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METHODS OF THE COORDINATE SYSTEMS TIEING OF TWO IMAGES OBTAINED IN DIFFERENT REFERENCE SYSTEMS

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Abstract

The aim of this work was to develop a method of binding coordinate systems of the two images obtained in different systems of reference data. The proposed method can be used in the analysis of coordinate-time support of objects moving at different speeds in the phase space. The method was tested in conjunction of lunar objects to the inertial coordinate system on the basis of the use of two images: the Moon and the star field. As receivers that allow to obtain such images, CCD-matrix systems were used. Precision of images binding in the system of equatorial coordinates in average equaled to in right ascension $0\overset{''}{.}04 \div 0\overset{''}{.}13$, declination – $0\overset{''}{.}05 \div 0\overset{''}{.}11$. The paper represents a comparative analysis of the methods for the preparation of the Moon images without others stars and with the stars. The practical results of a system of double images creation, which move at different speeds on the celestial sphere are given. Double image method makes it possible to obtain binding of lunar objects to the stars in an absolute way with high accuracy. These studies are of great practical application in the field of space astrometry and celestial mechanics.

Keywords: coordinate systems, method of separate images, optical observations, inertial coordinate system.

1. Introduction

Methods of astrometric observations of the Moon can be divided into two types [1]: getting large-scale images of the Moon without any surrounding stars and getting images of the Moon with the surrounding background of faint stars. Observations of the first kind are relatively simple [2]. They are usually held on the long-focus telescopes. Pictures of the Moon are obtained with a short exposure. The scale and orientation of images is determined by the points of the Moon itself with a priori known selenodetic coordinates. The informative value of such observations is comparatively high and, in particular, allows to study the orbital motion of the Moon.

Binding of pictures of the Moon to the background of faint stars is aimed at the study of movement, rotation and figure of the Moon in the "inertial" system of coordinates defined by the fundamental star catalog. Because of the significant proper motion (about $0\overset{''}{.}5$ per second), and the brightness of the Moon, the implementation of such observations is a difficult task for positional astrometry. In these cases, a short-term exposure of the Moon in the average time of receipt of star images is carried out.

Method of instantaneous exposure of the Moon in the average time of receipt of images of stars has the disadvantage – lack of simultaneity of images of the Moon and stars. Because of this, a significant impact on the accuracy of determining the coordinates of the Moon have: the unevenness of running of the clock drive of the telescope and atmospheric turbulence [3]. In these observations at the $8''$ telescope in order to best display its qualities corner flicker according to D. D. Maksutov should not exceed $0\overset{''}{.}175$ [4]. This state is rare even in mountainous areas with excellent visibility. For example, in the observatory Peak du Midi resolution $0\overset{''}{.}23$ has been obtained for only 10% of the Moon cinematic observations [3, 5].

2.1. Methods Of Instant Expositions

A significant contribution to the development of positional observations of the Moon using photographical means was made by B. Markowitz [5]. He designed a special chamber with two plane-parallel filters which were arranged in front of the photographic plate. Thus a simultaneous photographing the Moon and stars was achieved. Surveillances using Markowitz camera were usually made on the short-throw astrographs to determine the coordinates of the Moon by measuring the edge points. Large series of observations by this method were obtained in Engelhardt astronomical observatory (EAO) [6, 7]. Turning the filter about an axis parallel to the focal plane of the telescope, selecting speed of rotation and the position angle of the axis of rotation allowed to render the image of the Moon on a background of stars motionless. However, when using long lenses Markowitz method distorts macrofigure of the Moon. Further the method of instantaneous exposures replaced Markowitz approach.

The stars on the celestial sphere located near the orbit of the Moon, have not only just weak brightness, but also different speed of movement with respect to the Moon in the telescope's field because of the difference of their own movements.

In observations it is necessary to compensate not only the exposure of the stars to the Moon light, but also the difference between the movements. Previously, these problems were solved in part by using astrographs with small and medium focal length, while receiving the large-scale images of the Moon with the stars on long-focus astrographs it was necessary to develop a fundamentally new method of receiving lunar images. Long-focus horizontal telescope with coelostat appeared to be the most suitable telescope for the production of such observations.

For the first time, large-scale images of the Moon with the stars were obtained by EAO on the horizontal telescope by instantaneous exposures. It received about 60 shots, which were used to solve a number of selenodetic problems. Method of instant exposures is based on the fact that at exposure of stellar field there is a short-term exposure of the Moon image. Imaging is carried out using a specially designed gate, consisting of two shutters, mounted on a metal rod moving perpendicularly to the optical axis of the telescope. At the beginning of the exposure image of the Moon is closed by a curtain, then the rod shifts its location, allowing to fix the image of the Moon, and then it is closed with the second shutter. In the process of observation the same exposure over the entire image of the Moon is given, the change in the relative position of the blinds regulates the duration of exposure. This method gave satisfactory images of both the Moon and background stars [7].

2.2 Method Of Dual Image

In this paper, we propose the method of binding the two coordinate systems moving at different speeds. The novelty of this method lies in the fact that the celestial coordinate system defined by the provisions of the stars S_1, and the coordinate system tied to the Moon S_2 body, move at different speeds relative to each other. Using the methods, described in the preceding paragraph, the scientists have been trying to bind these coordinate systems with manipulations in a single shot, which naturally led to orientation of coordinate and time support errors. In the method of double images with two cameras, optical centers of which coincide, the images of the starry sky, and the Moon are made separately. As a result, two separate images are obtained: the stars and the Moon. Exposure of the sky is carried out at a speed that matches the diurnal motion of the celestial sphere, of the Moon - at a different speed, which coincides with the speed of own motion of the Moon. The coordinate systems of these two images are linked by artificial light labels. This allows to receive precise images of celestial objects and accurate binding of coordinate systems in different frames of reference. This method has been named the method of double images [7]. Trial tests of this method gave a very good result. The accuracy of determining the coordinates of the Moon according to the measurements on the plate of edge points equaled to $\pm 0\overset{''}{.}30$ in right ascension and $\pm 0\overset{''}{.}15$ in declination.

At the same time the problem of mutual geometric binding of objects, located on the image of the Moon to the supporting stars of stellar images is resolved. For this purpose, eight lighted markers are used. They are fixed in relation to the image of the stars. Using this light scheme a system of light marks on the images is built: four on star ones and four - on the Moon ones. Since the light-receiving system for lunar images moved in the process of observations in relation to the star system, the star image had only four points imprinted, and on the Moon one - a series of light marks, corresponding to the number of flashes of light markers. Binding of markers on the lunar and stellar images to a single coordinate system is the main stage of the observations reduction. Fig. 1 represents the Lunar image, Fig. 2 represents the stellar image.

Fig. 1: The Lunar image with markers.

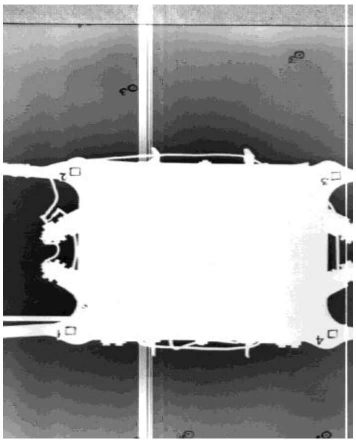

Fig. 2: The stellar image with the markers and the image of the moving system for lunar shots obtaining.

3. Algorithm of the Binding of Lunar Images to the Stars

The binding of the lunar and stellar images may be implemented using several algorithms [8, 9]. One of the methods is that a sector of sky with a lot of stars is projected at the same time on the Moon and star images. Let us consider the labels as the studied stars. Since the coordinates of the optical center for the both images are the same (α_0, δ_0), the ideal coordinates of stars on them belong to the same system, therefore, the ideal coordinates of light labels will also belong to the same system. Ideal coordinates of tags are used to convert measured coordinates of details pictures on the lunar image into the system of measured coordinates of star images.

Let us designate measured coordinates of tags $(i = 1 \div 4)$ and the coordinates of the stars $(j = 6 \div n)$ in the star image, and $(x_i^{\leftmoon}, y_i^{\leftmoon})$ and $(x_k^{\leftmoon}, y_k^{\leftmoon})$ (k = 6 ÷ m), respectively on the Moon image. Knowing the right ascension α_j^* and declination δ_j^* of stars on the star image, and right ascension and declination of stars $(\alpha_k^{\leftmoon}, \delta_k^{\leftmoon})$ on the lunar image standard coordinates of star are calculated (X_j^*, Y_j^*) and $(X_k^{\leftmoon}, Y_k^{\leftmoon})$. Further, the image orientation coefficients a, b, c, d, e, f are determined:

$$\begin{pmatrix} a^* & b^* & c^* \\ d^* & e^* & f^* \end{pmatrix} \begin{pmatrix} x_j^* \\ y_j^* \\ 1 \end{pmatrix} = \begin{pmatrix} X_j^* \\ Y_j^* \end{pmatrix}, \qquad (1)$$

$$\begin{pmatrix} a^{\leftmoon} & b^{\leftmoon} & c^{\leftmoon} \\ d^{\leftmoon} & e^{\leftmoon} & f^{\leftmoon} \end{pmatrix} \begin{pmatrix} x_k^{\leftmoon} \\ y_k^{\leftmoon} \\ 1 \end{pmatrix} = \begin{pmatrix} X_k^{\leftmoon} \\ Y_k^{\leftmoon} \end{pmatrix}. \qquad (2)$$

To determine the standard coordinates of labels on the star image (X_i^*, Y_i^*) and the image of the Moon $(X_i^{\leftmoon}, Y_i^{\leftmoon})$ we can write:

$$\begin{pmatrix} X_i^* \\ Y_i^* \end{pmatrix} = \begin{pmatrix} a^* & b^* & c^* \\ d^* & e^* & f^* \end{pmatrix} \begin{pmatrix} x_i^* \\ y_i^* \\ 1 \end{pmatrix}, \qquad (3)$$

$$\begin{pmatrix} X_i^{\leftmoon} \\ Y_i^{\leftmoon} \end{pmatrix} = \begin{pmatrix} a^{\leftmoon} & b^{\leftmoon} & c^{\leftmoon} \\ d^{\leftmoon} & e^{\leftmoon} & f^{\leftmoon} \end{pmatrix} \begin{pmatrix} x_i^{\leftmoon} \\ y_i^{\leftmoon} \\ 1 \end{pmatrix}. \qquad (4)$$

When we have the system of marks on the star and Moon images taken into a single coordinate system it is possible to transition from a system of measured coordinates on the Moon image to the system of measured coordinates on the star image, which is the main purpose of this reduction.

Connection of standard coordinates of labels and coordinates measured on the star image will look like:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \begin{pmatrix} X_i^* \\ Y_i^* \\ 1 \end{pmatrix} = \begin{pmatrix} x_i^* \\ y_i^* \end{pmatrix}. \qquad (5)$$

Solving (5) with least square method the coefficients of a_{ij} are determined. Let us mark coordinates of the labels on the Moon image in the measurement system of stellar plate as $(x_i^{m\leftmoon}, y_i^{m\leftmoon})$. We use the obtained coefficients a_{ij} for determining the coordinates of the Moon marks in the measurement system of stellar image:

$$\begin{pmatrix} x_i^{m\,☾} \\ y_i^{m\,☾} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \begin{pmatrix} X_i^{☾} \\ Y_i^{☾} \\ 1 \end{pmatrix}, \qquad (6)$$

Then

$$\begin{pmatrix} x_i^{m\,☾} \\ y_i^{m\,☾} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix} \begin{pmatrix} x_i^{☾} \\ y_i^{☾} \\ 1 \end{pmatrix}. \qquad (7)$$

From the expression (7) the values of b_{ij} coefficients are determined. The final step is to convert the coordinates of lunar objects measured on the Moon image into the S_2 system (we mark them as x_j^{S2}, y_j^{S2}) in the coordinates system of standard stars on the star image S_1 (x_j^{S1}, y_j^{S1}):

$$\begin{pmatrix} x_j^{S1} \\ y_j^{S1} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix} \begin{pmatrix} x_j^{S2} \\ y_j^{S2} \\ 1 \end{pmatrix}, \qquad (8)$$

4. Practical Use Of The Method

During observations on the eight-meter telescope for high-quality large-scale images of the Moon in the star system, ability to perform observations at small zenith distances has crucial role. We also need to be aware that it is very difficult to make a good variance of observations on optical librations in temperate zone. As it is known, we can solve a wide range of selenodetic tasks on the basis of the method of lunar images binding to the stars [10]. In contrast to the lunar images processing methods without the stars in the case of binding to the stars we have the absolute determination of orientation, the zero point of the coordinate system and its scale. However, when using the method of two separate images, one should pay special attention to the accuracy of coordinates of reference stars and ephemeris coordinates of the center of mass of the Moon, since they are fully included in the decision results as permanent components [11]. As a result of the method of obtaining images of the Moon and stars on two separate images, the processing of such observations should be carried out in two stages: reduction of the stellar image, and then – reduction of the Moon image. With regard to reduction of measurements of star images, during the adjustment of the optical system of the horizontal telescope its optical center is aligned with the geometric center of star marks (x_0, y_0). Given the strong position of the horizontal telescope components and precise alignment of autocollumating method we can consider that the position of the optical center on the star image was determined with an accuracy of less than ± 0.5 mm (millimeters). This means that errors due to inaccurate knowledge of the position of the optical center and the inclination of the image can be ignored.

Then the view of Turner equations at reduction of stellar plates will be the following:

$$\begin{pmatrix} X_i^{i*} \\ Y_i^{i*} \end{pmatrix} = \begin{pmatrix} a^* & b^* & c^* \\ d^* & e^* & f^* \end{pmatrix} \begin{pmatrix} x_i^* - x_0 \\ y_i^* - y_0 \\ 1 \end{pmatrix}. \quad (9)$$

In equation (9) X_i^{i*}, Y_i^{i*} – ideal coordinates of reference stars, calculated in spherical equatorial coordinates of reference stars α_i, δ_i and coordinates of the optical center or principal point of the image α_0, δ_0. In order for the system of perfect coordinates to correspond to the system of measured ones, it is necessary to calculate ideal coordinates of reference stars by their visible spherical coordinates given to the epoch and equinox of the date of observation. Accordingly, the orientation of the axes of the system of the measured coordinate is arbitrary, and the zero point (x_0, y_0) with high precision corresponds to the position of the optical center on the image. On the other hand, the position of the zero-point of the system of ideal coordinates (α_0, δ_0) is known approximately, and the orientation of the axes corresponds to the projection of the main circles of the celestial spherical coordinate system on the image plane.

Thus, in the system of measured coordinates there is a wrong orientation of axes, and in the ideal coordinate system - the position of the zero point. To improve the accuracy of determining the permanent values of the image, the zero-point of the system of ideal coordinates must be aligned with the position of the point (x_0, y_0), and axes x, y with the direction of the axes X, Y. To do this, follow these steps. First, constant images are found using the least squares method. Several iterations are made and before each subsequent iteration the system of measured coordinates of the stellar image objects is rotated by an angle $\varphi = \varphi_2 - \varphi_1$, where

$$tg\varphi_1 = \frac{Y_{i=1}}{X_{i=1}}, tg\varphi_2 = \frac{y_{i=1} - y_0}{x_{i=1} - x_0}. \quad (10)$$

After each iteration the values of coordinates of the main point of the image changed as follows:

$$\alpha_0^{(k)} = \alpha_0^{(k-1)} + c^{(k-1)} sec\delta_0^{(k-1)}, \delta_0^{(k)} = \delta_0^{(k-1)} + f^{(k-1)}, \quad (11)$$

where $k = 1, 2, 3, \ldots n$ – number of approximations. Usually there was enough three - four approximations for the values both of the rotation angle $\varphi^{(k)}$, and permanent ones of the image (9) $b^{(k)}, c^{(k)}, d^{(k)}, f^{(k)}$ have become practically equal to zero. After a certain approximation the constant ones of the image (9) $a^{(k)}, e^{(k)}$ will characterize the scale of the stellar image on the axes X, Y, and visible spherical equatorial coordinates of the main point of the image corresponding to the projection of the optical center of the image on the celestial sphere will be A $= \alpha_0^{(k-1)}, D = \delta_0^{(k-1)}$

The mean values of the coefficients of transition matrix b_{11} and b_{22} of the system of the measured coordinates on the Moon image to the measured coordinates system on the star image are almost equal to one 1.0000 ± 10^{-5} and 0.9999 ± 10^{-5}, and the coefficients $b_{12} \approx b_{21} \approx 0$. The values of the free members b_{13} and b_{23}, determining the difference between the positions of the geometric centers of the lunar and stellar marks also have small values.

If we talk about the accuracy of the method of separate images, then measuring of marks of the lunar images showed that their values while moving of one image in relation to the other image remain almost constant with an accuracy of σ = ± 0.5 micron, where σ – average square deviation. Therefore, since the errors of compensation of own motion of the Moon, and the binding between the lunar and stellar images are not dependent on the focal length, the accuracy of the observations is better on the long-focus instruments. It was important to establish the accuracy of the conversion of the measured coordinates on the Moon image into the coordinate system of the stellar image. Precision of images binding was investigated without the Moon.

On the Moon image a few reference stars as equivalents of craters were taken. Their coordinates were calculated according to the reference stars, located on the star image, using the connection between the Moon and star images. They were compared then with the reference positions. According to the difference of coordinates of control stars the observed value minus the calculated value (O - C) binding quality of the lunar image to the star image was defined. Similar differences are determined by the analysis of digital maps [11]. As a result, it was determined that on the average right ascension $(O - C)_\alpha = 0\overset{''}{.}04 \pm 0\overset{''}{.}13$ and in declination $(O - C)_\delta = 0\overset{''}{.}05 \pm 0\overset{''}{.}11$. Thus, it can be concluded that the binding of the system of two images with the help of labels is done with sufficiently high accuracy.

5. Conclusion

In this paper, the following results were obtained:

1) On the average for all lunar images the mean square error of measurements of lunar objects positions in the picture plane is about $20 \times 10^{-5} R_{☾}$.

2) With the height of the lunar mountain of 3 km tthe displacement caused by stereo effect on the lunar plates obtained using the telescope, with the difference of libration of 15^0, is approximately $50 \times 10^{-5} R_{☾}$.

3) The accuracy of separate images method when binding reference marks of two images have shown that their values during movement of one image in relation to the other one remain almost constant with an accuracy of $\sigma = \pm 0.5mk$.

4) Method of separate images can be successfully used in all cases where the observed objects have movement relative to each other and it is required to bind their coordinate systems having different orientation in space and different null points of reference data.

6. Summary

The method developed in this paper is to bind heterogeneous coordinate systems that are present on the two images obtained by the CCD-matrices and to determine the exact values of the spatial positions of objects on one image in the frame of reference of another image. In this paper an algorithm for reduction of images and their binding, evaluation of the accuracy of the determined data have been given. The results obtained can be used in the manufacture of various scientific and technical papers, that require a single coordinate-time provision to bind two moving in relation to each other systems.

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