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DOMINATION DOT STABLE TREES

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Abstract

A graph G is said to be domination dot stable (DDS) if γ (G$\bullet$ uv) = γ (G), for all u, v $\in$ V (G), u adjacent to v. In this paper, we provide a constructive characterization of DDS trees.

Key Words: Domination, DDS, Tree, Stable graphs.

Introduction

In [1], E. Shan et al have provided a tree characterization of total domination and paired domination with equal total domination and paired - domination numbers. In [6], M. Yamuna et al have introduced γ - stable graphs and they have established that γ - stable trees have a unique structure. In this paper we have provided a constructive characterization for DDS trees. We consider only simple connected undirected graphs G = (V,E). The open neighborhood of vertex v $\in$ V (G) is denoted by N (v) = { u $\in$ V (G) | (u v) $\in$ E (G) }. P_n denote a path with n edges. The number of edges incident on a vertex v_i, with self loops counted twice, is called the degree, d (v_i), of vertex vi. The diameter of a tree T is defined as the length of the longest path in T. We indicate that u is adjacent to v by writing u $\perp$ v. For properties related to graph theory we refer to F. Harary [2].

A set of vertices D in a graph G = (V, E) is a dominating set if every vertex of V – D is adjacent to some vertex of D. If D has the smallest possible cardinality of any dominating set of G, then D is called a minimum dominating set. The cardinality of any minimum dominating set for G is called the domination number of G and it is denoted by γ (G). γ - set denotes a dominating set for G with minimum cardinality. A vertex v is said to be selfish in the γ - set D, if v is needed only to dominate itself. A dominating set D is said to be an independent dominating set if no two vertices in D are adjacent. A vertex v is said to be good if there is a γ - set of G containing v. If there is no γ - set of G containing v, then v is said to be a bad vertex. For properties related to domination we refer to T. W. Haynes, S. T. Hedetniemi, and P. J. Slater [3].

Results and Discussions: For a pair of vertices v, u of G, we denote by $G_\bullet$ vu the graph obtained by identifying v and u. So $G_\bullet$ vu may be viewed as the graph obtained from G by deleting the vertices v and u and appending a new vertex, denoted by (vu), that is adjacent to all the vertices of G – v – u that were originally adjacent to either of v or u [4].

A graph G is said to be domination dot stable (DDS) if γ ($G_\bullet$ uv) = γ (G), for all u, v $\in$ V (G), u $\perp$ v. They proved the following [5].

R1. A graph G is DDS if and only if every γ - set of G is an independent dominating set.

R2. Let G be a DDS graph. Any v $\in$ V (G) is not selfish.

A Characterization of DDS – Trees:

In this section we provide a constructive characterization of DDS trees.

Theorem 1: Let u be a vertex of a DDS graph G. H is obtained from G by attaching a tree path P_1 at u. H is DDS if and only if u is a good vertex in G.

Proof: Assume that H is DDS. γ (H) $\geq$ γ (G). If u is bad vertex in G, then since there is no γ - set containing u in G, γ (H) = γ (G) + 1, say γ (H) = D' = D $\cup$ { u }, where D is a γ - set for G.

Since u is bad with respect to D, there exist one $u_1 \in$ N (u) such that $u_1 \in$ D. D' is a γ - set for H such that

1. u, $u_1 \in$ D'
2. u $\perp$ u_1

D' is a non independent γ - set for H, a contradiction as H is DDS. Hence u is a good vertex in G. Also γ (H) = γ (G).

Conversely assume that u is good with respect to G and D a γ - set for G containing u. γ (H) = γ (G) as D dominates H. Let w be the pendant vertex attached to u, to generate H. There is no γ - set containing u, w for H.

So, if H has a non – independent γ - set, then there is some x, y $\in$ V (G), x $\perp$ y, x, y $\in$ D'', where D'' is some γ - set for H. Since γ (H) = γ (G), D'' is a non independent γ - set for G also, a contradiction as G is DDS. So, every γ - set of H is independent, implies H is DDS.

Theorem 2

Let u be a vertex of a DDS graph G. H is obtained from G by attaching a tree path P_2 at u. H is DDS if and only if u is a bad vertex in G.

Proof:

Assume that H is DDS and u is a good vertex with respect to G. Let u ⊥ v and v ⊥ w in the tree path P_2 attached to u. Since we have assumed u is good there is a γ - set, say D for G containing u.

Since G is DDS, u is never a selfish vertex [R2], implies γ (G) ≥ γ (G) – { u }. Also w is pendant , implies γ (H) = γ (G) + 1 = γ (G) ∪ { v }. D' = D ∪ { v } is a γ - set for H such that u ⊥ v, a contradiction as H is DDS, implies u is bad with respect to G.

Conversely assume that u is a bad vertex with respect to G. If possible assume that H is not DDS, implies there is a non – independent γ - set say D for H. Since w is pendant both { v, w } ∉ H. Since u is bad vertex with respect to G both { u, v } ∉ H (else D – { v } is a γ - set for G containing u, a contradiction). So, there is some x, y ∈ V (G), { x, y } ≠ u such that

1. x, y ∈ D
2. x ⊥ y

D' = D – { v } is a non independent γ - set for G, a contradiction as G is DDS. So, every γ - set of H is independent, implies H is DDS.

Operation O1: Attach a path P1 to a good vertex v of T.

Operation O2: Attach a path P2 to a bad vertex v of T.

Let τ be the family defined by τ = { T / T is obtained from K1, by a finite sequence of operations O1 or O2 }.

From Theorem 1 and 2 we know that if T ∈ τ, then T is a DDS tree.

Theorem 3

If T is a DDS tree, then T ∈ τ.

Proof:

We proceed by induction on the order n ≥ 1. If T is a star, then T can be obtained from K_1, by repeated application of Operation O1. Hence we may assume that diam (T) ≥ 3. Assume that the theorem is true for all tree T' of order n ' < n.

Let T be rooted at a leaf r, of longest path r – u path P. Let v be the neighbor of u. Further let w denote the parent of v. By T_x, we denote the sub tree induced by vertex x and its descendants in the rooted tree T.

Let T' = T – T_u. Let d_T (v) ≥ 3, v is a support vertex with respect to T '. Let D' be a γ - set for T ' that contains all support vertices, implies v ∈ D'. Since γ (T) = γ (T ') [by Theorem 2], D' is a γ - set for T such that v is a good vertex. Hence T can be obtained from T ' by operation O1.

Let d_T (v) = 2. Since P is a longest r – u path T " = T_w' – { w } is either K_1 or K_2. Assume that T" contains K1. If D' is a γ - set for T ' containing all the support vertices, then w ∈ D'. Then D = D' ∪ { v } is a γ - set for T such that w, v ∈ D, w ⊥ v, a contradiction as T is DDS. Assume that T" contains K_2 (say kl). Let D' be a γ - set for T' containing all the support vertices, implies k, w ∈ D', k ⊥ w, a contradiction as T' is DDS. Hence d_T (w) = 2.

Let D' be a γ - set for T' containing all the support vertices. We claim that, v is a good vertex with respect to T '. Assume that v is a bad vertex with respect to T '.

1. Let γ (T) = γ (T'). Let D" be a γ - set for T containing all the support vertices. Then D" is γ - set for T' also such that v ∈ D", a contradiction as v is a bad vertex.
2. Let γ (T) = γ (T') + 1. Let D' be a γ - set for T' containing all support vertices, w ∈ D'. Then D = D' ∪ { v } is a γ - set for T such that w, v ∈ D, w ⊥ v , a contradiction as T is DDS.

So, v is a good vertex with respect to T'. Hence T can be obtained from T' by operation O1.

Let T' = T – T_v . Let d_T (w) ≥ 3. Since P is a longest path, T " = T_w' – { w } is either K_1 or K_2 . Assume that T " contains K_1. Let D' be a γ - set for T' containing all the support vertices. Then D = D' ∪ { v } is a γ - set for T such that w, v ∈ D, w ⊥ v, a contradiction as T is DDS. Assume that T " contains K_2 (say kl). Let D' be a γ - set for T' containing all support vertices. k ∈ D'. If w is good with respect to D', then w, k ∈ D', w ⊥ k, a contradiction as T' is DDS. So, if d_T (w) ≥ 3, then T" contains K_2 only, w is bad with respect to T'. Hence T can be obtain from T' by operation O2.

Let d_T (w) = 2, γ (T) = γ (T'). Let D be a γ - set for T containing all the support vertices. Then D' = D – { v } ∪ { w } is a γ - set for T' such that w selfish, a contradiction as T' is DDS, implies γ (T) = γ (T') + 1. If w is good with respect to T', D' a γ - set containing w, then D = D' ∪ { v } is a γ - set for T such that w, v ∈ D, w ⊥ v, a contradiction as T is DDS, implies w is bad with respect to T'.

Hence T can be obtain from T' by applying operation O2.

As a consequence of Theorem 1, 2 and 3 we have the following characterization for DDS trees.

Theorem 4

A tree T is DDS if and only if T $\in \tau$.

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