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MATHEMATICAL SIMULATION OF DYNAMICS FOR A RING ROTATING ON SUPPORTS

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Abstract.

The article describes two mathematical models of the elastic ring dynamics when it rotates on supports in the course of processing according with mobile technology. One model uses the extendable center line hypothesis, the second - non-extendable. In this case, solution of partial differential equations that describe the dynamics of the ring, is very challenging. Thus, for a ring rotating on supports, using the non-extendable center line hypothesis, we obtain a partial differential equation of the sixth order, on the right side of which there is the second derivative of the delta function. Therefore, to obtain the equations and solve them, we perform the transition from a system of partial differential equations with constraints describing the behavior of the ring, to ordinary differential equations. We used the Lagrange formalism for derivation of the equations. The dependences have been obtained for cutting forces and internal friction forces acting along the generalized coordinates, taking into account the precession motion of the excited standing waves. The calculation results obtained by the models of natural vibration frequencies of the ring rotating on the supports at different speeds of rotation, are presented.

Key words: Mobile technology for processing large rings and shells; dynamics of a ring rotating on supports; standing wave, oscillation frequency of a ring rotating on supports; effect of a supported ring rotation speed on its natural oscillation frequency; internal friction of the rotating ring.

Introduction. When processing large rings and shells according to the mobile technology, they are put on two vertically supporting rollers and are rotated by an attachable machining module for the purpose of to obtain their desired shape. One of the most important tasks in this technology is the choice of process parameters to ensure the best accuracy of shaping the body being processed. Rings and shells have finite characteristics of rigidity and are large in size, so in the course of processing the body oscillations occur; they are caused by the action of cutting forces and disturbing factors due to non-ideal geometrical and physical characteristics. It is necessary to carry out calculations on dynamics of rings and shells rotating on supports under the influence of various disturbing factors in order to account for those oscillations in the practice of mobile technology.

The issues of rotating rings dynamics are considered mainly for a freely rotating ring; it is due to scientific works on hemispherical resonator gyros [1] - [7]. The number of works on dynamics of elastic ring rotating on supports is much less. One of the new questions arising in the solution of this problem is to find out the nature of the precession motion of standing waves generated in the ring due to perturbation effect [8], [10] what should be considered in the equations. The article deals with the mathematical simulation of the dynamics of a ring with ideal strength and geometric characteristics that rotates on supports.

Its behavior is described by partial differential equations with the conditions of constraint specifying zero radial displacements of the ring at the points of support. Two approaches are possible in solving the simulation problem. The first is accounting for extensibility of the ring center line, and the second is when the center line of the ring should be considered as non-extendable. For these two approaches, the solution of partial differential equations of the ring dynamics with constraints is a very problematic issue. Therefore, it is advisable to use an approach based on the reduction of partial differential equations to ordinary differential equations by introducing the generalized coordinates using Fourier series. We used the Lagrange formalism to obtain the equations.

Main part.

Let's obtain dependences describing the behavior of the ring with extendable center line of constant radius r rotating at a constant speed Ω on two support rollers; the ring is exposed to action of external forces and internal friction forces. Let's make the following assumptions to derive the equations. Support rollers have the shape of a cylinder with a constant radius.

Axes of the rollers are parallel; their position during the rotation process is unchangeable. Rotation axes of the ring in disturbed and undisturbed movements are the same. Let's consider the behavior of the ring using the flat cross-section hypothesis.

To describe the ring behavior, let's define a fixed coordinate system X_1, Y_1, Z_1 (Figure 1). Axis O_1Y_1 is the rotation axis of the ring, axis O_1Z_1 is directed vertically, and axis O_1X_1 is orthogonal to them.

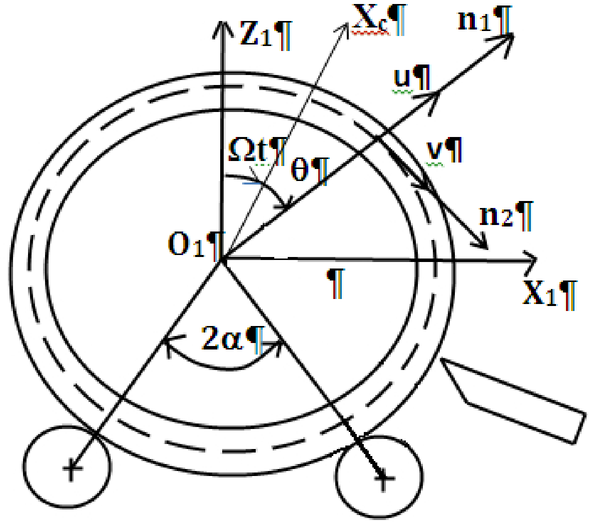

Figure 1. Coordinate systems.

Let's use a polar coordinate system θ, r to define the positions of center line points of the rotating ring where θ is the angle defining a point position on the ring center line relative to the axis O_1X_C associated with the rotating ring, r - the center line radius of the ring. The coordinate system origin is located in the center of the ring. The width of the ring is a, its thickness is h. Using this coordinate system, let's define the movement of any point of the ring during the deformation process by projections of its movement on axes n_1, n_2. The origin of coordinate system is located on the ring center line at a point defined by the angle θ. Axis n_1 is directed along the vector r, the axis n_2 is tangential to the center line and orthogonal to n_1. The angle between the supports is equal to 2α.

Let's denote the displacement projections for a point of the center line of the ring upon deformation to the vectors n_1, n_2, with $U(\theta), V(\theta)$, respectively

Let's set the values of displacements for points of the center line of the ring, taking into account its extensibility, in the form of Fourier series.

$$U = a_0 + \sum_{i=1}^{N} a_{ui} \cos\big(i(\theta + \varphi_{uj})\big) + \sum_{i=1}^{N} b_{ui} \sin\big(i(\theta + \varphi_{uj})\big), (1)$$

$$V = \sum_{i=1}^{N} a_{vi} \cos\big(i(\theta + \varphi_{vj})\big) + \sum_{i=1}^{N} b_{vi} \sin\big(i(\theta + \varphi_{vj})\big). (2)$$

Here $a_0, a_{ui}, b_{ui}, a_{vi}, b_{vi}$ are unknown functions of time t which is necessary to determine; they set amplitude of oscillations; $\varphi_{uj}, \varphi_{vj}$ are functions of time which set the precession of standing waves generated in the ring; N is the number of terms being considered of the Fourier series. Let's consider a dynamic problem that occurs when processing the rings according to the mobile technology. The locating chart for the ring and the machining module is shown in Fig. 2. During the processing, the machine and the ring are affected by cutting forces: radial, tangential and the one directed along the rotation axis of the ring. The values of those forces depend on the cutting depth t_R, the cutting speed which in turn depends on the angular speed of the ring rotation Ω and its radius r, and the material characteristics. When designing a mathematical models, it is assumed that the angular speed of the ring rotation is constant, and therefore the tangential cutting force causes displacement along the coordinate *V*, not affecting the ring rotation speed. The rings processed according to the mobile technology have a large mass, on the order of tons. When an axial cutting force affects the ring, its oscillations in the axial direction are substantially less than in radial and tangential, moreover, this force does not lead to a change in the cutting depth, and has a static effect on the ring behavior, so its effect on the dynamics can be ignored. Thus, when developing a mathematical model, it is necessary to take into account the dynamics of radial and tangential components of the cutting force.

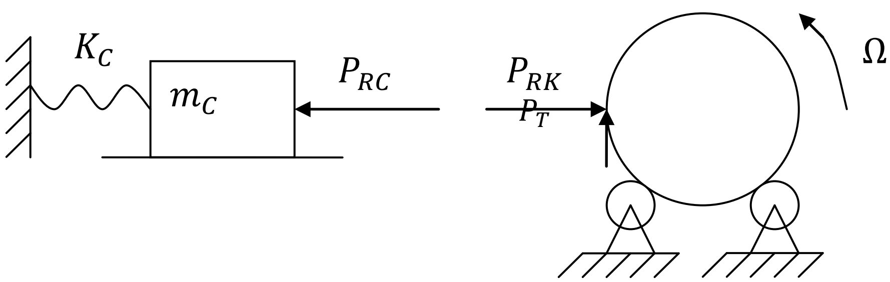

Fig. 2. Analytical model.

On the figure: K_C is the coefficient characterizing the degree of rigidity of the machining module with a cutter;

m_C - Equivalent weight of machining module;

P_{RC}, P_{RK} - Cutting forces affecting the machine tool and ring module, respectively;

P_T - Tangential cutting force;

Ω - Ring rotational speed.

One of the features of machining modules which are used for the processing of rings according to the mobile technology is their smaller rigidity, and smaller rigidity of technological devices wherein the tool is mounted, as compared with a stationary machine. The cause is that a machine for mobile processing should be portable, take a small volume, so it could be put in a place where the best accuracy is provided. These conditions impose restrictions on the size of the machine, and hence its characteristics of rigidity.

In the process of cutting, the ring and the machine are affected by cutting forces. Under their affecting, the rotating ring becomes deformed, and the cutter changes its position due to its deformation, deformation of its fastening system, the machine deformation, and deformation of the base for machine referencing. Deformation of the cutter compared to the other parts is substantially less, so it can be neglected. Let's replace the deformation of the remaining elements of the system with the total coordinate X_P. In this case, the analytical model for analysis of the dynamics could be presented in the form as depicted in Fig. 2. Formula of kinetic and potential energies of the system and their derivatives are necessary to obtain dynamic equations. According to Figure 2, the kinetic energy of the system consists of the kinetic energy of the machining module T_C and of the rotating ring T. Formula for potential energy of the system must take into account the potential energy of the elastic deformation of the machining module (coefficient K_C), and the potential energy of the ring.

The kinetic energy of the ring rotating on the supports should be defined according to the dependence [8]

$$T = \frac{r\rho F}{2}\int_0^{2\pi} \left[\left(\dot{V} + \Omega r + \Omega U\right)^2 + \left(\dot{U} - \Omega V\right)^2\right] d\theta, \ (3)$$

Where r, ρ $(po), F$ are center line radius of the ring, the specific density of the material, and the cross-sectional area, respectively.

The expression for the potential energy

$$\Pi_1 = \frac{\mu}{2}\int_0^{2\pi} \left(\frac{\partial V}{\partial\theta} - \frac{\partial^2 U}{\partial\theta^2}\right)^2 d\theta \, , \Pi_2 = \frac{\nu}{2}\int_0^{2\pi} \left(\frac{\partial V}{\partial\theta} + U\right)^2 d\theta \, , (4)$$

Where $\mu = \frac{EI}{r^3}$; $\nu = \frac{EF}{r}$; E is modulus of elasticity of the ring material, I - moment of inertia of the ring.

Let's designate the movement of the tool and machining module in the process of cutting X_P. Then the kinetic and potential energies of the machining module are

$$T_c = \frac{m_c \dot{X}_P^2}{2}, \ \Pi_c = \frac{K_c X_P^2}{2}$$

Let us find the dependences to account for the affect of cutting forces on the dynamics of the system. Analytical model of this system is shown in Fig. 2.

The cutting force value in the radial direction is determined from the formula [9]

$$P_R = 10C_p^p t_R^{XP} S^{YP} V^{np},$$

and in the tangential direction

$$P_T = 10C_p^T t_R^{XT} S^{YT} V^{nT},$$

Where: t_R - depth of cut; S - feed rate; V - cutting speed; $C_p^p, XP, YP, nP, C_p^T, XT, YT, nT$ - coefficients. In general, these relationships are non-linear.

A cutting depth change has the greatest influence on the dynamics. During processing, the cutting depth of the ring depends on its set point t_{R0}, the magnitude of the radial oscillations of the ring $U(t,\theta_R)$ at the cutter location which we will designate with the angular coordinate θ_R relative to the axis OZ $_1$ in the fixed system of coordinates, and the magnitude of change the position of the machine and the cutter under the action of radial force designated with the coordinate X_P .

We have the following to determine a cutting depth (Fig. 2)

$$t_R = t_{R0} + U(t,\theta_R) + X_P. \tag{5}$$

Let's designate the values of the cutting force in the radial and tangential directions, respectively, with $P_R(t_R), P_\tau(t_R)$. In accordance with (1), (2) and (5), the magnitude of their elementary work could be described with the dependence

$$\delta A^0 = \delta A_R^0 + \delta A_\tau^0 = -P_R\big(\delta X_P + \delta U(\theta_R)\big) - P_\tau \delta V(\theta_R) = -P_R\delta X_P - P_R\delta a_0 -$$

$$-\sum_{j=1}^{N} P_R \delta a_{uj} \cos\Big(j\big(\theta_R + \Omega t + \varphi_{uj}\big)\Big) - \sum_{j=1}^{N} P_R \delta b_{uj} \sin\Big(j\big(\theta_R + \Omega t + \varphi_{uj}\big)\Big) -$$

$$-\sum_{j=1}^{N} P_\tau \delta a_{vj}\, cos\Big(j\big(\theta_R + \Omega t + \varphi_{vj}\big)\Big) - \sum_{j=1}^{N} P_\tau \delta b_{vj}\, sin\Big(j\big(\theta_R + \Omega t + \varphi_{vj}\big)\Big).$$

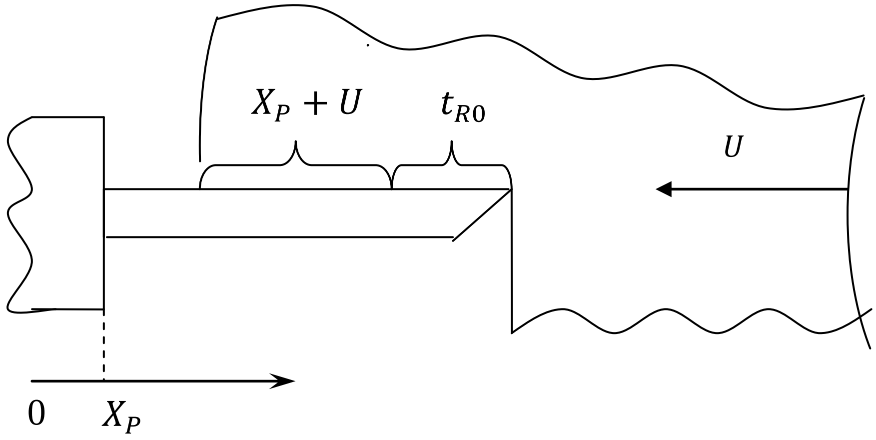

Fig. 3. Displacement of the system elements in the course of a cutting process.

From here, we get the following expression for the generalized cutting forces applied along the generalized coordinates.

$$Q_{XP}^{R} = -P_R, Q_{a0}^{R} = -P_R, Q_{auj}^{R} = -P_R \cos\left(j\left(\theta_R + \Omega t + \varphi_{uj}\right)\right),$$

$$Q_{buj}^{R} = -P_R \sin\left(j\left(\theta_R + \Omega t + \varphi_{uj}\right)\right), Q_{avj}^{\tau} = -P_\tau \cos\left(j\left(\theta_R + \Omega t + \varphi_{vj}\right)\right),$$

$$Q_{bvj}^{\tau} = -P_\tau \sin\left(j\left(\theta_R + \Omega t + \varphi_{vj}\right)\right)$$

Cutting force is generally a nonlinear function of the depth of cut.

In the formula for cutting depth,

$$U(t,\theta_R) = a_0 + \sum_{i=1}^{N} a_{ui} \cos\left(i\left(\theta_R + \Omega t + \varphi_{Uj}\right)\right) +$$

$$+ \sum_{i=1}^{N} b_{ui} \sin\left(i\left(\theta_R + \Omega t + \varphi_{Uj}\right)\right).$$

Let us find the dependence of the generalized internal friction forces applied along the generalized coordinates q_j. To calculate the stresses occurred in the material of the ring, we will use the Voigt law $\sigma = EK_T \frac{\partial \varepsilon}{\partial t}$, where K_T is an internal friction coefficient, and ε is a deformation.

When bending the ring, deformation of its layer at a distance z from the center line is related with the change of curvature by the dependence $\varepsilon = \frac{z\gamma}{r^2}$,

where $\gamma = \left(\frac{\partial V}{\partial \theta} - \frac{\partial^2 U}{\partial \theta^2}\right)$ is the change in the ring curvature. Hence we have $\frac{\partial \varepsilon}{\partial t} = \frac{z}{r^2}\frac{\partial \gamma}{\partial t}$.

Elastic moment of internal friction forces in a section of the ring

$$M = \int_S z\sigma dS = \frac{IEK_T}{r^2}\frac{\partial\gamma}{\partial t}$$

Where $I = \int_S z^2 dS$ is the second area moment of the ring

Elementary work of this moment on the curve of the elementary part of the ring is $\delta A^{\Delta\theta} = M\delta a,$ where $\delta a = \frac{\delta\gamma}{r^2} r\Delta\theta$ is a change in the angle value. Let's express the variation δγ through the variation of the generalized coordinate $q_j \delta\gamma = \frac{\partial\gamma}{\partial q_j}\delta q_j$. Then $\delta A_{uj}^{\Delta\theta} = \mu K_T \frac{\partial\gamma}{\partial t}\frac{\partial\gamma}{\partial q_j}\delta q_j \Delta\theta$ where $\mu = \frac{EI}{r^3}$.

The total work of the internal friction force around the ring corresponding to the variation of the coordinate q_j ,$\delta A_{q_j}^u = \mu K_T \int_0^{2\pi} \frac{\partial\gamma}{\partial t}\frac{\partial\gamma}{\partial q_j} d\theta \delta q_j$

Value $F_{q_j}^u = \mu K_T \int_0^{2\pi} \frac{\partial\gamma}{\partial t}\frac{\partial\gamma}{\partial q_j} d\theta$ is a generalized internal friction force applied alon the generalized coordinate q_j in the course of bending.

Similarly, we'll obtain the formula for the elementary work of the internal friction forces under tension of the ring

$$\delta A_{q_j}^P = \nu K_T \int_0^{2\pi} \left(\frac{\partial\beta}{\partial t}\frac{\partial\beta}{\partial q_j}\right) d\theta \delta q_j, где\ \beta = \frac{\partial V}{\partial\theta} + U, \nu = \frac{EF}{r}.$$

Generalized internal friction force applied along q_j under tension of the ring is determined by the formula $F_{q_j}^P = \nu K_T \int_0^{2\pi} \left(\frac{\partial\beta}{\partial t}\frac{\partial\beta}{\partial q_j}\right) d\theta.$

Let's represent in general terms the frictional force applied along the generalized coordinates a_0, a_{uj}, b_{uj} under tension. We can write

$$\frac{\partial\gamma}{\partial q_j} = \frac{\partial\gamma}{\partial V'}\frac{\partial V'}{\partial q_j} + \frac{\partial\gamma}{\partial U''}\frac{\partial U''}{\partial q_j}$$

Using these relationships, we have

$$F_{a_{uj}}^u = \mu K_T \int_0^{2\pi} \frac{\partial\gamma}{\partial t}\left[-\frac{\partial\gamma}{\partial U''} j^2 cos\ (j(\theta + \varphi_{uj}))\right] d\theta,$$

$$F_{b_{uj}}^u = \mu K_T \int_0^{2\pi} \frac{\partial\gamma}{\partial t}\left[-\frac{\partial\gamma}{\partial U''} j^2 sin\ (j(\theta + \varphi_{uj}))\right] d\theta,\ F_{a_0}^u = 0$$

Similarly, we'll obtain the formula for the frictional force $F_{q_j}^P$ caused by the ring tension. We have: $\beta = \frac{\partial V}{\partial Q} + U$; $\nu = \frac{EF}{r}$; $\frac{\partial \beta}{\partial a_0} = 1$

For generalized variables a_0, a_{uj}, b_{uj} we get:

$$F_{a_0}^P = \nu K_T \int_0^{2\pi} \frac{\partial \beta}{\partial t} d\theta;\ F_{au_j}^P = \nu K_T \int_0^{2\pi} \frac{\partial \beta}{\partial t}\frac{\partial \beta}{\partial U} \cos\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta;$$

$$F_{b_{uj}}^P = \nu K_T \int_0^{2\pi} \frac{\partial \beta}{\partial t}\frac{\partial \beta}{\partial V'} \sin\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta$$

When calculating the generalized friction forces $F_{q_j}^u$,$F_{q_j}^P$. in the case if the ring is non-rotating, the magnitudes $\frac{\partial \gamma}{\partial q_j}, \frac{\partial \beta}{\partial q_j}$ do not depend on time, so we can write

$$\int_0^{2\pi} \frac{\partial \gamma}{\partial t}\frac{\partial \gamma}{\partial q_j} d\theta = \frac{\partial}{\partial t}\int_0^{2\pi} \gamma \frac{\partial \gamma}{\partial q_j} d\theta = \frac{1}{2}\frac{\partial}{\partial t}\int_0^{2\pi} \frac{\partial}{\partial q_j}\gamma^2\, d\theta, (6)$$

$$\int_0^{2\pi} \frac{\partial \beta}{\partial t}\frac{\partial \beta}{\partial q_j} d\theta = \frac{1}{2}\frac{\partial}{\partial t}\int_0^{2\pi} \frac{\partial}{\partial q_j}\beta^2\, d\theta. (7)$$

Hence, the well-known result follows that generalized forces of friction can be calculated for the non-rotating ring by differentiating with respect to time the expressions for the derivatives of the potential energy in generalized coordinates. In this case, if the there is a standing wave procession, magnitudes $\frac{\partial \gamma}{\partial q_j}, \frac{\partial \beta}{\partial q_j}$ depend on functions $\varphi_{uj}, \varphi_{vj}$ and consequently on time. Therefore, the use of dependencies (6), (7) is incorrect in our case, and the formulas for $F_{q_j}^u$, $F_{q_j}^P$. should be used.

Summarizing the relations for the friction forces, we get:

$$F_{a_{uj}}^u + F_{a_{uj}}^P = \mu K_T \int_0^{2\pi} \frac{\partial \gamma}{\partial t}\left[-\frac{\partial \gamma}{\partial U''} j^2 \cos\left(j\left(\theta + \varphi_{uj}\right)\right)\right] d\theta\ +$$

$$+ \nu K_T \int_0^{2\pi} \frac{\partial \beta}{\partial t}\frac{\partial \beta}{\partial U} \cos\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta;$$

$$F^u_{b_{uj}} + F^P_{b_{uj}} = \mu K_T \int_0^{2\pi} \frac{\partial \gamma}{\partial t}\left[-\frac{\partial \gamma}{\partial U"} j^2 \sin\left(j\left(\theta + \varphi_{uj}\right)\right)\right] d\theta +$$

$$+ \nu K_T \int_0^{2\pi} \frac{\partial \beta}{\partial t}\frac{\partial \beta}{\partial V'} \sin\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta;$$

Calculating the integrals in the formulas for the friction forces, we get the final dependencies to obtain bending values

$$F^u_{a0} = 0, F^u_{auj} = \mu K_T \pi(-\dot{a}_{vj} j^3 K^{vu}_j - a_{vj} j^4 \dot{\varphi}_{vj} C^{vu}_j + \dot{b}_{vj} j^3 C^{vu}_j - b_{vj} j^4 \dot{\varphi}_{vj} K^{vu}_j +$$

$$+\dot{a}_{Uj} j^4 + b_{Uj} j^5 \dot{\varphi}_{Uj}),$$

$$F^u_{buj} = \mu K_T \pi(-\dot{a}_{vj} j^3 C^{vu}_j - a_{vj} j^4 \dot{\varphi}_{vj} K^{vu}_j + \dot{b}_{vj} j^3 K^{vu}_j - b_{vj} j^4 \dot{\varphi}_{vj} C^{vu}_j -$$

$$-a_{uj} j^5 \dot{\varphi}_{uj} + \dot{b}_{uj} j^4).$$

For the friction forces caused by extension of the ring, we get

$$F^P_{a0} = 2\nu K_T \pi \dot{a}_0, F^P_{auj} = \nu K_T \pi(-\dot{a}_{vj} j K^{vu}_j - a_{vj} j^2 \dot{\varphi}_{vj} C^{vu}_j + \dot{b}_{vj} j C^{vu}_j -$$

$$-b_{vj} j^2 \dot{\varphi}_{vj} K^{vu}_j + \dot{a}_{Uj} + b_{Uj} j \dot{\varphi}_{Uj}),$$

$$F^P_{buj} = \nu K_T \pi(-\dot{a}_{vj} j C^{vu}_j - a_{vj} j^2 \dot{\varphi}_{vj} K^{vu}_j + \dot{b}_{vj} j K^{vu}_j - b_{vj} j^2 \dot{\varphi}_{vj} C^{vu}_j +$$

$$+\dot{b}_{uj} - a_{uj} j \dot{\varphi}_{uj})$$

For internal friction forces applied along the generalized coordinates a_{vj}, b_{vj} we obtain for the bending and extension, similarly

$$F^u_{a_{vj}} = \mu K_T \int_0^{2\pi} \frac{\partial \gamma}{\partial t}(-j sin\left(j\left(\theta + \varphi_{vj}\right)\right)) d\theta = \pi \mu K_T(\dot{a}_{vj} j^2 + b_{vj} j^3 \dot{\varphi}_{vj} -$$

$$-\dot{a}_{uj} j^3 K^{uv}_j + a_{uj} j^4 \dot{\varphi}_{uj} C^{uv}_j - \dot{b}_{uj} j^3 C^{uv}_j - b_{uj} j^4 \dot{\varphi}_{uj} K^{uv}_j),$$

$$F^u_{b_{vj}} = \mu K_T \int_0^{2\pi} \frac{\partial \gamma}{\partial t} j cos\left(j\left(\theta + \varphi_{vj}\right)\right) d\theta = \pi \mu K_T(-a_{vj} j^3 \dot{\varphi}_{vj} + \dot{b}_{vj} j^2 +$$

$$+\dot{a}_{uj} j^3 C^{uv}_j - a_{uj} j^4 \dot{\varphi}_{uj} K^{uv}_j + \dot{b}_{uj} j^3 K^{uv}_j + b_{uj} j^4 \dot{\varphi}_{uj} C^{uv}_j),$$

$$F^p_{a_{vj}} = \nu K_T \int_0^{2\pi} \frac{\partial \beta}{\partial t}\left(- j sin\left(j\left(\theta + \varphi_{vj}\right)\right)\right) d\theta =$$

$$= \pi\nu K_T(\dot{a}_{vj}j^2 + b_{vj}j^3\dot{\varphi}_{vj} - \dot{a}_{uj}jK_j^{uv} +$$

$$+a_{uj}j^2\dot{\varphi}_{uj}C_j^{uv} - \dot{b}_{uj}jC_j^{uv} - b_{uj}j^2\dot{\varphi}_{uj}K_j^{uv}),$$

$$F_{b_{vj}}^p = \nu K_T\int_0^{2\pi}\frac{\partial\beta}{\partial t}j\cos\left(j\left(\theta+\varphi_{vj}\right)\right)d\theta = \pi\nu K_T(-a_{vj}j^3\dot{\varphi}_{vj} + \dot{b}_{vj}j^2 +$$

$$+\dot{a}_{uj}jC_j^{uv} - a_{uj}j^2\dot{\varphi}_{uj}K_j^{uv} + \dot{b}_{uj}jK_j^{uv} + b_{uj}j^2\dot{\varphi}_{uj}C_j^{uv})$$

When calculating the integrals we have used the dependencies

$$C_i^{pq} = \cos\left(i\left(\varphi_{pi}-\varphi_{qi}\right)\right),\ K_i^{pq} = \sin\left(i\left(\varphi_{pi}-\varphi_{qi}\right)\right)$$

Constraints are superimposed on the generalized coordinates rings due to the availability of support points. The constraint equations are of the form

$$a_0 + \sum_{i=1}^{N} a_{ui}\cos\left(i(\pi-\propto+\Omega t+\varphi_{Uj})\right) + \sum_{i=1}^{N} b_{ui}\sin\left(i(\pi-\propto+\Omega t+\varphi_{Uj})\right) = 0,$$

$$a_0 + \sum_{i=1}^{N} a_{ui}\cos\left(i(\pi+\propto+\Omega t+\varphi_{Uj})\right) + \sum_{i=1}^{N} b_{ui}\sin\left(i(\pi+\propto+\Omega t+\varphi_{Uj})\right) = 0.$$

Using the Lagrange formalism, conditions of constraints, the formulas (3), (4) for the forces, we'll get the system of differential equations with undetermined Lagrange multipliers describing the dynamics of the machine unit and the processed ring, and taking into account the precession motion of standing waves generated in the ring due to the affect of disturbing factors. To obtain final dynamics equations for the ring, let's exclude from this system the undetermined Lagrange multipliers, take into account the conditions of constraints, and use dependence relations [10] for the precession motion of standing waves $\varphi_{uj} = \varphi_{vj} = -\varphi_k, (\phi u)\ \dot{\varphi}_{uj} = \dot{\varphi}_{vj} = -\Omega$ (j = 1, ..., N), where φ_k is the ring rotation angle, Ω - the ring rotational speed. The system takes the form

$$\ddot{a}_0 - \frac{1}{2\cos(\pi-\alpha)}\ddot{a}_{u1} + \frac{K_T\nu}{æ}\dot{a}_0 - \frac{K_T(\mu+\nu)}{2æ\cos(\pi-\alpha)}\dot{a}_{u1} + \frac{\Omega}{\cos(\pi-\alpha)}\dot{b}_{u1} +$$

$$+\frac{\Omega}{\cos(\pi-\alpha)}\dot{a}_{v1} - \frac{K_T(\mu+\nu)}{2æ\cos(\pi-\alpha)}\dot{b}_{v1} + \frac{(\nu-æ\Omega^2)}{æ}a_0 - \frac{(\nu+\mu-2æ\Omega^2)}{2æ\cos(\pi-\alpha)}a_{u1} +$$

$$+\frac{K_T\Omega(\mu+\nu)}{2æ\cos(\pi-\alpha)}b_{u1} - \frac{K_T\Omega(\mu+\nu)}{2æ\cos(\pi-\alpha)}a_{v1} - \frac{(\nu+\mu+2æ\Omega^2)}{2æ\cos(\pi-\alpha)}b_{v1} =$$

$$= \Omega^2 r + \frac{P_R}{2\pi æ}\left(\frac{cos(\theta_R)}{cos(\pi - \alpha)} - 1\right),$$

$$c_j \ddot{a}_{u1} + \ddot{a}_{uj} + K_T c_j \frac{(\mu + \nu)}{æ}\dot{a}_{u1} + K_T \frac{(\mu j^4 + \nu)}{æ}\dot{a}_{uj} - 2\Omega c_j \dot{b}_{u1} - 2\Omega j \dot{b}_{uj} -$$

$$-2\Omega c_j \dot{a}_{v1} - 2\Omega \dot{a}_{vj} + K_T \frac{(\mu + \nu)}{æ} c_j \dot{b}_{v1} + K_T j \frac{(\mu j^2 + \nu)}{æ}\dot{b}_{vj} +$$

$$+\frac{(\nu + \mu - 2æ\Omega^2)}{æ} c_j a_{u1} + \frac{\left(\nu + \mu j^4 - æ\Omega^2(1 + j^2)\right)}{æ} a_{uj} - K_T \Omega \frac{(\mu + \nu)}{æ} c_j b_{u1} -$$

$$-K_T \Omega j \frac{(\mu j^4 + \nu)}{æ} b_{uj} + K_T \Omega \frac{(\mu + \nu)}{æ} c_j a_{v1} + K_T \Omega j^2 \frac{(\mu j^2 + \nu)}{æ} a_{vj} +$$

$$+\frac{(\nu + \mu + 2æ\Omega^2)}{æ} c_j b_{v1} + j\frac{(\nu + \mu j^2 + 2æ\Omega^2)}{æ} b_{vj} =$$

$$= -\frac{P_R}{\pi æ}\left(c_j\, cos(\theta_R) + cos(j\theta_R)\right),$$

$$s_j \ddot{b}_{u1} + \ddot{b}_{uj} + 2\Omega s_j \dot{a}_{u1} + 2\Omega j \dot{a}_{uj} + K_T \frac{(\mu + \nu)}{æ} s_j \dot{b}_{u1} + K_T \frac{(\mu j^4 + \nu)}{æ}\dot{b}_{uj} -$$

$$-K_T \frac{(\mu + \nu)}{æ} s_j \dot{a}_{v1} - K_T j \frac{(\mu j^2 + \nu)}{æ}\dot{a}_{vj} - 2\Omega s_j \dot{b}_{v1} - 2\Omega \dot{b}_{vj} +$$

$$+K_T \Omega \frac{(\mu + \nu)}{æ} s_j a_{u1} + K_T j \Omega \frac{(\mu j^4 + \nu)}{æ} a_{uj} + \frac{(\nu + \mu - 2æ\Omega^2)}{æ} s_j b_{u1} +$$

$$+\frac{\left(\nu + \mu j^4 - æ\Omega^2(1 + j^2)\right)}{æ} b_{uj} - \frac{(2æ\Omega^2 + \nu + \mu)}{æ} s_j a_{v1} -$$

$$-j\frac{(2æ\Omega^2 + \nu + \mu j^2)}{æ} a_{vj} + K_T \Omega \frac{(\mu + \nu)}{æ} s_j b_{v1} + K_T \Omega j^2 \frac{(\mu j^2 + \nu)}{æ} b_{vj} =$$

$$= -\frac{P_R}{\pi æ}\left(s_j\, sin(\theta_R) + sin(j\theta_R)\right), (j = 2,3, \dots, N-1),$$

$$c_N \ddot{a}_{u1} + W_1 \ddot{p} + K_T c_N \frac{(\mu + \nu)}{æ}\dot{a}_{u1} + K_T \frac{(\mu N^4 + \nu)}{æ} W_1 \dot{p} - 2\Omega c_N \dot{b}_{u1} -$$

$$-2\Omega N W_2 \dot{p} - 2\Omega c_N \dot{a}_{v1} - 2\Omega \dot{a}_{vN} + K_T \frac{(\mu + \nu)}{æ} c_N \dot{b}_{v1} + K_T N \frac{(\mu N^2 + \nu)}{æ}\dot{b}_{vN} +$$

$$+\frac{(\nu + \mu - 2æ\Omega^2)}{æ} c_N a_{u1} + \frac{\left(\nu + \mu N^4 - æ\Omega^2(1 + N^2)\right)}{æ} W_1 p -$$

$$-K_T \Omega \frac{(\mu + \nu)}{æ} c_N b_{u1} - K_T \Omega N \frac{(\mu N^4 + \nu)}{æ} W_2 p + K_T \Omega \frac{(\mu + \nu)}{æ} c_N a_{v1} +$$

$$+K_T\Omega N^2\frac{(\mu N^2+\nu)}{æ}a_{vN}+\frac{(\nu+\mu+2æ\Omega^2)}{æ}c_N b_{v1}+N\frac{(\nu+\mu N^2+2æ\Omega^2)}{æ}b_{vN}=$$

$$=-\frac{P_R}{\pi æ}(c_N\cos(\theta_R)+\cos(N\theta_R)),$$

$$s_N\ddot{b}_{U1}+W_2\ddot{p}+2\Omega s_N\dot{a}_{u1}+2\Omega N W_1\dot{p}+K_T\frac{(\mu+\nu)}{æ}s_N\dot{b}_{u1}+K_T\frac{(\mu N^4+\nu)}{æ}W_2\dot{p}-$$

$$-K_T\frac{(\mu+\nu)}{æ}s_N\dot{a}_{v1}-K_T N\frac{(\mu N^2+\nu)}{æ}\dot{a}_{vN}-2\Omega s_N\dot{b}_{v1}-2\Omega\dot{b}_{vN}+$$

$$+K_T\Omega\frac{(\mu+\nu)}{æ}s_N a_{u1}+K_T N\Omega\frac{(\mu N^4+\nu)}{æ}W_1 p+\frac{(\nu+\mu-2æ\Omega^2)}{æ}s_N b_{u1}+$$

$$+\frac{\left(\nu+\mu N^4-æ\Omega^2(1+N^2)\right)}{æ}W_2 p-\frac{(2æ\Omega^2+\nu+\mu)}{æ}s_N a_{v1}-$$

$$-N\frac{(2æ\Omega^2+\nu+\mu N^2)}{æ}a_{vN}+K_T\Omega\frac{(\mu+\nu)}{æ}s_N b_{v1}+K_T\Omega N^2\frac{(\mu N^2+\nu)}{æ}b_{vN}=$$

$$=-\frac{P_R}{\pi æ}(s_N\sin(\theta_R)+\sin(N\theta_R)),(j=2,3,\dots,N-1),$$

$$\ddot{a}_{vj}+2\Omega\dot{a}_{uj}-K_T j\frac{(\mu j^2+\nu)}{æ}\dot{b}_{uj}+K_T j^2\frac{(\mu+\nu)}{æ}\dot{a}_{vj}-2j\Omega\dot{b}_{vj}-$$

$$-K_T j^2\Omega\frac{(\mu j^2+\nu)}{æ}a_{uj}-j\frac{\left(2æ\Omega^2+(\mu j^2+\nu)\right)}{æ}b_{uj}+$$

$$+\frac{\left(j^2(\nu+\mu)-æ\Omega^2(1+j^2)\right)}{æ}a_{vj}-K_T j^3\Omega\frac{(\mu+\nu)}{æ}b_{vj}=-\frac{P_\tau\cos(j\theta_R)}{\pi æ},$$

$$\ddot{b}_{vj}+K_T j\frac{(\mu j^2+\nu)}{æ}\dot{a}_{uj}+2\Omega\dot{b}_{uj}+2j\Omega\dot{a}_{vj}+K_T j^2\frac{(\mu+\nu)}{æ}\dot{b}_{vj}+$$

$$+j\frac{(\mu j^2+\nu+2æ\Omega^2)}{æ}a_{uj}-K_T j^2\Omega\frac{(\mu j^2+\nu)}{æ}b_{uj}+K_T j^3\Omega\frac{(\mu+\nu)}{æ}a_{vj}+$$

$$+\frac{\left(j^2(\nu+\mu)-æ\Omega^2(1+j^2)\right)}{æ}b_{vj}=-\frac{P_\tau\sin(j\theta_R)}{\pi æ},(j=1,2,\dots,N-1),$$

$$\ddot{a}_{vN}+2\Omega W_1\dot{p}-K_T N\frac{(\mu N^2+\nu)}{æ}W_2\dot{p}+K_T N^2\frac{(\mu+\nu)}{æ}\dot{a}_{vN}-2N\Omega\dot{b}_{vN}-$$

$$-K_T N^2\Omega\frac{(\mu N^2+\nu)}{æ}W_1 p-N\frac{\left(2æ\Omega^2+(\mu N^2+\nu)\right)}{æ}W_2 p+$$

$$+\frac{\left(N^2(\nu+\mu)-æ\Omega^2(1+N^2)\right)}{æ}a_{vN}-K_T N^3\Omega\frac{(\mu+\nu)}{æ}b_{vN}=-\frac{P_\tau\cos(N\theta_R)}{\pi æ},$$

$$\ddot{b}_{\nu N} + K_T N\frac{(\mu N^2+\nu)}{æ}W_1\dot{p} + 2\Omega W_2\dot{p} + 2N\Omega\dot{a}_{\nu N} + K_T N^2\frac{(\mu+\nu)}{æ}\dot{b}_{\nu N} +$$

$$+N\frac{\left(\mu N^2+\nu+2æ\Omega^2\right)}{æ}W_1 p - K_T N^2\Omega\frac{(\mu N^2+\nu)}{æ}W_2 p + K_T N^3\Omega\frac{(\mu+\nu)}{æ}a_{\nu N} +$$

$$+\frac{\left(N^2(\nu+\mu) - æ\Omega^2(1+N^2)\right)}{æ}b_{VN} = -\frac{P_\tau\sin(N\theta_R)}{\pi æ},$$

$$m_C\ddot{x}_P + K_{xP}^T\dot{x}_P + K_C x_P = -P_R.$$

Let's get similar equations using the hypothesis of non-extendable center line of the ring. To this end, the radial displacement value should be represented as

$$U = \sum_{i=1}^{N} a_{ui}\cos\left(i\left(\theta+\varphi_{ui}(t)\right)\right) + \sum_{i=1}^{N} b_{ui}\,sin\left(i\left(\theta+\varphi_{ui}(t)\right)\right).$$

The tangential movement value for the points of the ring will be found from the dependence which is widely used in the elasticity theory [2]$V \approx -\int U d\theta$. Then we get

$$V = -\sum_{i=1}^{N}\frac{a_{ui}}{i}\sin\left(i\left(\theta+\varphi_{ui}(t)\right)\right) + \sum_{i=1}^{N}\frac{b_{ui}}{i}\cos\left(i\left(\theta+\varphi_{ui}(t)\right)\right)$$

Using the same approach as for the derivation of equations for extendable center line ring, we'll obtain a final set of equations for behavior of a non-extendable centerline ring

$$2C_j\ddot{a}_{u1} + L_j\ddot{a}_{uj} + \nu K_T n_j^2\dot{a}_{uj} - 8\mathit{\Omega} C_j\dot{b}_{u1} - m_j\mathit{\Omega}\dot{b}_{uj} -$$

$$-8C_j\mathit{\Omega}^2 a_{u1} + \left(\nu n_j^2 - \mathit{\Omega}^2 l_j\right)a_{uj} - \nu K_T j n_j^2\mathit{\Omega} b_{uj} =$$

$$= (\pi\varkappa)^{-1}[C_j F_{a1} - P_R\,cos(j\theta_R) + P_\tau j^{-1}\,sin(j\theta_R)],$$

$$2S_j\ddot{b}_{u1} + L_j\ddot{b}_{uj} + 8\mathit{\Omega} S_j\dot{a}_{u1} + m_j\mathit{\Omega}\dot{a}_{uj} + \nu K_T n_j^2\dot{b}_{uj} +$$

$$+\nu K_T j n_j^2\mathit{\Omega} a_{uj} - 8S_j\mathit{\Omega}^2 b_{u1} + (\nu n_j^2 - \mathit{\Omega}^2 l_j)\,b_{uj} =$$

$$= (\pi\varkappa)^{-1}[S_j F_{b1} - P_R\sin(j\theta_R) - P_\tau j^{-1}\cos(j\theta_R)],$$

$$(j = 2, 3, \ldots, N-1),$$

$$2C_N\ddot{a}_{u1} + L_N w_1\ddot{p} + (\nu K_T n_N^2 w_1 - m_N\mathit{\Omega} W_2)\dot{p} - 8C_N\mathit{\Omega}\dot{b}_{u1} -$$

$$-8C_N\mathit{\Omega}^2 a_{u1} + \left((\nu n_N^2 - \mathit{\Omega}^2 l_N\right)w_1 - \nu K_T N n_N^2\mathit{\Omega} w_2)P =$$

$= (\pi\varkappa)^{-1}[C_N F_{a1} - P_R\, cos(N\theta_R) + P_\tau N^{-1}\, sin(N\theta_R)],$

$$2S_N \ddot{b}_{u1} + L_N w_2 \ddot{p} \quad + 8S_N \Omega \dot{a}_{u1} + (\nu K_T n_N^2 w_2 + m_N \Omega w_1)\dot{p} \quad +$$

$$+\left((\nu n_N^2 - \Omega^2 l_N\,)w_2 + \nu K_T N n_N^2 \Omega w_1\right)P - 8S_N \Omega^2 b_{u1} =$$

$= (\pi\varkappa)^{-1}[S_N F_{b1} - P_R \sin(N\theta_R) - P_\tau N^{-1}\, cos(N\theta_R)],$

$$m_c \ddot{X}_p + K_{Xp}^T \dot{X}_p + K_c X_p = -P_R$$

Where $L_j=1+1/j^2$, $K_j=j+1/j$, $m_j=1+j^2$, $n_j=j^2-1$,

$$p^T = [a_{u1} a_{u2} \dots a_{u,N-1} \quad b_{u1} b_{u2} \dots b_{u,N-1}],$$

$$C_j = \frac{(-1)^{j+1}\, cos(j\alpha)}{cos(\pi - \alpha)}, S_j = \frac{(-1)^j\, sin(j\alpha)}{sin(\pi - \alpha)},$$

w_1, w_2 are the first and second line of R and M matrixes product, respectively, where

$$R = \begin{bmatrix} cos\left(N(\pi-\alpha)\right) & sin\left(N(\pi-\alpha)\right) \\ cos\left(N(\pi+\alpha)\right) & sin\left(N(\pi+\alpha)\right) \end{bmatrix}^{-1},$$

$$M = \begin{bmatrix} -cos\left(j(\pi-\alpha)\right), (j = 1, 2, \dots, N-1) & -sin\left(j(\pi-\alpha)\right), (j = 1, 2, \dots, N-1) \\ -cos\left(j(\pi+\alpha)\right), (j = 1, 2, \dots, N-1) & -sin\left(j(\pi+\alpha)\right), (j = 1, 2, \dots, N-1) \end{bmatrix}$$

There have been carried out studies of natural frequencies values for a ring rotating on support that were calculated according to different models. Ring parameters: the center line radius - 3m; width of the ring - 1 m; ring thickness - 0.05m; modulus of elasticity - 2 10 ^{11} N / m ^{2}. The angle between supports - 80 degrees. Results of the studies are presented in the tables. The values of the ring rotational speed and its natural oscillation frequencies are given in radians per second.

Table 1. Consideration of the ring center line extensibility.

Rotational speed of the ring	0	2.	4	6	8	10
Natural oscillation frequency	31.1	30.6	29.4	28.8	28.0	16.57

Table 2. Consideration of the ring center line non-extensibility.

Rotational speed of the	0	2.	4	6	8	10

ring						
Natural oscillation frequency	30.6	26.9	15.5	7.7	3.9	$2\ 10^{-5}$

Analysis of the results shows that at low ring rotation speeds the calculation results for different models are close to each other. With the increase of the ring rotation speed, a difference between the values of the calculated natural frequencies increases. The cause is probably the fact that upon obtaining the equations for the ring with non-extendable centerline an approximate linear relationship between the variables U and V has been taken from their constraint equation. More correct is to obtain such equations from the equations of the ring with extendable center line by integrating into them the conditions of constraint.

At the same time, the resulting equations for the ring with non-extendable center line may be used for calculations at low rotational speeds of the ring. Their use is particularly convenient due to ease in the study of the influence of various disturbing factors such as variable ring thickness along its perimeter, curvature, modulus of elasticity, etc., on the dynamics of the ring in the course of shaping process.

Resume

The dynamic equations for a ring rotating on the supports have been obtained using hypotheses of extendable and non-extendable center line. Upon obtaining the equations, the transition from a system of partial differential equations to ordinary differential equations by the introduction of the generalized coordinates was used. This has simplified the process on obtaining the decision.

The equations have been obtained with the use of the Lagrange formalism. The equations considered the cutting forces applied to the ring, and internal friction forces which take into account the precessional motion of standing waves within the ring. The system was converted in order to eliminate Lagrange multipliers. The calculations according to the obtained models have been carried out.

They have shown that at low ring rotation speeds the results of calculations are close to each other, and with an increase in the rotational speed difference substantially increases, too. This is due to the method used for taking into account the relation between radial and tangential movements of the ring points.

Conclusions

The mathematical models for calculating the dynamics of a ring rotating on the supports when it is processed according to the mobile technology with the purpose of shaping have been developed. The calculation results according to the obtained models are represented. Results of the research may be used for mathematical simulation of shaping quality upon the processing of rings using the mobile technology.

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References

1. Bryan, G. H., 1890. On the beats in the vibrations of a revolving cylinder or bell. Proceedings of the Cambridge Philosophical Society, 7: 101 -111.
2. Zhuravlev V.F. and D. M. Klimov, 1985. The hemispherical resonator gyro. M .: Nauka, pp. 126.
3. Zhuravlev V.F. 1993. Theoretical foundations of hemispherical resonator gyros (HRG). RAN Bulletin. MTT, 3: 8 - 17.
4. Egarmin, N.E. 1993. Nonlinear effects in the dynamics of a rotating circular ring. RAN Bulletin. MTT. 3: 50 - 59.
5. Wilke, V.G. 1988. Nonlinear oscillations of an elastic extendable rotating ring. MGU Bulletin. Mathematics, Mechanics. 5: 31 – 35
6. Basarab, M.A., Kravchenko V.F., Matveev V.A., 2005. Mathematical simulation of physical processes in gyroscopy. M .: Radio Engineering, pp. 176.
7. Donnik, A. S. 2003. Natural oscillations of a hemispherical shell of a hemispherical resonator gyro. Proceedings of international conference "Information technologies and means". In 3 volumes, V. 2. M .: Yanus - K, pp. 60 - 63.
8. Polunin, A., 2011.Dynamics of rotating shells. Mathematic modeling of dynamics of an elastic rotating shell with supports. LAP LAMBERT Academic Publishing, Saarbrucken, Germany, pp: 148.
9. Handbook for Production and Mechanical Engineer: In 2 volumes / ed. by A.G. Kosilova, R.K. Meshcheryakova. M.: Mechanical Engineering, 1985, pp: 496.
10. Polunin, A., 2015. About The Motion Nature Of Standing Waves With Supporting Ring Variable Speed. International Journal of Applied Engineering Research , 10 (24) 44992-45002.