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FULLY ROUGH INTEGER INTERVAL TRANSPORTATION PROBLEMS

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Abstract

A new method namely, Slice-Sum method for solving fully rough integer interval transportation problems is proposed. The optimal values of decision rough variables and rough objective function for the problem obtained by the proposed method are rough integer intervals. Numerical examples in Pharmaceutical Logistics are presented to understand the solution procedure of the slice-sum method.

Keywords: Transportation problem, Rough integer intervals, Optimal solution, Slice-Sum method.

1. Introduction

Transportation problem is one of the popular and most important applications of the linear programming problem. Many efficient algorithms have been developed for solving transportation problems having deterministic parameters. In many real life situations, some or all parameters of the transportation problem are not deterministic always, but they are uncertain. Many researchers have studied the transportation problem in various uncertain environments such as fuzzy, random etc.. Pawlak [8] initiated the rough set theory. Then, many researchers have developed the rough set theory both in theoretical and applied. The concept of rough variable which is a measurable function from rough space to the set of real numbers was proposed by Liu [5]. A two-person zero-sum matrix games with rough payoffs was studied by Xu and Yao [11]. A rough programming problem considering the decision set as a rough set was introduced and solved by Youness [13]. A rough DEA model to solve a supply chain performance evaluation problem with rough parameters was presented by Xu et al.[12]. Shu Xiao and Edmund M-K. Lai [9] solved a power-aware VLIW instruction scheduling problem with power consumption parameters as rough variables. Some solid transportation models with crisp and rough costs were presented and solved by Kundu et al.[4]. Subhakanta Dash and Mohanty [10] have proposed a compromise solution method for transportation problems considering the unit cost of transportation from a source to a destination as a rough integer interval. Akilbasha et al.[1] have proposed a new method namely, split and separation method for finding an optimal solution for integer transportation problems with rough nature.

This paper is organized as follows: Section 2 gives some basic concepts of rough integer intervals which are used in this paper. In Section 3, a fully rough integer transportation problem is considered and a relation between optimal solutions of the fully rough integer transportation problem and its four induced integer transportation problems is established. Section 4 presents a new method namely, slice-sum method for solving fully rough integer transportation problems and numerical examples in Pharmaceutical Logistics for demonstrating the solution procedure of the proposed method and finally, the conclusion is given in Section 5.

2. Preliminaries

We need the following definitions of the basic arithmetic operators and partial ordering on a set of all rough intervals which can be found in Hongwei Lu et al.[2] .

Let D denote the set of all rough intervals on the real line R. That is,

$$D = \{ [[b,c],[a,\mathrm{d}]],\ a \le b \le c \le d \text{ and } a, b, c \text{ and } d \text{ are in } R \}.$$

Note that (i) if $a = b$ and $c = d$ in D, then D becomes the set of all real intervals and

(ii) if $a = b = c = d$ in D, then D becomes the set of all real numbers.

Definition 2.1: Let $A = [[a_2, a_3],[a_1, a_4]]$ and $B = [[b_2, b_3],[b_1, b_4]]$ be in D. Then,

(i) $A \oplus B = [[a_2 + b_2, a_3 + b_3],[a_1 + b_1, a_4 + b_4]]$;

(ii) $kA = [[\mathrm{k}\,a_2, \mathrm{k}\,a_3],[\mathrm{k}\,a_1, \mathrm{k}\,a_4]]$ if k is a positive real interval and

(iii) $A \otimes B = [[a_2, a_3][b_2, b_3],[a_1, a_4][b_1, b_4]]$.

Definition 2.2: Let $A = [[a_2, a_3],[a_1, a_4]]$ and $B = [[b_2, b_3],[b_1, b_4]]$ be in D. Then,

(i) $A \le B$ if $a_i \le b_i, i = 1, 2, 3, 4$;

(ii) $A \ge B$ if $B \le A$, that is, $a_i \ge b_i, i = 1, 2, 3, 4$ and

(iii) $A = B$ if $A \le B$ and $B \le A$, that is, $a_i = b_i, i = 1, 2, 3, 4$.

Definition 2.3: Let $A=[[a_2,a_3],[a_1,a_4]]$ be in D. Then, A is said to be non-negative , that is,

$A \geq 0$ if $a_1 \geq 0$.

Remark 2.1: **If** $A=[[a_2,a_3],[a_1,a_4]]$ and $B=[[b_2,b_3],[b_1,b_4]]$ in D are non-negative, then, $A \otimes B =$ $[[a_2 b_2, a_3 b_3],[a_1 b_1, a_4 b_4]]$.

Definition 2.4: Let $A=[[a_2,a_3],[a_1,a_4]]$ be in D. Then, A is said to be rough integer if $a_i, i=1,2,3,4$ are integers.

3. Fully rough integer transportation problems

Consider the following fully rough integer transportation problem:

(P) Minimize $[[z_2,z_3],[z_1,z_4]] = \sum_{i=1}^{m} \sum_{j=1}^{n} [[c_{ij}^2, c_{ij}^3],[c_{ij}^1, c_{ij}^4]] \otimes [[x_{ij}^2, x_{ij}^3],[x_{ij}^1, x_{ij}^4]]$

Subject to

$$\sum_{j=1}^{n} [[x_{ij}^2, x_{ij}^3],[x_{ij}^1, x_{ij}^4]] = [[a_i^2, a_i^3],[a_i^1, a_i^4]],\ i \in I \quad (1)$$

$$\sum_{i=1}^{m} [[x_{ij}^2, x_{ij}^3],[x_{ij}^1, x_{ij}^4]] = [[b_j^2, b_j^3],[b_j^1, b_j^4]],\ j \in J \quad (2)$$

$x_{ij}^1 \geq 0$, $i \in I$ and $j \in J$ and $x_{ij}^1, x_{ij}^2, x_{ij}^3$ and x_{ij}^4, $i \in I$ and $j \in J$ are integers (3)

Where $I=\{1,2,3,...,m\}$, $J=\{1,2,3,...,n\}$, $c_{ij}^1, c_{ij}^2, c_{ij}^3$ and c_{ij}^4 are positive integers for all $i \in I$ and $j \in J$, a_i^1, a_i^2, a_i^3 and a_i^4 are positive integers for all $i \in I$ and b_j^1, b_j^2, b_j^3 and b_j^4 are positive integers for all $j \in J$. The problem (P) is said to be balanced if the total supply is equal to the total demand.

Definition 3.1:

A set of rough intervals $\{[[x_{ij}^2, x_{ij}^3],[x_{ij}^1, x_{ij}^4]]$, for all $i \in I$ and $j \in J\}$ is said to be a feasible solution to the problem (P) if it satisfies the equations (1), (2) and (3).

Definition 3.2:

A feasible solution $\{[[x_{ij}^2, x_{ij}^3],[x_{ij}^1, x_{ij}^4]]$, for all $i \in I$ and $j \in J\}$ to the problem (P) is said to be an optimal solution of the problem (P) if the feasible solution minimizes the objective function of the problem (P), that is,

$$\sum_{i=1}^{m} \sum_{j=1}^{n} [[c_{ij}^2, c_{ij}^3],[c_{ij}^1, c_{ij}^4]] \otimes [[x_{ij}^2, x_{ij}^3],[x_{ij}^1, x_{ij}^4]] \leq \sum_{i=1}^{m} \sum_{j=1}^{n} [[c_{ij}^2, c_{ij}^3],[c_{ij}^1, c_{ij}^4]] \otimes [[u_{ij}^2, u_{ij}^3],[u_{ij}^1, u_{ij}^4]],$$

for all feasible $\{[[u_{ij}^2, u_{ij}^3],[u_{ij}^1, u_{ij}^4]]$, for $i \in I$ and $j \in J\}$.

Now, the problem (P) is sliced into four integer transportation problems namely, upper approximation upper bound integer transportation (UAUBIT) problem, lower approximation upper bound integer transportation (LAUBIT) problem, lower approximation lower bound integer transportation (LALBIT) problem and upper approximation lower bound integer transportation (UALBIT) problem which are given below:

(UAUBIT) Minimize $z_4 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^4 x_{ij}^4$

subject to $\sum_{j=1}^{n} x_{ij}^4 = a_i^4$, $i \in I$; $\sum_{i=1}^{m} x_{ij}^4 = b_j^4$, $j \in J$;

$x_{ij}^4 \geq 0$, $i \in I$ and $j \in J$ and are integers ,

(LAUBIT) Minimize $z_3 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^3 x_{ij}^3$

Subject to $\sum_{j=1}^{n} x_{ij}^3 = a_i^3$, $i \in I$; $\sum_{i=1}^{m} x_{ij}^3 = b_j^3$, $j \in J$;

$x_{ij}^3 \geq 0$, $i \in I$ and $j \in J$ and are integers,

(LALBIT) Minimize $z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^2 x_{ij}^2$

Subject to $\sum_{j=1}^{n} x_{ij}^2 = a_i^2$, $i \in I$; $\sum_{i=1}^{m} x_{ij}^2 = b_j^2$, $j \in J$;

$x_{ij}^2 \geq 0$, $i \in I$ and $j \in J$ and are integers ,

and

(UALBIT) Minimize $z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^1 x_{ij}^1$

subject to $\sum_{j=1}^{n} x_{ij}^1 = a_i^1$, $i \in I$; $\sum_{i=1}^{m} x_{ij}^1 = b_j^1$, $j \in J$;

$x_{ij}^1 \geq 0$, $i \in I$ and $j \in J$ and are integers.

Now, we establish a relation between optimal solutions of the fully rough integer transportation problem (P) and its four induced integer transportation problems (UAUBIT) (LAUBIT) (LALBIT) and (UALBIT). The established relation is used in the proposed method, namely, slice-sum method.

Theorem 3.1:

If the set { $\overline{x}_{ij}^{4}$, for all $i \in I$ and $j \in J$ } is an optimal solution for the (UAUBIT) problem of the problem (P) with the minimum transportation cost $\overline{z}_4$, the set { $\overline{x}_{ij}^{3}$, for all $i \in I$ and $j \in J$ } is an optimal solution for the (LAUBIT) problem of the problem (P) with the minimum transportation cost $\overline{z}_3$, the set { $\overline{x}_{ij}^{2}$, for all $i \in I$ and $j \in J$ } is an optimal solution for the (LALBIT) problem of the problem (P) with the minimum transportation cost $\overline{z}_2$ and the set { $\overline{x}_{ij}^{1}$, for all $i \in I$ and $j \in J$ } is an optimal solution for the (UALBIT) problem of the problem (P) with the minimum transportation cost $\overline{z}_1$, then the set of rough integer intervals $\{[[\overline{x}_{ij}^{2}, \overline{x}_{ij}^{3}],[\overline{x}_{ij}^{1}, \overline{x}_{ij}^{4}]]$, for all $i \in I$ and $j \in J$ } is an optimal solution for the problem (P) with the minimum transportation cost $[[\overline{z}_2, \overline{z}_3],[\overline{z}_1, \overline{z}_4]]$ provided $\overline{x}_{ij}^{1} \le \overline{x}_{ij}^{2} \le \overline{x}_{ij}^{3} \le \overline{x}_{ij}^{4}$, for all $i \in I$ and $j \in J$.

Proof:

Now, since { $\overline{x}_{ij}^{1}$, for all $i \in I$ and $j \in J$ } , { $\overline{x}_{ij}^{2}$, for all $i \in I$ and $j \in J$ } , { $\overline{x}_{ij}^{3}$, for all $i \in I$ and $j \in J$ } and { $\overline{x}_{ij}^{4}$, for all $i \in I$ and $j \in J$ } are optimal solutions for the problems (UALBIT), (LALBIT), (LAUBIT) and (UAUBIT) respectively and $\overline{x}_{ij}^{1} \le \overline{x}_{ij}^{2} \le \overline{x}_{ij}^{3} \le \overline{x}_{ij}^{4}$, for all $i \in I$ and $j \in J$, then we can conclude that the set of rough integer intervals { $[[\overline{x}_{ij}^{2}, \overline{x}_{ij}^{3}],[\overline{x}_{ij}^{1}, \overline{x}_{ij}^{4}]]$, for all i and j } is a feasible solution to the problem (P).

Let $\{[[u_{ij}^{2}, u_{ij}^{3}],[u_{ij}^{1}, u_{ij}^{4}]]$, for all $i \in I$ and $j \in J$ } be a feasible solution to the problem (P).

Therefore, { u_{ij}^{1}, for all $i \in I$ and $j \in J$ } , { u_{ij}^{2}, for all $i \in I$ and $j \in J$ } , { u_{ij}^{3}, for all $i \in I$ and $j \in J$ } and { u_{ij}^{4}, for all $i \in I$ and $j \in J$ } are feasible solutions to the problems (UALBIT), (LALBIT), (LAUBIT) and (UAUBIT) respectively.

Since { $\overline{x}_{ij}^{1}$, for all $i \in I$ and $j \in J$ } , { $\overline{x}_{ij}^{2}$, for all $i \in I$ and $j \in J$ } , { $\overline{x}_{ij}^{3}$, for all $i \in I$ and $j \in J$ } and { $\overline{x}_{ij}^{4}$, for all $i \in I$ and $j \in J$ } are optimal solutions for the problems (UALBIT), (LALBIT), (LAUBIT) and (UAUBIT) respectively, we have

$$\bar{z}_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^1 \bar{x}_{ij}^1 \le \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^1 u_{ij}^1 \; ; \qquad \bar{z}_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^2 \bar{x}_{ij}^2 \le \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^2 u_{ij}^2 \; ;$$

$$\bar{z}_3 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^3 \bar{x}_{ij}^3 \le \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^3 u_{ij}^3 \quad \text{and} \quad \bar{z}_4 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^4 \bar{x}_{ij}^4 \le \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^4 u_{ij}^4$$

This implies that, $[[\bar{z}_2, \bar{z}_3],[\bar{z}_1, \bar{z}_4]] = \sum_{i=1}^{m}\sum_{j=1}^{n} [[c_{ij}^2, c_{ij}^3],[c_{ij}^1, c_{ij}^4]] \otimes [[\bar{x}_{ij}^2, \bar{x}_{ij}^3],[\bar{x}_{ij}^1, \bar{x}_{ij}^4]]$ and

$$\sum_{i=1}^{m}\sum_{j=1}^{n} [[c_{ij}^2, c_{ij}^3],[c_{ij}^1, c_{ij}^4]] \otimes [[\bar{x}_{ij}^2, \bar{x}_{ij}^3],[\bar{x}_{ij}^1, \bar{x}_{ij}^4]] \le \sum_{i=1}^{m}\sum_{j=1}^{n} [[c_{ij}^2, c_{ij}^3],[c_{ij}^1, c_{ij}^4]] \otimes [[u_{ij}^2, u_{ij}^3],[u_{ij}^1, u_{ij}^4]].$$

Therefore, the set of rough integer intervals $\{[[\bar{x}_{ij}^2, \bar{x}_{ij}^3],[\bar{x}_{ij}^1, \bar{x}_{ij}^4]]$, for all $i \in I$ and $j \in J$ $\}$ is an optimal solution for the problem (P) with the minimum transportation cost $[[\bar{z}_2, \bar{z}_3],[\bar{z}_1, \bar{z}_4]]$. Hence, the theorem is proved.

Remark 3.1 The above theorem is an extension of the Theorem 1. in Pandian and Natarajan [7].

4 Slice-sum method

We, now propose a new method namely, slice-sum method for solving the fully rough integer interval transportation problem (P).

The slice-sum method proceeds as follows.

Step 1. Check that the given problem (P) is balanced. If not, make it into balanced.

Step 2. Construct the (UAUBIT) problem of the given problem (P).

Step 3. Solve the (UAUBIT) problem using a transportation algorithm [3] / the zero-point method [6] . Let $\{\bar{x}_{ij}^4$, for all $i \in I$ and $j \in J\}$ be an optimal solution of the (UAUBIT) problem with the minimum transportation cost $\bar{z}_4$

Step 4. Construct the (LAUBIT) problem of the given problem (P).

Step 5. Solve the (LAUBIT) problem with the upper bound constraints $x_{ij}^3 \le \bar{x}_{ij}^4$, for all $i \in I$ and $j \in J$ using the zero point method [6] / the integer linear programming technique [3] . Let $\{\bar{x}_{ij}^3$, for all $i \in I$ and $j \in J\}$ be an optimal solution of the (LAUBIT) problem with the minimum transportation cost $\bar{z}_3$.

Step 6. Construct the (LALBIT) problem of the given problem (P).

Step 7. Solve the (LALBIT) problem with the upper bound constraints $x_{ij}^2 \le \overline{x}_{ij}^3$, for all $i \in I$ and $j \in J$ using the zero point method [6] / the integer linear programming technique [3]. Let { $\overline{x}_{ij}^2$, for all $i \in I$ and $j \in J$} be an optimal solution of the (LALBIT) problem with the minimum transportation cost $\overline{z}_2$.

Step 8. Construct the (UALBIT) problem of the given problem (P).

Step 9. Solve the (UALBIT) problem with the upper bound constraints $x_{ij}^1 \le \overline{x}_{ij}^2$, for all $i \in I$ and $j \in J$ using the zero point method [6] / the integer linear programming technique [3]. Let { $\overline{x}_{ij}^1$, for all $i \in I$ and $j \in J$} be an optimal solution of the (UALBIT) problem with the minimum transportation cost $\overline{z}_1$.

Step 10. The optimal solution of the given problem (P) is $\{[[\overline{x}_{ij}^2, \overline{x}_{ij}^3],[\overline{x}_{ij}^1, \overline{x}_{ij}^4]]$ for all i and j }

With the minimum transportation cost $[[\overline{z}_2, \overline{z}_3],[\overline{z}_1, \overline{z}_4]]$ (by the Theorem 3.1.).

The proposed method for solving the fully rough integer interval transportation problem is illustrated by the following example.

Example 4.1: A pharmaceutical company produces a product in its three factories F1, F2 and F3. The product will be sent to three destinations D1, D2 and D3 from the three factories. Determine a shipping plan for the company from factories to the destination such that total shipping cost should be minimum using the following data:

The minimum supply ranges of F1, F2 and F3 are [13,15], [11,13] and [14,16] respectively and the maximum supply ranges of F1, F2 and F3 are [12,16], [10,14] and [13,18] respectively. The minimum demand ranges of D1, D2 and D3 are [20,22], [11,13] and [7,9] respectively and the maximum demand ranges of D1, D2 and D3 are [19,24], [10,14] and [6,10] respectively. The minimum unit shipping cost range from each supply point to each demand point is given below:

	D1	D2	D3
F1	[7,9]	[12,14]	[10,11]
F2	[4,5]	[3,4]	[5,7]
F3	[5,6]	[2,3]	[10,11]

And the maximum unit shipping cost range from each supply point to each demand point is given below:

	D1	D2	D3
F1	[6,10]	[11,15]	[8,12]
F2	[3,6]	[2,7]	[4,9]
F3	[3,7]	[1,4]	[9,12]

Now, the given problem can be modeled as a fully rough integer interval transportation problem as follows:

	D1	D2	D3	Supply
F1	[[7,9], [6,10]]	[[12,14], [11,15]]	[[10,11], [8,12]]	[[13,15],[12,16]]
F2	[[4,5], [3,6]]	[[3,4], [2,7]]	[[5,7], [4,9]]	[[11,13],[10,14]]
F3	[[5,6], [3,7]]	[[2,3], [1,4]]	[[10,11], [9,12]]	[[14,16],[13,18]]
Demand	[[20 ,22],[19,24]]	[[11,13],[10,14]]	[[7,9],[6,10]]	

Now, since the total supply = the total demand = [[38,44],[35,48]], the given problem is balanced.

Now, by using the Step 2. and the Step 3., the optimal solution to the (UAUBIT) problem is $\bar{x}_{11}^4 = 6$, $\bar{x}_{13}^4 = 10$, $\bar{x}_{21}^4 = 14$, $\bar{x}_{31}^4 = 4$ and $\bar{x}_{32}^4 = 14$ with the minimum transportation cost 348.

Now, using the Step 4. and the Step 5., the optimal solution to the (LAUBIT) problem with the upper bound constraints $x_{ij}^3 \le \bar{x}_{ij}^4$, $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ and are integers, is $\bar{x}_{11}^3 = 6$, $\bar{x}_{13}^3 = 9$, $\bar{x}_{21}^3 = 13$, $\bar{x}_{31}^3 = 3$ and $\bar{x}_{32}^3 = 13$ with the minimum transportation cost 275.

Now, by the Step 6. and the Step 7., the optimal solution to the (LALBIT) problem with the upper bound constraints $x_{ij}^2 \le \bar{x}_{ij}^3$, $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ and are integers, is $\bar{x}_{11}^2 = 6$, $\bar{x}_{13}^2 = 7$, $\bar{x}_{21}^2 = 11$, $\bar{x}_{31}^2 = 3$ and $\bar{x}_{32}^2 = 11$ with the minimum transportation cost 193.

Now, by using the Step 8. And the Step 9., the optimal solution to the (UALBIT) problem with the upper bound constraints $x_{ij}^1 \le \bar{x}_{ij}^2$, $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ and are integers, is $\bar{x}_{11}^1 = 6$, $\bar{x}_{13}^1 = 6$, $\bar{x}_{21}^1 = 10$, $\bar{x}_{31}^1 = 3$ and $\bar{x}_{32}^1 = 10$ with the minimum transportation cost 133.

Now, by the Step 10., the optimal solution of the given problem is given below:

$[[\bar{x}_{11}^2, \bar{x}_{11}^3],[\bar{x}_{11}^1, \bar{x}_{11}^4] = [[6,6],[6,6]]$, $[[\bar{x}_{13}^2, \bar{x}_{13}^3],[\bar{x}_{13}^1, \bar{x}_{13}^4] = [[7,9],[6,10]]$,

$[[\bar{x}_{21}^2, \bar{x}_{21}^3],[\bar{x}_{21}^1, \bar{x}_{21}^4] = [[11,13],[10,14]]$, $[[\bar{x}_{31}^2, \bar{x}_{31}^3],[\bar{x}_{31}^1, \bar{x}_{31}^4] = [[3,3],[3,4]]$ and

$[[\bar{x}_{32}^2, \bar{x}_{32}^3],[\bar{x}_{32}^1, \bar{x}_{32}^4] = [[11,13],[10,14]]$ With the minimum shipping cost [[193,275],[133,348]].

Using the following numerical example, the proposed method is better than the split and separation method proposed by Akilbasha et al.[1].

Example 4.2. A pharmaceutical company has three plants at three locations S1, S2 and S3 which supply to three warehouses D1, D2 and D3. Monthly plant capacities are 14, 12 and 15 units respectively. Monthly warehouse requirements are 21, 12 and 8 respectively. Determine an optimal distribution for the company in order to minimize the total shipping cost given that the minimum range and maximum range unit transportation costs are given below:

	D1	D2	D3
S1	[[4,5], [3,10]]	[[12,14], [11,15]]	[[10,11], [8,12]]
S2	[[4,5], [3,6]]	[[3,4], [2,7]]	[[5,7], [4,9]]
S3	[[5,6], [3,7]]	[[2,3], [1,4]]	[[7,8], [6,12]]

Now, the given problem is represented as a rough integer interval transportation problem with integer interval decision variables as follows:

	D1	D2	D3	Supply
S1	[[4,5], [3,10]]	[[12,14], [11,15]]	[[10,11], [8,12]]	[[14,14],[14,14]]
S2	[[4,5], [3,6]]	[[3,4], [2,7]]	[[5,7], [4,9]]	[[12,12],[12,12]]
S3	[[5,6], [3,7]]	[[2,3], [1,4]]	[[7,8], [6,12]]	[[15,15],[15,15]]
Demand	[[21,21],[21,21]]	[[12,12],[12,12]]	[[8,8],[8,8]]	

Now, using the slice-sum method, the optimal solution for the given problem is obtained as:

$[[\bar{x}^2_{11}, \bar{x}^3_{11}],[\bar{x}^1_{11}, \bar{x}^4_{11}]] = [[6,6],[6,6]]$, $[[\bar{x}^2_{13}, \bar{x}^3_{13}],[\bar{x}^1_{13}, \bar{x}^4_{13}]] = [[8,8],[8,8]]$,

$[[\bar{x}^2_{21}, \bar{x}^3_{21}],[\bar{x}^1_{21}, \bar{x}^4_{21}]] = [[12,12],[12,12]]$, $[[\bar{x}^2_{31}, \bar{x}^3_{31}],[\bar{x}^1_{31}, \bar{x}^4_{31}]] = [[3,3],[3,3]]$ and

$[[\bar{x}^2_{32}, \bar{x}^3_{32}],[\bar{x}^1_{32}, \bar{x}^4_{32}]] = [[12,12],[12,12]]$ with the minimum shipping cost [[191,232],[139,297]].

Remark 4.1 The optimal value of the objective function of the Example 4.2, obtained by the slice-sum method is [[191,232], [139,297]] , but it has no solution by the split and separation method [1] since the optimal solutions of the lower approximation problem and the upper approximation problem are not the same.

5. Conclusion

Transportation problem having all or some parameters as rough integer intervals is considered in this paper. A new method namely, slice-sum method is proposed to solve a rough integer interval transportation problem in which all or some of the parameters, that is, cost of transportation, supply and demand are rough integer intervals. The proposed method is a systematic procedure, both easy to understand and to apply and also, it is a crisp method and provides an exact optimal solution to the given problem. The solution procedure of the slice-sum method is illustrated with numerical examples in Pharmaceutical Logistics. The proposed method can be served an important tool for the decision makers when they are handling various types of logistic models for vreal life situations having rough integer interval parameters.

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