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DETERMINATION OF THE AERODYNAMIC DRAG FACTOR OF THE INCIDENT PARTICLES OF A MULTIFRACTIONAL MATERIAL

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Abstract.

A significant part of processes in the mining and processing industry involves feeding of loose material in a variety of containers and bunkers. Charging the silo bunkers with the help of self-discharging cars causes a particularly acute problem of dusting from the feeding openings. This paper is aimed at improving the calculation methods of aspiration for charging the silo bunkers with a polydisperse material. A new statistical approach has been proposed to the consideration of the effect of constraint conditions on the drag factor of the particles in the free stream of a multifractional material. During free fall of multifractional material jet, the larger particles obscure small and dust particles, which aerodynamic drag is insignificant in these conditions. We performed an analytical derivation of the active aerodynamic drag factor of the particles being outside the aerodynamic shadow, which allowed us to obtain a methodology for assessing the ejecting ability of the particles flow at silo bunker dedusting. Considering the dispersability of the transshipped material will allow calculating the optimum amount of aspirated air and reducing energy consumption of the aspiration systems.

Keywords: Bunker aspiration, bunker charging, polydisperse material, air ejecting, particles dynamics, aerodynamics, dedusting ventilation

Introduction. A significant part of processes in the mining and processing industry involves feeding of loose material in a variety of containers and bunkers.

Aspirated air flow from the local suction systems is the main parameter that determines the energy consumption and operating costs of the exhaust ventilation system.

Existing methods for determining the ejected air flow suggest replacing the real polydisperse material with a monofractional one with the particles of medium diameter. Most of the processed materials are polydisperse, which causes the need for the trial coefficients.

In the case of a large area of the feeding openings, which is typical of the silo bunkers, a protective rate is usually applied equal to 0.5-1 m/sec, preventing from dusting from the bunker. This approach results in significant aspiration air flow rate, taking into account neither the features of the transshipped material nor aerodynamic processes occurring inside the bunker. This urgently requires clarification of the existing methods of calculation and design of aspiration systems at the enterprises of the mining and processing industry.

The fundamentals of the theory of air ejection with uniformly accelerated flow of incident particles were established by S.E. Butakov [1] and developed by O.D. Neikov [2]. In recent years in Russia, V.A. Minko, I.N. Logachev [3], K.I. Logachev [4] have dealt with the problem of reducing the volume of aspirated air and studied the patterns of movement of bulk materials through the channels. The same problems of bunkers aspiration were considered by Semineko A.S., Goltsov A.B. [5, 6]. Foreign researches were Olifer V.D. [7], Ze Qin Liu [8]. However, these researchers consider mainly the monofractional flows, while most of the recyclable materials are multifractional.

Main part. Aerodynamic interaction of the collective particles in the jet of material and ejected air, as is known [4], is determined by the sum of the aerodynamic forces of all the collective particles per unit volume $dV = S \cdot dx$:

$$dA = c \cdot R \cdot S \cdot dx, \qquad (1)$$

where $R = \psi \cdot F_{м} \cdot \rho \frac{(\upsilon - u)^2}{2}$ - the aerodynamic force of a single particle, H; [rho] - air density, kg/m^3; c - particles count, 1/m^3; dV - elementary volume of a two-component medium "solids - air", m^3; $F_{м}$ - the mid-section area of the particle, m^2; u - ejected air velocity, m/s; [psi] - particle drag factor depending generally on the geometry, the relative motion of the Reynolds number and the volume concentration of the particles.

Unlike an adequately studied mechanism of intercomponent interaction in the flow of size-average particles, we will consider the aerodynamic interaction of the flow of particles with different equivalent diameters $d_1 > d_2 > ...d_N$, i.e., a multifractional flow of bulk material, consisting of *N* fractions.

We will consider an accelerated flow of freely falling particles, which volume concentration decreases as the material moves from the "bulk" (during transportation on the conveyor or at the time of its dropping)

$$\beta_{н} = \frac{\rho_{нч}}{\rho_{ч}}$$

to "sparse" (during dropping)

$$\beta = \frac{G}{S \cdot \rho_m \cdot \upsilon}, \qquad (2)$$

where G - the mass flow of falling particles, kg/sec; S - cross-sectional area of the jet of particles, m^2; υ - particle falling velocity, m/s; [rho-m], [rho-nh] - respectively, density of the particulate material and its bulk density, kg/m^3.

To avoid introducing the empirical corrections for coefficient ψ *, associated with the tightness of the particles during their fall we shall use a probabilistic approach. Since these corrections are usually associated with the volume concentration of the monofractional particles flow rather than actual flow of particles of different sizes. Let us suggest that the drag factor of the fraction *i* particles is proportional to the probability of the active aerodynamic interaction of the particle P_{ai}:

$$\psi_i = K_p \cdot \psi_{0i} \cdot P_{ai}; \qquad (3)$$

$$P_{ai} = 1 - P_i(T_i),$$

where $K_p \le 1$ - proportionality coefficient (hereinafter the proportionality coefficient K_p is equal to unity); $P_i(T_i)$ - the probability of presence of the particle in the aerodynamic shadow; [psi-0i] - drag factor of a single (free, not shaded) the fraction *i* particle.

Thus, a probability P_{ai} shall mean the ratio of the number of particles outside the aerodynamic shadow to the total number of particles in the considered volume of the two-component mixture. Its value in the process of particles falling will increase from almost zero (at the moment of dropping, when all particles are in the aerodynamic shadow of neighboring particles) up to a value close to unity (at a very sparse collective particles falling at a high speed).

We shall consider the case of the fall of the collective particles of the same weight.

We assume that the position of the particle in the elementary volume is equally probable and the volume concentration of the particles is equal to:

$$\beta = \frac{G}{S \cdot \rho_m \cdot \upsilon}, \qquad (4)$$

where G - particle mass flow, kg/sec; S - a jet cross-section, m^2; [rho-m] - particles density, kg/m^3; υ - particle velocity, m/sec.

Thus, the probability of presence of a particle in the aerodynamic shadow $P(T)$ of other particles in this elementary volume is:

$$P(T)=\frac{\Omega\cdot c\cdot dV}{dV-W\cdot c\cdot dV}, \qquad (5)$$

where c - particles count in the elementary volume $dV=S\cdot dx$, un/m^3, obviously equal to

$$c=\frac{\beta}{W}, \qquad (6)$$

W - volume of a single particle, m^3, [omega] - volume of aerodynamic shadow of a single particle, m^3

Then, considering (6), we have:

$$P(T)=\frac{\frac{\Omega}{W}\cdot\beta}{1-\beta}, \qquad (7)$$

The probability of the active particle effect on the air, which can be considered as the opposite event to the presence of the particle in the aerodynamic shadow, is equal to:

$$P(A)=1-P(T)=1-\frac{K\cdot\beta}{1-\beta}, \qquad (8)$$

where

$$K=\frac{\Omega}{W} \qquad (9)$$

The aerodynamic shadow volume depends on the particle shape and the Reynolds number. When Re> 10^5 according to Academician A.A. Skochinskii, given in the reference book by I.E. Idelchik [9]: a) for the case of a timber cross-flow by a cross-section a*b (air velocity vector is directed along the long side a): $K=30$ when a/b = 2.2 ([9], page 407); b) for the case of the cross-flow of a cylinder with diameter d: $K=100$; c) for the case of the cross-flow of a disc with diameter d and thickness δ (air velocity vector is directed perpendicular to the plane of the disc): $K=4{,}8\frac{d}{\delta}$, ([9], page 4067) ($K=48$ when δ = 0.1 d) ([9], page 397).

In this case, the length of aerodynamic shadow was taken as the shortest distance between two identical bodies, when the total drag factor becomes equal to the doubled ratio of a single body drag. If we take more stringent conditions for the aerodynamic shadow, for example, assuming that when the particles are in the "thick" shade, the total factor is equal to the a single particle drag factor, then the ratio $\Omega/W = 10$ - for the case "a"; $\Omega/W = 3$ - for the case "b"; and $\Omega/W = 2 \cdot d/\delta$ - for the case "c".

We shall take a weighted average K for the considered bodies at doubled value of the sum of the factors of local resistance as the reference calculation $K = \dfrac{30+100+48}{3} = 63$.

Obviously, the probability of presence of two or more particles (n> 2) in the aerodynamic shadow will be the product of the probabilities

$$P_n(A) = \left(\frac{K \cdot \beta}{1-\beta}\right)^n, \qquad (10)$$

or since $\beta \ll 1$

$$P_n(A) \approx (K \cdot \beta)^n (1 + n \cdot \beta), \qquad (11)$$

For free-falling flow of particles from the conveyor at a height of H = 1 m, bulk concentration is often $\beta < 0{,}01$, and therefore the probability of presence of a particle in a "dense" shadow of the second particle, for example, in the case "b" (if $K = 3$), it is $P_2(T) \le 0{,}001$. We may consider this case hardly probable, since its probability in comparison with the case of presence of the first particle in the shadow is at least sequence higher.

However, in the case of a multifractional particles flow, this probability can not be ignored. Suppose, we have a case of equiprobable allocation in this elementary volume $dV = S \cdot dx$ of particles $d_1,\ d_2,\ \ldots d_N$ (and $d_1 > d_2 > \ldots d_N$, respectively) with a volume concentration β_i (*i* = 1, 2, ..., *N*):

$$\beta_i = \frac{G_i}{S \cdot \rho_m \cdot \upsilon},\quad G_i = g_i G, \text{ while } \beta = \sum_{i=1}^{N} \beta_i \qquad (12)$$

where g_i - weight content of particles of size d_i (in fractions) in the flow.

The probability of presence of a single particle of size d_i in the shadow of the other particle of the same size will still be determined by the type of relation (7):

$$P_i\left(A_i\right)=\frac{\Omega_i}{W_i}\cdot\frac{\beta_i}{1-\beta_i}, \qquad (13)$$

We still can ignore the probability of presence of the second particle of size d_i in the shadow. But the probability of presence of these particles in the shadow of larger particles d_{i-1} cannot be neglected, at least because

$$\frac{\Omega_{i-1}}{W_i} \gg \frac{\Omega_i}{W_i}, \qquad (14)$$

We shall find this probability, assuming that the probability of the location of the particle with diameter d_i in the aerodynamic shadow of a single particle d_{i-1} is determined by the type of relation (5):

$$P_i\left(T\right)=\frac{\Omega_{i-1}\cdot c_{i-1}dV}{dV\left(1-\beta\right)}, \qquad (15)$$

We shall consider the case of the fall of the collective particles of the same weight (Figure 1).

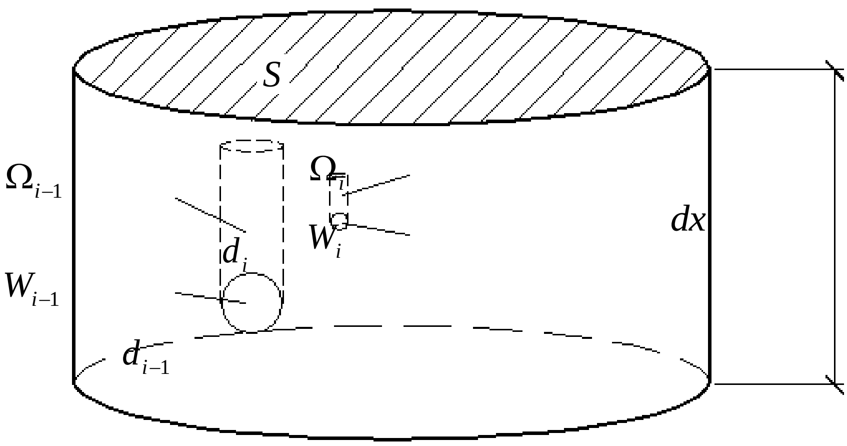

Fig. 1. The elementary volume of the jet of falling particles

Volume concentration of particles with diameter d_i in the elementary volume $dV = S\cdot dx$ is obviously equal to:

$$\beta_i=\frac{g_i\cdot G}{S\cdot\rho_m\cdot\upsilon_i}, \qquad (16)$$

where υ_i - velocity of the particles of size d_i, m/sec

Then, the number of these particles in the elementary volume is $S\cdot dx$

$$\frac{\beta_i \cdot S \cdot dx}{W_i} = c_i \cdot S \cdot dx \,, \qquad (17)$$

where c_i - particles count of the particles with diameter d_i, un/m^3, equal to

$$c_i = \frac{\beta_i}{W_i} \,, \qquad (18)$$

The total volume of the aerodynamic shadow of these elementary particles in the elementary volume is $S \cdot dx$, taking into account that

$$\Omega_i = K_i \cdot W_i \quad \left(K_i \approx 10 \div 100\right); \qquad (19)$$

It can be expressed as

$$T_i = c_i \cdot K_i \cdot W_i \quad \cdot S \cdot dx \,; \qquad (20)$$

where

$$\frac{T_i}{S \cdot dx} = c_i \cdot K_i \cdot W_i \quad = \frac{\beta_i}{W_i} K_i \cdot W_i = K_i \cdot \beta_i \,. \qquad (21)$$

We shall divide all fraction *i* particles into two classes: aerodynamically active particles that with equal probability are located throughout the elementary volume not occupied by particles of all factions ($\left(1-\beta\right) \cdot S \cdot dx$, where $\beta = \sum_{i=1}^{n} \beta_i$), and a class of aerodynamically inactive particles, which are placed in the aerodynamic shadow of bot the fraction *i* particles and larger particles. We believe that the active particles are involved in inter-component dynamic interaction, which total value is the sum of the aerodynamic forces of free particles (excluding the effect of constraint). As for the particles in the aerodynamic shadow, it is believed that they are not involved in this interaction for two reasons. Firstly, the drag factor of the shaded particles is much less than that of free particles, and secondly, because of their low relative fall velocity in the shade in comparison with the relative velocity $\upsilon_i - u$ of the active particles.

We shall find the probability of presence of the particle of size d_i in the aerodynamic shadow. By virtue of the known geometric probability [10], the event of presence of the particle in the shadow is the ratio of areas favoring the occurrence of this event (in our case, the volume A_i) to the entire area (the volume T_i free of the particles) will be with regard to (21):

$$P_i(T_i) = \frac{T_i}{S \cdot dx - \beta \cdot S \cdot dx} = \frac{T_i}{(1-\beta) \cdot S \cdot dx} = K_i \frac{\beta_i}{1-\beta}. \qquad (22)$$

We shall find the probability of presence of the particles with diameter d_i in the aerodynamic shadow of the larger particle d_{i-1} ($d_{i-1} > d_i$);

Total volume of the aerodynamic shadow of particles with diameter d_{i-1} is

$$T_{i-1} = c_{i-1} \cdot K_{i-1} \cdot W_{i-1} \cdot S \cdot dx = K_{i-1} \cdot \beta_{i-1} \cdot S \cdot dx. \qquad (23)$$

Then, the probability of this event will be:

$$P_i(T_i) = \frac{T_i}{(1-\beta) S \cdot dx} = K_{i-1} \frac{\beta_{i-1}}{1-\beta}. \qquad (24)$$

The sum of all the probabilities of presence of a particle with the minimum diameter d_N in the shadow of all larger particles ($d_1 > d_2 > ... > d_N$) will be:

$$\sum_{i=1}^{N} P_i(T_{i-1}) = K_1 \cdot \frac{\beta_1}{1-\beta} + K_2 \cdot \frac{\beta_2}{1-\beta} + ... + K_N \cdot \frac{\beta_N}{1-\beta}, \qquad (25)$$

or, if we apply $K_1 = K_2 = ... = K_N = K$;

$$\sum_{i=1}^{N} P_i(T_{i-1}) = K \cdot \frac{\sum_{i=1}^{N} \beta_i}{1-\beta} = K \frac{\beta}{1-\beta}, \qquad (26)$$

where $\beta = \beta_1 + \beta_2 + ... + \beta_N$ - total volume concentration of particles of different sizes in the volume $S \cdot dx$.

Then the ratio of the active aerodynamic drag of particles (outside the shadows) with a diameter d_N will be:

$$\psi_{(d_N)} = \left(1 - K \cdot \frac{\sum_{1}^{N} \beta_{(d_i)}}{1-\beta} \right) \cdot \psi_0, \qquad (27)$$

where [psi-0] – drag factor of a single (free) particle with diameter d_N.

Accordingly, drag factor of the particles with diameter d_{N-1} will be:

$$\psi_{(d_{N-1})} = \left(1 - K \cdot \frac{\sum_{1}^{N-1} \beta_{(d_i)}}{1-\beta} \right) \cdot \psi_0 \qquad (28)$$

or in the general form, for the particles with diameter $d_j > d_N$:

$$\psi_{(d_i)} = \left(1 - K \cdot \frac{\sum_{1}^{i} \beta_{(d_i)}}{1-\beta} \right) \cdot \psi_0 \qquad (29)$$

It may be that the total aerodynamic shadow of the larger particles $K \cdot \sum_{1}^{i} \beta_{(d_i)}$ will be more than the free (from all particles) part of the air volume $1-\beta$. Then the factor $\psi_{(d_i)}$ will be less than zero (due to the fact that $K \cdot \sum_{1}^{i} \beta_{(d_i)} > (1-\beta_i)$), and the calculations should take $\psi_{(d_i)} = 0$ for these particles. That is, we have a case of "crossing" of aerodynamic shadows of large particles, while all small particles pass into the class of inactive particles.

Aerodynamic effect of the falling collective particles in the jet of multifractional material (the sum of the aerodynamic forces) on the air determines in one-dimensional approximation the change of momentum of ejected air by (20):

$$d\left((1-\beta)\cdot \rho \cdot u \cdot S\right)\cdot u = \left(\sum_{i=1}^{N} R_i \cdot c_i \right) \cdot S \cdot dx, \qquad (30)$$

where c_i - the particles count in the elementary volume of the jet $S \cdot dx$, 1/m^3; R_i - aerodynamic force of the dynamic interaction of a single particle with diameter d_i, [m], determined with the help of aerodynamic drag factor:

$$R_i = \psi_i \cdot F_{мi} \cdot \rho \cdot \frac{(\upsilon_i - u)^2}{2}. \qquad (31)$$

Further, we shall take $S = const$, with a uniformly accelerated falling of particles:

$$\upsilon_i = \upsilon = \sqrt{2 \cdot g \cdot x + \upsilon_0^2}, \qquad (32)$$

where g=9.8 – gravity acceleration, m/sec^2; x – free fall height, m; υ_0 – initial fall rate (when x = 0), m/sec.

Drag factor is determined by the relation (29).

We shall set the expression in parenthesis of the right side of the equation (30)

$$S_a = \sum_{i=1}^{N} R_i \cdot c_i \text{, H/m}^3 \qquad (33)$$

representing the sum of the aerodynamic forces of all particles of the multifractional material per unit volume of the flow of these particles. We shall compare this value with the average sum of the forces of inter-component communication in a unit volume of the particle flow with a weight-average diameter d_y,

$$d_y = \sum_{i=1}^{N} g_i \cdot d_i \text{, m} \qquad (34)$$

that is, with an aerodynamic force of the flow of the accelerated particles of size d_y, and the average drag factor determined by the empirical relation [4]:

$$\psi^* = \frac{\psi_0}{\exp\left(-\frac{1,8}{d_{cp} \cdot 10^3} \cdot \sqrt{\beta_y \cdot 10^3}\right)}, \qquad (35)$$

where ψ_0 - drag factor of a single particle; d_y - average diameter of the particles, m; β_y - average volume concentration of particles in the flow, equal to:

$$\beta_y = \frac{G}{S \cdot \rho_m \cdot 0,5(\upsilon_{1n} + \upsilon_{1k})}. \qquad (36)$$

In the formula (36), $\upsilon_{1н}$, $\upsilon_{1к}$ - respectively to the particles velocity at the beginning and at the end of the channel, m/sec.

The final formula for the calculation of the total force of inter-component interaction in this case will be:

$$S_y = \beta_y \frac{6}{\pi d_y^3} \cdot \psi^* \cdot \frac{\pi d_y^2}{4} \cdot \rho \cdot \frac{(\upsilon - u)^2}{2} \text{, H/m}^3$$

or

$$S_u = \psi^* \cdot \beta_y \frac{6}{4 \cdot d_{cp}} \cdot \rho \cdot \frac{(\upsilon - u)^2}{2} \text{. H/m}^3 \qquad (37)$$

We shall come back to formula (33). Considering that

$$R_i = \psi_i \cdot \frac{\pi d_i^2}{4} \cdot \rho \cdot \frac{(\upsilon - u)^2}{2}, \qquad (38)$$

$$c_i = \frac{\beta_i}{\frac{\pi d_i^3}{6}}, \qquad (39)$$

The calculation formula for S_a will be:

$$S_a = \psi_c \cdot \frac{6}{4} \cdot \frac{\beta_y}{d_y} \rho \cdot \frac{(\upsilon - u)^2}{2}, \qquad (40)$$

where taking $\psi_{0i} = \psi_0 = const$, $\psi_c = \frac{d_y}{\beta_y} \sum_{i=N}^{1} \psi(d_i) \frac{\beta(d_i)}{d_i}$

$$\psi(d_i) = \psi_0 \cdot \left(1 - K \frac{\sum_{i=N}^{j} \beta(d_i)}{1 - \beta} \right) \qquad (41)$$

and assuming that the jet cross-section is $\upsilon,\ u,\ \rho = const$, we will have:

$$S_a = \frac{3}{4} \rho (\upsilon - u)^2 \cdot \sum_{i=1}^{N} \frac{\beta_i}{d_i} \cdot \psi_i. \qquad (42)$$

Thus, we obtain the dependence for determining the force of aerodynamic drag of the polydisperse material flow.

Conclusion.

The falling flow of polydispersed material is conventionally divided into the largest, aerodynamically active particles, and smaller dust particles, located in the aerodynamic shadow and not involved in the forces of aerodynamic interaction when ejecting the air in free jets of a polydisperse material.

Summary. We have analytically determined the drag factor of the particles in the free jet of the multifractional material, which allows us to determine the ejecting ability of the jet without using the empirical relations. An

improved method for calculating the aspirated air flow for charging the silo bunkers will reduce the energy intensity of the aspiration systems.

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