems.

Preposition 3.1

If G is TDDS, then a pendant vertex is always a level vertex.

Proof

Let G be a TDDS graph. Let v be a pendant vertex and $e = uv \in E(G)$. Then $G_{\bullet} uv = G - uv$ as G is TDDS, ie., $\gamma_t (G) = \gamma_t (G_{\bullet} uv) = \gamma_t (G - v)$. Hence v is a level vertex. □

Preposition 3.2

If $u, v \in \gamma_t (G)$ such that u adjacent to v, $B(\{u, v\}) \cap D = \phi$ and v is selfish, then $N(u) - \{v\} \in Pn [u, D]$

Proof

If $\exists$ one $w \in N(u) - \{v\}$ such that w is 2 – dominated say w adjacent to u, x where $x \in D$ where D is a γ_t – set of G. Then $D - \{v\} \cup \{w\}$ is a γ_t – set such that u adjacent to w adjacent to x, ie., $B(\{u, w\}) \cap D \neq \phi$. □

Observation 3.3

Let G be TDDS and let $D = \{u_1, u_2, \ldots, u_k\}$ be a γ_t – set for G. Let $x, y \in V(G)$.

Case 1 $x, y \in D$

Let $x = u_i$ and $y = u_j$. Then $D' = D - \{u_i\} - \{u_j\} \cup \{u_i u_j\} \cup \{w\}$ where $w \in B\{u_i, u_j\} \cap D$ is a γ_t – set for $G_{\bullet}$ xy.

Case 2 $x \in D, y \notin D$

Let $x = u_i$. Then $D'' = D - \{u_i\} \cup \{u_i y\}$ is a γ_t – set for $G_{\bullet}$ xy.

Case 3 $x, y \notin D$

Then D itself is a γ_t – set for $G_{\bullet}$ xy.

By the above cases we observe that if G is a TDDS graph, then $\forall \ x, y \in V(G)$ either D or D' or D'' is a γ_t – set for $G_{\bullet}$ xy. This is true $\forall \ x, y \in V(G)$. □

Preposition 3.4

Let G be TDDS and let u be a bad vertex. Let $G_{\bullet}$ uv be TDDS. In $G_{\bullet}$ uv $\exists$ no vertex w such that $uv, w \in D$, uv adjacent to w, w is selfish, where D is a γ_t – set for $G_{\bullet}$ uv.

Proof

Let G be TDDS and u be a bad vertex. Let G$\bullet$ uv be TDDS. In G$\bullet$ uv, let uv be a good vertex. This is possible by observation 3.3, If possible assume that $\exists$ a γ_t – set D such that w adjacent to uv, w $\in$ D, w is selfish. Then $D' = D - \{uv\} - \{w\} \cup \{u\} \cup \{v\}$ is a γ_t – set for G such that u is a good vertex, which is a contradiction to our assumption that u is a bad vertex. □

Theorem 3.5

Let G be a graph such that γ_t (G) = γ_{2t} (G), γ_t (G) > 2. Then G is not TDDS.

Proof

Let G be a graph such that γ_t (G) = γ_{2t} (G), γ_t (G) > 2. Let u $\in$ V – D. $\exists$ two vertices x, y $\in$ D such that x, y dominates u and v, w $\in$ D such that v adjacent to x and w adjacent to y. Now (D – v) $\cup$ {u} and (D – w) $\cup$ {u} are γ_t – set of G, which are not TDDS. This implies that B ({u, x}) $\cap$ D $\neq \phi$, which is a contradiction. □

Theorem 3.6

Let G be a TDDS graph,

1. Let u,v $\in$ D such that u adjacent to v. If x is 2– dominated for some x $\in$ N (u) or N (v) but x $\notin$ N (u) $\cap$ N (v), then G$\bullet$ ux and G$\bullet$ vx are not TDDS.
2. If u,v $\in$ γ_t (G) such that u adjacent to v, B ({v, u}) $\cap$ D = ϕ such that v is selfish, then G$\bullet$ uw is not TDDS $\forall$ w $\in$ N (u) – {v}.
3. If v is a selfish vertex as in case (2), then G$\bullet$ vw is not TDDS $\forall$ w $\in$ N (v).
4. If | Pn [u, D] | =1 say Pn [u, D] = {x} and | Pn [v, D] | =1say Pn [v, D] ={y} such that x adjacent to y, then G$\bullet$ uv is not TDDS if $\exists$ at least one w$\in$ N (x) or w$\in$ N (y), w $\neq$ u, v, x, y.

Proof

Let G be TDDS graph and D be a γ_t – set for G,

1. Let x be 2 – dominated and let x $\in$ N (u). Then $\exists$ one y in D such that x adjacent to y. In G$\bullet$ ux, B ({v, ux}) $\cap$ D = y. Hence G is not TDDS.
2. By preposition 3.2 we know that N (u) – {v} $\in$ Pn [u, D]. Let w $\in$ Pn [u, D]. If $\exists$ one x such that x adjacent to w, then x $\notin$ γ_t (G). So there is one y such that y $\in$ γ_t (G) and y adjacent to x. In G$\bullet$ uw, $D' = D - \{u\} - \{v\} \cup \{uw\} \cup \{x\}$ is a γ_t – set such that B ({uw, x}) $\cap$ D$'$ = y. Hence G$\bullet$ uw is not TDDS. This is true $\forall$ w $\in$ N (u) – {v}.

3. If v is selfish we know that any $w \in N(v)$, $w \neq u$ is 2 – dominated. Hence by case 1 $G_\bullet vw$ is not TDDS.

4. If | Pn [u, D] | =1 say Pn [u, D] = {x} and | Pn [v, D] | =1say Pn [v, D] ={y} such that x adjacent to y. Let D' be a γ_t – set for $G_\bullet uv$. Since $y \in$ Pn [v, D] $\exists$ one z such that y adjacent to z adjacent to w where $w \in D$. Now $D' = \gamma_t (G_\bullet uv) = D - \{v\} - \{u\} \cup \{y\} \cup \{z\}$ is a γ_t – set for $G_\bullet uv$, such that B ({y, z}) $\cap D' = w \neq \phi$. Hence $G_\bullet uv$ is not TDDS.

Remark

1. Let G be a TDDS graph such that $\gamma_t (G) > 2$. If Pn [u, D] = ϕ = Pn [v, D] such that u adjacent to v, then G is not TDDS.

Proof

Any $x \in B(\{u, v\}) \cap D$ is 2 – dominated. $D' = D - \{v\} - \{u\} \cup \{ux\}$ is a γ_t – set for $G_\bullet ux$. $\gamma_t (G_\bullet ux) \neq \gamma_t (G)$. Hence G is not TDDS. □

2. If $\gamma_t (G) = 2$, then it is possible that Pn [u, D] = ϕ = Pn [v, D] and G is TDDS.

Example: C_3, K_n.

4. Total Excellent and TDDS Graphs.

In this section, we proved the resulting graph of TDDS is total excellent, just total excellent and total very excellent graphs or not.

Theorem 4.1

Let G be a TDDS and total excellent graph. Then $G_\bullet uv$ is total excellent $\forall$ u, v $\in$ V (G).

Proof

Let G be a TDDS and total excellent graph.

Case 1 If $u \in \gamma_t (G)$ and $v \notin \gamma_t (G)$.

Let D be a γ_t - set such that $u \in \gamma_t (G)$ and $v \notin \gamma_t (G)$. Consider $G_\bullet uv$. $D' = D - \cup \{uv\}$ is a γ_t - set for $G_\bullet uv$. D' is a γ_t - set containing uv.

Case 2 If u, $v \in \gamma_t (G)$.

We know that G is γ_t – excellent. Let D be a γ_t – set for G such that u, $v \in D$. Since G is TDDS, B ({u, v}) $\cap$ D = ϕ . $\gamma_t (G_\bullet uv) = D'' = D - \{u\} - \{v\} \cup \{uv\} \cup \{w\}$ where $w \in B\{u, v\}$. D'' is a γ_t – set containing uv.

Case 3 If u, $v \notin \gamma_t (G)$.

Since G is TDDS and total excellent. If u, v $\notin \gamma_t$ (G), since G is total excellent $\exists$ a γ_t – set for G containing u and v. If D is a γ_t – set for G containing u then $D''' = D - \{u\} \cup \{uv\}$ is a γ_t – set for G$\bullet$ uv containing uv. If D is a γ_t – set for G containing v then $D^{iv} = D - \{v\} \cup \{uv\}$ is a γ_t – set for G$\bullet$ uv containing uv.

In all three cases $\exists$ a γ_t – set containing uv. All other γ_t – sets for G and G$\bullet$ uv are the same. This is true $\forall$ u, v$\in$ V (G). Hence G$\bullet$ uv is total excellent. □

Preposition 4.2

If G is total non – excellent and TDDS, then the number of bad vertices in G$\bullet$ uv is atleast one less than the number of bad vertices in G.

Proof

Let G be a TDDS and total non – excellent graph. Let D = $\{u_1, u_2,\ldots,u_k\}$ be a γ_t – set for G. Let v be a bad vertex. Since v is bad $\exists$ one u_i adjacent to v. Then $\{u_1,\ldots u_{i-1}, u_i v, u_{i+1}, \ldots,u_k\}$ is a γ_t – set for G$\bullet$ u_i v [by observation(3.3)] where u_i v is a good vertex, ie., G$\bullet$ u_i v has atleast one bad vertex less than number of bad vertices in G. □

Remark

If G is a TDDS and total non – excellent graph with exactly one bad vertex u, then we can generate N (u) total excellent graphs.

Preposition 4.3

Let G be a TDDS and just total excellent graph. Then G$\bullet$ uv is not just total excellent $\forall$ u, v $\in$ V (G).

Proof

Let G be TDDS and JTE graph. We know that JTE graph is total excellent. By the theorem 4.1, we know that G$\bullet$ uv is γ_t – excellent. D', D'' are two γ_t –sets for G$\bullet$ uv containing the vertex uv . Hence $\exists$ no unique γ_t – set containing uv. Hence G$\bullet$ uv is not JTE. □

Theorem 4.4

A JTE graph is not total dot - critical.

Proof

Let G be JTE and total dot - critical. Then by preposition 2.1[2], $\forall$ u, v $\in$ V (G) either u or v is a γ_t – critical vertex or $\exists$ a γ_t – set D containing both u and v such that B ({u, v}) $\cap$ D $\neq \phi$.

Let u, v $\in$ V (G) such that u adjacent to v.

Claim: A JTE graph has no down vertex.

Proof

If G has a down vertex u then γ_t (G – u) = γ_t (G) – 1. Let D' be a γ_t – set for G – u. If ∃ atleast one v ∈ N (u) such that v ∈ D', then D'' = $D' \cup$ {u} is a γ_t – set for G such that Pn [u, D''] = ϕ. If ∃ no v ∈ N (u) such that v ∈ D', ∃ one w ∈ D' such that w adjacent to v. Then D'' = $D' \cup$ {v} where v ∈ N (u) is a γ_t – set for G such that | Pn [v, D''] | = {u} = 1. By the theorem 2.3[5], if G is JTE then Pn [u, D] ≥ 2. Hence Pn [u, D''] = ϕ and | Pn [v, D''] | = 1 are not possible, ie., a JTE graph has no down vertex. G is total dot – critical and has no down vertex. By the preposition 2.1[2]∀ u, v ∈ V (G), u adjacent to v ∃ a γ_t – set containing u and v ie., if $v_1, v_2, \ldots, v_n$ ∈ N (u), ∃ γ_t – set containing $uv_1, uv_2, \ldots, uv_n$ such that B ({u, v_1}) ∩ D_i ≠ ϕ. Since G is JTE ∃ a unique γ_t – set containing u which implies N [u] belongs to a common MDS set say D. In D, Pn [u, D] = ϕ, which is not possible by the Remark in[5], ie.,∃ no common MDS set containing uv ∀ u, v ∈V (G).

Hence ∀ u, v ∈ V (G) such that u ∈ D and v ∉ D neither u nor v is critical, which is a contradiction. Hence a JTE graph is not total dot – critical. □

Corollary

If G is JTE, then G has no selfish vertex.

Proof

Let G be just total excellent. Let D be a γ_t – set for G. Suppose ∃ a selfish vertex say u then Pn [u, D] = ϕ , which is not possible. Hence G has no selfish vertex. □

Preposition 4.5

If G is a TVE graph such that x ∈ V (G) is k – dominated, k ≥ 3 then G is not TDDS.

Proof

If x is k – dominated, k ≥ 3 say x is 3 – dominated. Let u, v, w ∈ γ_t (G) such that u, v, w adjacent to x. If G is TVE then ∃ one y ∈ γ_t (G) such that D = G – {y} ∪ {x} is a γ_t – set for G. (y may be one of u, v, w).Then D is a γ_t – set where u adjacent to x, x adjacent to v and B ({u, x}) ∩ D ≠ ϕ . Hence G is not TDDS. □

Theorem 4.6

Let G be TVE and TDDS graph, u is a γ_t - selfish vertex in G then,

a) u cannot be a γ_t – selfish vertex such that w adjacent to u adjacent to v where u, v, w ∈ γ_t (G).

b) If u, v $\in$ D such that u adjacent to v, then u can be used for vertex exchange only with y where y $\in$ Pn [v, D].

c) If w is a 2 – dominated vertex such that u adjacent to w adjacent to v and u adjacent to v, then u can be used for vertex exchange with w,

where D is a TVE γ_t – set for G.

Proof

Let G be TVE and TDDS graph. Let D be a TVE γ_t – set for G.

a) If u is a γ_t – selfish vertex such that w adjacent to u adjacent to v, then B ({u, v}) $\cap$ D $\neq \phi$, which is a contradiction as G is not TDDS.

b) $D' = D - \{u\} \cup \{y\}$, where y $\in$ Pn [v, D] is also a γ_t – set for G such that B ({u, v}) $\cap$ $D' = \phi$ $\forall$ u, v $\in D'$. u can be used for vertex exchange with y. Suppose u can be used for vertex exchange with any x such that x $\in$ N (v), x $\notin$ Pn [v, D], ie., $\exists$ one w $\in$ D such that v adjacent to x adjacent to w. Then $D'' = D - \{u\} \cup \{x\}$ is a γ_t – set for G such that v adjacent to x adjacent to w, which is a contradiction as G is TDDS. Hence u can be used for vertex exchange only with y.

c) $D' = D - \{u\} \cup \{w\}$ is a γ_t – set for G such that B ({w, v}) $\cap$ $D' = \phi$. Hence u can be used for vertex exchange with w. □

Remark: 1. If G is TVE and TDDS as in case (b) of theorem 4.6, then y can be interchanged only with vertex u.

Proof

Suppose $\exists$ one x $\neq$ u $\in$ D such that $D' = D - \{x\} \cup \{y\}$ is a γ_t – set for G.

Then u, v, y$\in D'$ such that u adjacent to v adjacent to y, which is a contradiction as G is TDDS. □

2. If u is a selfish vertex, then D – {u} $\cup$ {x} is not TDDS if x $\in$ V (G) such that x $\notin$ Pn [v, D] or x is not a 2 – dominated vertex as in (c) or u is a selfish vertex as in (a).

Preposition 4.7

Let G be TVE and TDDS. Let D be a TVE γ_t – set for G. Let u, v $\in$ D such that u adjacent to v. Let u be a vertex that is not used for vertex exchange. Then v can be used for vertex exchange only with w, where w $\in$ Pn [u, D].

Proof

Let u, v $\in$ D such that u adjacent to v. Let u be a vertex that is not used for vertex exchange. Let w $\in$ Pn [u, D] $D' = D - \{v\} \cup \{w\}$ is a γ_t – set for G such that B ({w, u}) $\cap$ $D' = \phi$.

Suppose v can be used for vertex exchange with some $x \in N(u)$, $x \notin Pn[u, D]$.Then ∃ one $y \in D'$ such that x adjacent to u, y. Then $D'' = D - \{v\} \cup \{x\}$ is aγ_t – set for G such that u adjacent to x adjacent to y, which is a contradiction.

Remark

If G is TVE and TDDS as in above preposition 4.7, then w can be interchanged only with vertex v.

Proof

Suppose ∃ one $y \neq v \in D$ such that $D' = D - \{y\} \cup \{w\}$ is a γ_t –set for G such that w adjacent to u adjacent to v, $B(\{u, v\}) \cap D' \neq \phi$. Hence G is not TDDS. □

Preposition 4.8

Let G be TVE. If w is a 2 – dominated vertex not like the one discussed in part (c) of theorem 4.6, then G is not TDDS.

Proof

If w is 2 – dominated vertex not like the one discussed in part (c) of theorem 4.6, ie., ∃ one $v, x \in D$ such that v adjacent to w adjacent to x. ∃ one $u, y \in D$ such that u adjacent to v, x adjacent to y. Also since G is TVE ∃ one $z \in D$ such that $D - \{z\} \cup \{w\}$ is a γ_t – set for G. (z may be one of u, v, x, y). Then G is not TDDS by the theorem 2.2 in [4] is not satisfied. □

Remark

If G is TDDS and TVE then,

1. If G has a selfish vertex u, it can be interchanged with Pn [v, D] where v adjacent to u or any vertex w such that w adjacent to u, v.
2. G has no 2 – dominated vertex w such that $w \in B(\{v, u\}) \cap D$ and $w \in B(\{x, y\}) \cap D$.
3. If u is a vertex that is not used for vertex exchange, then any $w \in N(u) - \{v\}$ can be interchanged with v if $w \in Pn[u, D]$, u adjacent to v.

From the remark we observe that if G is TVE and TDDS then ∀ $u, v \in D$ such that u adjacent to v, N (u) can be exchanged with v only and N (v) can be exchanged with u only.

Preposition 4.9

If G is TDDS graph such that $\gamma_t(G) = 2$, and if G has at least one selfish vertex, then G is TVE.

Proof

Since G is TDDS such that γ_t (G) = 2 . Let u, v $\in$ D, u adjacent to v such that u is selfish. Then N [v] = V (G) as u is selfish. Let N [v] = u, (w_1, w_2, …, w_k). D – {u} $\cup$ {w_i} i = 1, 2, …, k are γ_t – sets for G. Hence G is TVE. □

Remark

If G is TVE such that γ_t (G) = 2 and G is TDDS, then G need not have a selfish vertex.

Example: Complete bipartite graph $K_{m, n}$.

5. Conclusion

In this paper we studied about TDDS graph, also we proved the resulting graph of TDDS is total excellent, just total excellent and total very excellent graphs or not. Characterizations for total domination dot stable trees are still open.

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